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A Full Adaptive Observer for DC Servo Motors

Ata SEVİNÇ
Kırıkkale University, Department of Electrical and Electronics Engineering,
71450 Kırıkkale-TURKEY
e-mail: A.Sevinc@kku.edu.tr

Abstract

An adaptive observer estimating all parameters and load torque is proposed for DC servo motors. The observer uses no direct feedback but the adaptation schemes use current and speed measurements. Both the observer and adaptations are simple to implement for real-time applications. Simulation results are satisfactory for the full adaptive observer. If the observer works in parallel with only load torque and armature resistance adaptations, the results are very good even if very low-quality sensors are used. In this simulation, only a single hall sensor is used as a rotational transducer, which produces a single pulse per revolution, and very high level noise and disturbance are added in order to provide a more realistic simulation.

Key Words: *DC servo motors, adaptive observers, DC motor drives, parameter estimations.*

1. Introduction

Even though feedback has been considered essential for observers, using observers without direct feedback was proposed in a previous study [1] for simplicity and more accurate estimations. Convergence is achieved in such observers by means of adaptation schemes which use feedback. The application of this technique to DC servo motors was presented in [1] for position/speed-sensorless operations. However, all motor parameters were assumed to be known and the adaptation was restricted to load torque estimation due to the lack of rotational transducer measurements.

In this paper, none of the motor parameters is assumed to be known, but speed is assumed to be a measured state. The adaptation technique is applied to all the motor parameters as well as the load torque. However, the full adaptation is restricted to variable operating conditions because of the DC servo motor's dynamics. Under constant speed and current, only two of these adaptations can work due to theoretical restrictions.

One of the aims of this paper is to present a way of designing a full adaptive observer for DC servo motors. Using observers may seem redundant as both the speed and current feedback are available for DC servo motors; however, using the observer's state estimations increases the controller's performance, especially when the feedback is provided by low-quality sensors. In this context, conventional observers using feedback directly have a disadvantage. At least some components of their state estimations include the measurement signals in their first derivatives. Therefore, control algorithms requiring the derivative of some states, e.g. PID control, may not be suitable with such observers due to the measurement noise. The proposed observer in this paper is adaptive and it provides state estimations less sensitive to noise. The simplicity of this observer is another advantage.

In practice, such full adaptation is usually not needed. Estimating some of the parameters and states is usually enough. This paper suggests designs of some simple parameter estimation algorithms with an observer which does not use any direct feedback. Therefore, in some cases designers can use only some of the proposed algorithms with this observer as it may not be necessary to measure both states. When the observer works in parallel with only load torque and armature resistance adaptations, the proposed adaptation schemes do not use a real speed measurement. The time derivative of the position measurement from a pulse encoder is used instead. In practical motion control, a speed observer is widely used to get a better speed signal than the time derivative of pulse counts. Direct use of the derivative of the pulse counts as a speed feedback would considerably reduce the performance, especially at low speeds causing vibrations. The proposed observer fulfils this demand too.

2. Observer Design

The model of a DC servo motor [2,3] is given by

$$\begin{bmatrix} \dot{\omega}_r \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} -f_d/J & K_t/J \\ -K_t/L_a & -R_a/L_a \end{bmatrix} \begin{bmatrix} \omega_r \\ i_a \end{bmatrix} + \begin{bmatrix} -T_L/J \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_a \end{bmatrix} v_a \tag{1}$$

where ω_r, i_a, and v_a are rotor speed, armature current and armature voltage respectively, T_L is load torque, R_a and L_a are armature resistance and inductance, K_t is torque constant, which is equal to back emf constant, and f_d and J are dynamic friction constant and inertia respectively.

Based on the fact that the states of two DC servo motors with identical parameters converge to the same trajectories under the same input voltage and load torque regardless of their initial conditions, an observer [1] which is exactly in the same form as the actual motor model can be designed without using any feedback as

$$\begin{bmatrix} \dot{\hat{\omega}}_r \\ \dot{\hat{i}}_a \end{bmatrix} = \begin{bmatrix} -\hat{f}_d/\hat{J} & \hat{K}_t/\hat{J} \\ -\hat{K}_t/\hat{L}_a & -\hat{R}_a/\hat{L}_a \end{bmatrix} \begin{bmatrix} \hat{\omega}_r \\ \hat{i}_a \end{bmatrix} + \begin{bmatrix} -\hat{T}_L/\hat{J} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/\hat{L}_a \end{bmatrix} v_a \tag{2}$$

where the observer variables and parameters are shown with the addition of hats (^) to their symbols in the actual system. Such an observer is called a *natural observer* [1] since it has the natural characteristics of the actual motor, e.g., its convergence is as fast as the actual motor in reaching its steady state with the same parameters. In that case, the convergence is based on the *detectability* [4,5] rather than observability [5,6] of the system model.

The convergence is guaranteed with the condition that all the observer parameters and load torque are the same as the actual motor. However, the load torque is usually very difficult to measure in practice. In addition, motor parameters can be inaccurately known and some of them may change during operation. Therefore, the observer (2) gains a practical value if some adaptation algorithms accompany it. In a previous work [1], the observer (2) with a load torque adaptation was shown to be very successful for speed sensorless operations using a load torque adaptation. This paper considers operations with a speed sensor and proposes some parameter adaptations as well as another load torque adaptation.

3. Adaptation Algorithms

If all the parameters and the load torque in (2) are the same as in the actual system, error dynamics is linear time-invariant and asymptotically stable. If some parameter adaptation algorithms accompany the observer (2), then the error dynamics become nonlinear and it becomes difficult to analyse the asymptotic stability. However, limiting the load torque and parameter estimations in reasonable ranges, bounded-input bounded-state (*bibs*) stability is guaranteed since the actual system (1) is *bibs* stable and the observer (2) has exactly the same structure as the actual system model without any direct feedback.

In order to achieve convergence starting with wrong parameters and load torque, some adaptation algorithms will be proposed forcing some error terms towards zero. Both the speed and armature current are assumed to be accessible. When all the parameters are estimated simultaneously, the dynamics of the error between the actual system (1) and the observer (2) should be analysed. The armature voltage-current relation of (1) can be expressed as

$$\ddot{i}_a + p_1\dot{i}_a + p_0 i_a = b_1\dot{v}_a + b_0 v_a - c_1 \tag{3}$$

where

$$\left.\begin{aligned} p_1 &= \frac{f_d}{J} + \frac{R_a}{L_a} \quad , \quad p_0 = \frac{K_t^2 + f_d R_a}{JL_a} \quad , \\ b_1 &= \frac{1}{L_a} \quad , \quad b_0 = \frac{f_d}{JL_a} \quad , \\ c_1 &= \frac{K_t}{JL_a} T_L \end{aligned}\right\} \tag{4}$$

Here, since the load torque is assumed to be slow-varying or piece-wise constant, T_L is also considered as a parameter. A similar differential equation to (3) can be derived for the observer (2) as

$$\ddot{\hat{i}}_a + \hat{p}_1\dot{\hat{i}}_a + \hat{p}_0\hat{i}_a = \hat{b}_1\dot{v}_a + \hat{b}_0 v_a - \hat{c}_1 + v_a\frac{d}{dt}\left(\frac{1}{\hat{L}_a}\right) - \hat{\omega}_r\frac{d}{dt}\left(\frac{\hat{K}_t}{\hat{L}_a}\right) - \hat{i}_a\frac{d}{dt}\left(\frac{\hat{R}_a}{\hat{L}_a}\right) \tag{5}$$

where $\hat{p}_1$, $\hat{p}_0$, $\hat{b}_1$, $\hat{b}_0$ and $\hat{c}_1$ are the quantities calculated as in (4) but using the estimated parameters. Note that for any p, r, $\hat{p}$ and $\hat{r}$ quantities,

$$\hat{p}\hat{r} - pr = \hat{p}\hat{r} - pr + (p\hat{r} - p\hat{r}) = p(\hat{r} - r) + (\hat{p} - p)\hat{r} \tag{6}$$

an error differential equation for $e_i = \hat{i}_a - i_a$ can be obtained as

$$\begin{aligned} \ddot{e}_i + p_1\dot{e}_i + p_0 e_i = \delta_1 &= -\dot{\hat{i}}_a e_{p1} - \hat{i}_a e_{p0} + \dot{v}_a e_{b1} + v_a e_{b0} - e_{c1} \\ &\quad + v_a\frac{d}{dt}\left(\frac{1}{\hat{L}_a}\right) - \hat{\omega}_r\frac{d}{dt}\left(\frac{\hat{K}_t}{\hat{L}_a}\right) - \hat{i}_a\frac{d}{dt}\left(\frac{\hat{R}_a}{\hat{L}_a}\right) \end{aligned} \tag{7}$$

by subtracting (3) from (5), where

$$e_{p1} = \hat{p}_1 - p_1\ , \quad e_{p0} = \hat{p}_0 - p_0\ , \quad e_{b1} = \hat{b}_1 - b_1\ , \quad e_{b0} = \hat{b}_0 - b_0\ , \quad e_{c1} = \hat{c}_1 - c_1 \tag{8}$$

A similar error differential equation can be written for $e_\omega = \hat{\omega}_r - \omega_r$. The armature voltage-rotor speed relation of (1) can be expressed as

$$\ddot{\omega}_r + p_1\dot{\omega}_r + p_0\omega_r = c_2 v_a - c_3 - \dot{c}_4 \tag{9}$$

where

$$c_2 = \frac{K_t}{JL_a} \quad , \qquad c_3 = \frac{R_a T_L}{JL_a} \quad , \qquad c_4 = \frac{T_L}{J} \tag{10}$$

Similarly, for the observer (2),

$$\ddot{\hat{\omega}}_r + \hat{p}_1\dot{\hat{\omega}}_r + \hat{p}_0\hat{\omega}_r = \hat{c}_2 v_a - \hat{c}_3 - \dot{\hat{c}}_4 - \hat{\omega}_r \frac{d}{dt}\left(\frac{\hat{f}_d}{\hat{J}}\right) + \hat{i}_a \frac{d}{dt}\left(\frac{\hat{K}_t}{\hat{J}}\right) \tag{11}$$

where $\hat{c}_2$, $\hat{c}_3$ and $\hat{c}_4$ are the quantities calculated as in (10) but using the estimated parameters. Subtracting (9) from (11) gives

$$\ddot{e}_\omega + p_1\dot{e}_\omega + p_0 e_\omega = \delta_2 = -\dot{\hat{\omega}}_r e_{p1} - \hat{\omega}_r e_{p0} + v_a e_{c2} - e_{c3} - \dot{e}_{c4} - \hat{\omega}_r \frac{d}{dt}\left(\frac{\hat{f}_d}{\hat{J}}\right) + \hat{i}_a \frac{d}{dt}\left(\frac{\hat{K}_t}{\hat{J}}\right) \tag{12}$$

where

$$e_{c2} = \hat{c}_2 - c_2\ , \quad e_{c3} = \hat{c}_3 - c_3\ , \quad e_{c4} = \hat{c}_4 - c_4 \tag{13}$$

δ_1 and δ_2 are defined as the right hand sides of the error differential equations (7) and (12) respectively and they become zero if all the observer parameters are constant and equal to the corresponding values in the actual motor. Therefore, δ_1 and δ_2 can be regarded as small disturbances if v_a and e_{c4} are not changed suddenly, the observer parameters are sufficiently close to their corresponding values in the actual motor and change slowly enough. The left hand sides of (7) and (12) are in the same form as those of the actual motor's dynamics (3) and (9). Since the motor is stable, i.e. the motor states quickly approach zero from any initial condition for zero applied voltage and zero load torque, the error dynamics are also stable, i.e. e_i, $\dot{e}_i$, e_ω and $\dot{e}_\omega$ quickly approach zero from any initial condition for $\delta_1 = 0$ and $\delta_2 = 0$, or they decay to small levels for small disturbances of δ_1 and δ_2. Alternatively, if e_i and e_ω asymptotically approach zero when using some parameter adaptation schemes in parallel with the observer (2) and estimated parameters asymptotically approach some constants, this means all the parameters are converging to their actual values provided that the input v_a consists of sufficiently rich frequency components [7].

If v_a consists of only a DC component, then two equations for the parameters can be obtained from (1) in steady state. Obviously, these are not enough to get sufficient information to find all six parameters.

For each frequency component of v_a, four extra equations can be established: two from the coefficients of sine terms and two from the coefficients of cosine terms. Therefore, that v_a consists of sufficiently rich frequency components in order to estimate all six parameters means that v_a consists of at least a DC component plus an AC component. This is a theoretical requirement for the estimation of all six parameters; however, an attempt to solve the six parameters from such equations will usually fail in practice, and even in simulations. This is because such a method is extremely sensitive to noise; even a simulation with $1\ \mu s$ of time steps is not accurate enough to solve the parameters for the example presented in this paper.

In this paper, some PI adaptation schemes will be proposed. Each adaptation scheme will be developed separately using either e_ω, e_i or $e_{\omega i} = \hat{\omega}_r \hat{i}_a - \omega_r i_a$ as a correction term. For each parameter to be estimated, the derivative of the correction term is analysed in order to see the estimated parameter's effect on the correction term. For the adaptations presented in this paper, these derivatives are in the form of

$$\dot{e} = \rho \hat{p} + \cdots \tag{14}$$

where e is the correction term, $\hat{p}$ is the parameter estimation, ρ is the constant or variable coefficient of $\hat{p}$, and " ... " summarises all the other terms not including $\hat{p}$ explicitly. As an approximate method, whether a small increase in $\hat{p}$ has an incremental or decremental effect on the correction term is determined by the sign of ρ. Preventing overly fast changes in v_a and adaptations, a small increase in $\hat{p}$ causes e to increase in steady state if $\rho > 0$ and causes e to decrease if $\rho < 0$. Therefore, a PI adaptation scheme based on this approximation can be developed for $\hat{p}$ as

$$\hat{p} = K_p e + \int K_i e \, dt \tag{15}$$

selecting the signs of the PI gains, K_p and K_i, as

$$sign(K_p) = sign(K_i) = -sign(\rho) \tag{16}$$

For less noise-sensitive estimations $K_p = 0$ is preferred because $\hat{p}$ contains only filtered noise by the integration and state estimations contain double filtered noise in this case.

This technique will be illustrated for each adaptation in the following subsections. e_ω is going to be used as a correction term for $\hat{T}_L$, $\hat{J}$ and $\hat{f}_d$ since they appear in its derivative, and e_i is going to be used as a correction term for $\hat{R}_a$ and $\hat{L}_a$ since they appear in its derivative. $\hat{K}_t$ appears in the derivatives of both $\hat{\omega}_r$ and $\hat{i}_a$; therefore, using $e_{\omega i} = \hat{\omega}_r \hat{i}_a - \omega_r i_a$ as a correction term for the torque constant adaptation will be a reasonable choice.

3.1. Load torque adaptation

$\hat{T}_L$ appears in the derivative of e_ω as

$$\dot{e}_\omega = \dot{\hat{\omega}}_r - \dot{\omega}_r = -\frac{\hat{T}_L}{\hat{J}} + \ldots \tag{17}$$

where "..." denotes all the other terms not including $\hat{T}_L$. Then, as explained above, a PI scheme for $\hat{T}_L$ can be proposed as

$$\hat{T}_L = K_{1p} e_\omega + \int K_{1i} e_\omega dt \tag{18}$$

with

$$sign(K_{1p}) = sign(K_{1i}) = sign(\hat{J}) \tag{19}$$

(18) with (19) describes the proposed load torque adaptation scheme working in parallel with the observer (2). Limiting $\hat{T}_L$ within a reasonable range as $[T_{\min}, T_{\max}]$ guarantees the *bibs* stability and limiting the slope of $\hat{T}_L$ may ensure a smoother response.

3.2. Armature resistance adaptation

$\hat{R}_a$ appears in the derivative of e_i as

$$\dot{e}_i = \dot{\hat{i}}_a - \dot{i}_a = -\frac{\hat{i}_a}{\hat{L}_a}\hat{R}_a + \cdots \tag{20}$$

where "..." denotes all the other terms not including $\hat{R}_a$. Then a PI scheme for $\hat{R}_a$ is proposed as

$$\hat{R}_a = K_{2p} e_i + \int K_{2i} e_i dt \tag{21}$$

with

$$sign(K_{2p}) = sign(K_{2i}) = sign\left(\frac{\hat{i}_a}{\hat{L}_a}\right) \tag{22}$$

Limiting $\hat{R}_a$ within a reasonable range as $[R_{\min}, R_{\max}]$ guarantees the *bibs* stability and limiting the slope of $\hat{R}_a$ may ensure a smoother response.

3.3. Inertia adaptation

$\hat{J}$ appears in the derivative of e_ω as

$$\dot{e}_\omega = \dot{\hat{\omega}}_r - \dot{\omega}_r = -\left(\hat{f}_d\hat{\omega}_r - \hat{K}_t\hat{i}_a + \hat{T}_L\right)\cdot\frac{1}{\hat{J}} + \ldots \tag{23}$$

where "..." denotes all the other terms not including $\hat{J}$. Then, a PI scheme for $\hat{J}$ is proposed as

$$\frac{1}{\hat{J}} = K_{3p} e_\omega + \int K_{3i} e_\omega dt \tag{24}$$

$$sign(K_{3p}) = sign(K_{3i}) = sign\left(\hat{f}_d\hat{\omega}_r - \hat{K}_t\hat{i}_a + \hat{T}_L\right) \tag{25}$$

Limiting $1\big/\hat{J}$ within a reasonable range as $[1/J_{\max}, 1/J_{\min}]$ guarantees the *bibs* stability and limiting the slope of $1\big/\hat{J}$ may ensure a smoother response.

3.4. Armature inductance adaptation

$\hat{L}_a$ appears in the derivative of e_i as

$$\dot{e}_i = \dot{\hat{i}}_a - \dot{i}_a = -\left(\hat{K}_t\hat{\omega}_r + \hat{R}_a\hat{i}_a - v_a\right) \cdot \frac{1}{\hat{L}_a} + \cdots \tag{26}$$

where "…" denotes all the other terms not including $\hat{L}_a$. Then, a PI scheme for $\hat{L}_a$ is proposed as

$$\frac{1}{\hat{L}_a} = K_{4p}e_i + \int K_{4i}e_i dt \tag{27}$$

$$sign(K_{4p}) = sign(K_{4i}) = sign\left(\hat{K}_t\hat{\omega}_r + \hat{R}_a\hat{i}_a - v_a\right) \tag{28}$$

Limiting $1\big/\hat{L}_a$ within a reasonable range as $[1/L_{\max}, 1/L_{\min}]$ guarantees the *bibs* stability and limiting the slope of $1\big/\hat{L}_a$ may ensure a smoother response.

3.5. Friction constant adaptation

$\hat{f}_d$ appears in the derivative of e_ω as

$$\dot{e}_\omega = \dot{\hat{\omega}}_r - \dot{\omega}_r = -\left(\frac{\hat{\omega}_r}{\hat{J}}\right) \cdot \hat{f}_d + \cdots \tag{29}$$

where "…" denotes all the other terms not including $\hat{f}_d$. Then, a PI scheme for $\hat{f}_d$ is proposed as

$$\hat{f}_d = K_{5p}e_\omega + \int K_{5i}e_\omega dt \tag{30}$$

$$sign(K_{5p}) = sign(K_{5i}) = sign\left(\frac{\hat{\omega}_r}{\hat{J}}\right) \tag{31}$$

Limiting $\hat{f}_d$ within a reasonable range as $[f_{\min}, f_{\max}]$ guarantees the *bibs* stability and limiting the slope of $\hat{f}_d$ may ensure a smoother response.

3.6. Torque (or back emf) constant adaptation

$\hat{K}_t$ appears in the derivative of $e_{\omega\,i}$ as

$$\dot{e}_{\omega\,i} = \dot{\hat{\omega}}_r\,\hat{i}_a + \hat{\omega}_r\,\dot{\hat{i}}_a - \dot{\omega}_r\,i_a + \omega_r\,\dot{i}_a = \left(\frac{\hat{i}_a^2}{\hat{J}} - \frac{\hat{\omega}_r^2}{\hat{L}_a}\right)\hat{K}_t + \ldots \tag{32}$$

where "…" denotes all the other terms not including $\hat{K}_t$. Then, a PI scheme for $\hat{K}_t$ is proposed as

$$\hat{K}_t = K_{6p} e_{\omega\,i} + \int K_{6i} e_{\omega\,i} dt \tag{33}$$

$$sign(K_{6p}) = sign(K_{6i}) = sign\left(\frac{\hat{\omega}_r^2}{\hat{L}_a} - \frac{\hat{i}_a^2}{\hat{J}}\right) \tag{34}$$

Limiting $\hat{K}_t$ within a reasonable range as $[K_{\min}, K_{\max}]$ guarantees the *bibs* stability and limiting the slope of $\hat{K}_t$ may ensure a smoother response.

3.7. Re-initiation of integrals at limits and sign changes

When a sign change occurs in a PID adaptation scheme, the estimation jumps to another value due to the proportional and derivative terms. This causes another transient error peak and a delay for convergence. In addition, when the estimation tends to exceed its limits, just limiting the estimation and/or freezing the integral term may also cause a delay for convergence until the estimation comes back into the limited range. In order to prevent such delays and jumps, re-initiation of the integrals [1] is a good solution when the signs of the PID gains change or when the estimation tends to exceed the limits. Re-initiation is performed in such a way that the new gains and the new integral value produce the last value of the estimation. In general, denoting the integral values just before and after the re-initiation with ξ^- and ξ^+ respectively (assuming the integral gain is taken into account inside the integral), the re-initiation is performed as

$$\xi^+ = \hat{p}^- - K_d^+\dot{e} - K_p^+ e \tag{35}$$

where $\hat{p}^-$ is the limited estimation value just before the re-initiation, K_d^+ and K_p^+ are new derivative and proportional gains respectively and e is the correction term value before the re-initiation. (35) can also be applied for the same purpose when the gains change.

3.8. Simultaneous operation of all the adaptation schemes

Even though the adaptation schemes proposed in this paper have been derived separately, simulation studies show that they can be applied simultaneously. In that case, convergence is not guaranteed; however, finding suitable gains which result in convergence is not very difficult by trial and error. Therefore, the proposed method may be regarded as an empirical method in some respects. However, this is still very useful since it is simple and requires no prior information about the system parameters. Whether the parameter estimations have converged to the actual parameters or not can be ascertained by checking if the estimations settle at values other than their limits.

In practice, adaptation of all the parameters is not required in most operations. For example, once the armature inductance and torque constant are estimated in a test operation, fixed values can be used in most other operations for these parameters. The demand speed and armature current of the servo motor are usually constant over some periods. It is remarkable that adaptations of the inertia and armature inductance are not possible over such periods since $\dot{\omega}_r = 0$ and $\dot{i}_a = 0$ can be satisfied for any nonzero values of them. Therefore, fixed values should be used for inertia and armature inductance for such operations. In addition, only two of the other quantities can be simultaneously adapted as $\dot{\omega}_r = 0$ and $\dot{i}_a = 0$ since the dynamic equations of the system then become two static equations with speed and armature current measurement. As a reasonable choice, the load torque, which can vary with external effects, and the armature resistance, which can vary with temperature, will be estimated simultaneously with the state variables using the proposed method.

The proposed observer with only load torque and armature resistance adaptations is so insensitive to noise that even if the speed feedback is calculated by differentiating the pulse counts obtained from a very low-resolution encoder providing one pulse per revolution, quite accurate estimations can be obtained provided that the direction of rotation is also known.

4. Speed Control of the DC Servo Motor

The proposed estimation schemes do not require a specific control scheme. Using the estimated speed allows the designer to apply a PID control [1] with suitable PID gains, K_P, K_I and K_D,

$$v_a = K_D \frac{d}{dt}\left(\omega_{ref} - \hat{\omega}_r\right) + K_P\left(\omega_{ref} - \hat{\omega}_r\right) + \int K_I\left(\omega_{ref} - \hat{\omega}_r\right) dt \tag{36}$$

where ω_{ref} is the reference speed, which is assumed to be piecewise constant, any desired eigenvalues of the third order speed error dynamics can be obtained since the system is second order linear time-invariant, disregarding the term T_L/J, which is also assumed to be piecewise constant and disappears in the third order speed error differential equation when it is constant. $\dot{\hat{\omega}}_r$ can be substituted from (2). Then, (36) becomes

$$v_a = -K_D \frac{\hat{K}_t}{\hat{J}} \hat{i}_a + K_D \frac{\hat{f}_d}{\hat{J}} \hat{\omega}_r + K_D \frac{\hat{T}_L}{\hat{J}} + K_P\left(\omega_{ref} - \hat{\omega}_r\right) + \int K_I\left(\omega_{ref} - \hat{\omega}_r\right) dt \tag{37}$$

It is of note that if such a control is applied with a conventional observer, a feedback term appears in (37) since $\dot{\hat{\omega}}_r$ probably include a feedback term. Alternatively, if all the proportional adaptation gains shown in Section 3 are selected as zero, no feedback term directly appears in (37) with the proposed method. Therefore, the proposed method allows a less noisy control.

5. Simulation Results

5.1. Test operation to estimate all the parameters simultaneously

The model (1), the observer (2) and all the adaptation algorithms given in Section 3 have been simulated for $R_a = 3.2\,\Omega$, $L_a = 8.6$ mH, $K_t = 0.0319$ Nm/A, $f_d = 0.00012$ Nm·s/rad and $J = 3 \times 10^{-5}$ kgm^2. Applied load torque is $T_L = 0.01$ Nm and the applied armature voltage is $v_a = (1 + 5\sin\pi t + 4\sin 10\pi t)\ V$, which contains a DC and two distinct frequency components to estimate the parameters faster. Estimations are limited within quite large ranges so that it can be easily guessed that the actual values must be within these ranges: $\hat{T}_L \in [-0.05,\ 0.05]$ Nm, $\hat{R}_a \in [0.01,\ 10]\,\Omega$, $\hat{J} \in \left[10^{-6},\ 10^{-3}\right]$ kgm^2, $\hat{L}_a \in [0.001,\ 0.1]$ H, $\hat{f}_d \in \left[10^{-6},\ 1\right]$ Nm·s/rad and $\hat{K}_t \in [0.001,\ 0.2]$ Nm/A (or Vs/rad). A successful set of adaptation gains has been found with a few trials as $K_{1i} = 0.0025$ Nm/rad, $K_{2i} = 0.6\,\Omega$ A^{-1}s^{-1}, $K_{3i} = 80$ kg^{-1}m^{-2}rad^{-1}, $K_{4i} = 30$ H^{-1}A^{-1}s^{-1}, $K_{5i} = 1 \times 10^{-6}$ Nm·s·rad^{-2}, $K_{6i} = 9 \times 10^{-5}$ NmA^{-2}rad^{-1} and all the proportional gains are selected as zero in order to have less noise-sensitive estimations. Initial conditions are all zero for the actual system, $\hat{\omega}_r = 50$ rad/s and $\hat{i}_a = 1$ A for the observer, and zero for all the integral terms of the adaptations. All the adaptation schemes given in Section 3 and the observer (2) in parallel with the actual motor model (1) have been simulated with 1 ms of time steps using 4-step Runga-Kutta method and the results shown in Figures 1-3 have been obtained.

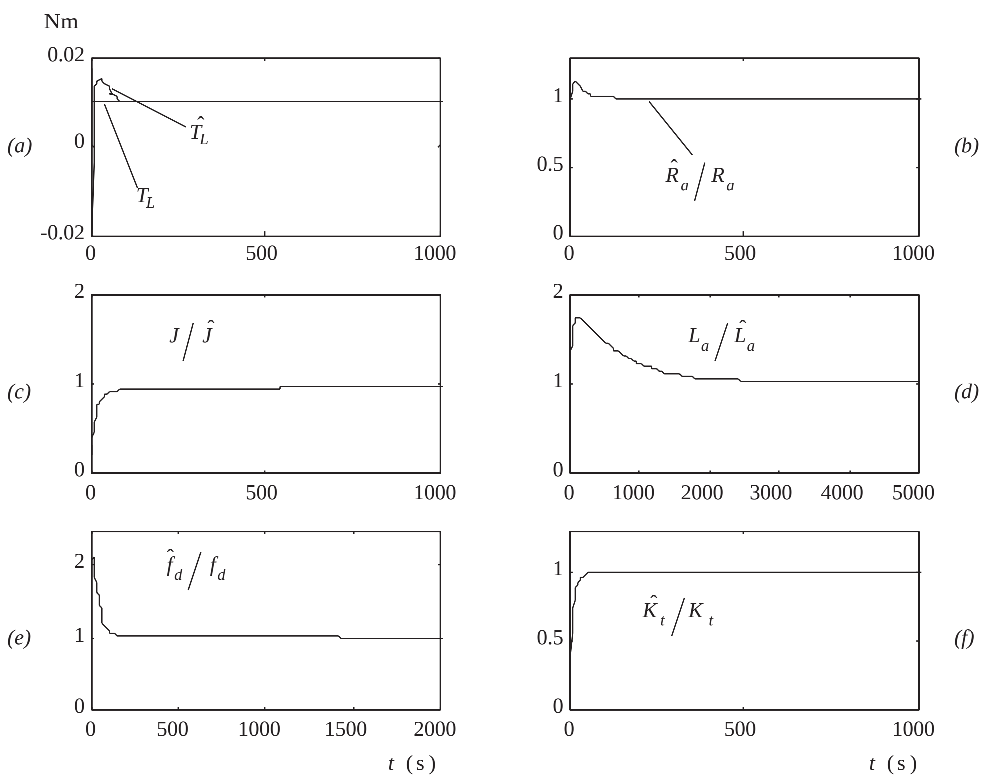

Figure 1. Simultaneous estimations of all the parameters in parallel with the observer (2): **a.** Load torque estimation. **b.** Armature resistance estimation. **c.** Inertia estimation. **d.** Armature inductance estimation. **e.** Dynamic friction constant estimation. **f.** Torque (or back emf) constant estimation.

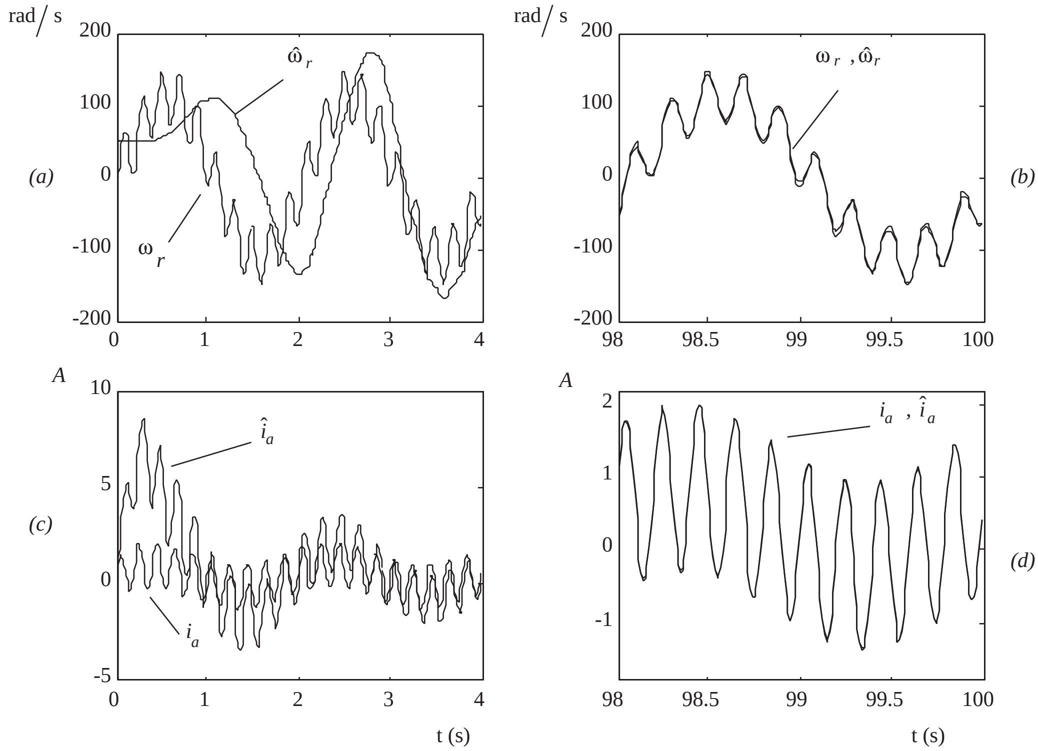

Figure 2. Convergence of state estimations at the beginning of simultaneous operation of all the proposed schemes: **a-b.** Speed and its estimation. **c-d.** Armature current and its estimation.

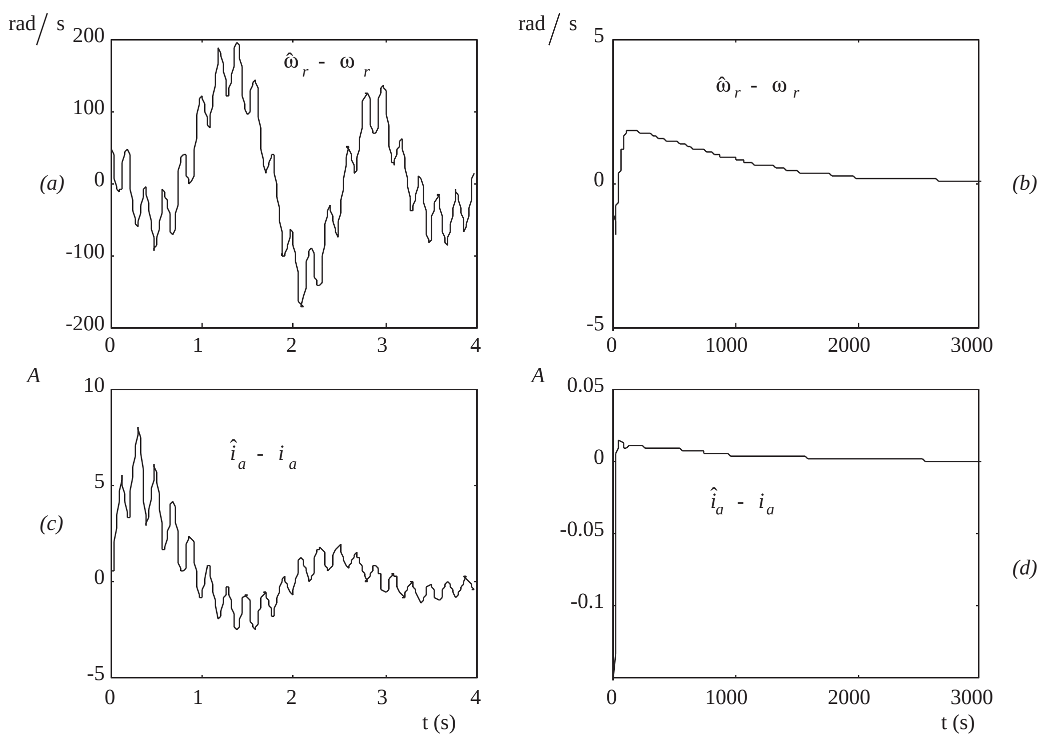

Figure 3 State estimation errors for simultaneous operation of all the proposed schemes: **a-b.** Speed error. **c-d.** Armature current error.

In general, for any adaptation method, as the number of parameters to be adapted from each correction term increases, the required time for the estimation also increases and the adaptations become more sensitive to noise. Similarly, as can be seen from the figures, estimating all the parameters simultaneously takes quite a long time in this application. Since the state estimation errors quickly decrease, the correction terms have reduced signal-to-noise ratios and less effects on the adaptations. The slowest adaptation is armature inductance adaptation due to the low current frequency.

5.2. Operation with load torque and armature resistance adaptations

The observer (2) in parallel with the proposed load torque and armature resistance adaptations has been simulated under excessively noisy conditions to exhibit the performance for more realistic conditions. First, the actual speed measurement is not available; instead, a single hall sensor is assumed to exist providing a single pulse per revolution and the direction of rotation is assumed to be known. The speed feedback is calculated from the derivative of the pulse counts. In addition, noise signals (5 rad/s) $\times$ randn(1) and (50 mA) $\times$ randn(1) are added to the calculated speed and armature current feedbacks, where "randn(1)" is MATLAB's scalar normal random number generating function with zero mean and unity variance. The observer and the adaptation schemes use these noisy quantities and the noise-free input voltage command provided by the controller. As a more realistic condition, this command voltage is applied to the actual motor model after adding a noise of (300 mV) $\times$ randn(1). This noise can also be considered as a modelling disturbance. These conditions are much worse than expected in a usual experimental study for a motor with given ratings.

The same limits and the initial conditions as in the previous simulations are used; however the gains are changed as $K_{1i} = 0.01$Nm/rad and $K_{2i} = 6\,\Omega\ \mathrm{A}^{-1}\mathrm{s}^{-1}$. All the speed reversal, sudden loading and unloading tests have been applied in this simulation. The speed reference is normally $\omega_{ref} = 100$rad/s but it is reversed during 10 s $\leq t < 20$ s. The load torque has been kept at $T_L = 0.01$ Nm for $t < 22$ s, which means the machine works in generating mode during the reverse speed. The load torque is suddenly increased to $T_L = 0.03$ Nm at $t = 22$ s and suddenly decreased to zero at $t = 35$ s. Simulation results are shown in Figures 4-5.

In this operation, each armature current error and speed error is used to adapt only one quantity. Therefore, the estimations quickly converge to the actual values. Once the convergence is achieved, the state estimations follow the actual states accurately even if they change very fast. However, when the actual torque suddenly changes, the speed and armature resistance estimations are affected. Then they quickly converge to the actual values.

This method is less sensitive to noise. The ripples in the load torque estimation are due to the large noise in the current measurement and they are filtered further by the observer. The noise between the command voltage and applied voltage to the actual system, which can be considered modelling disturbance, slightly affects the estimations. However, the measurement noise is filtered very well and its effects on the estimations are at very low levels with respect to the noise level.

6. Conclusions

An adaptive observer without direct feedback which was proposed for position/speed-sensorless DC servo motors has been applied to DC servo motors with some extra adaptation schemes using a speed sensor. It has been shown that designing an observer without direct feedback with some adaptation algorithms is

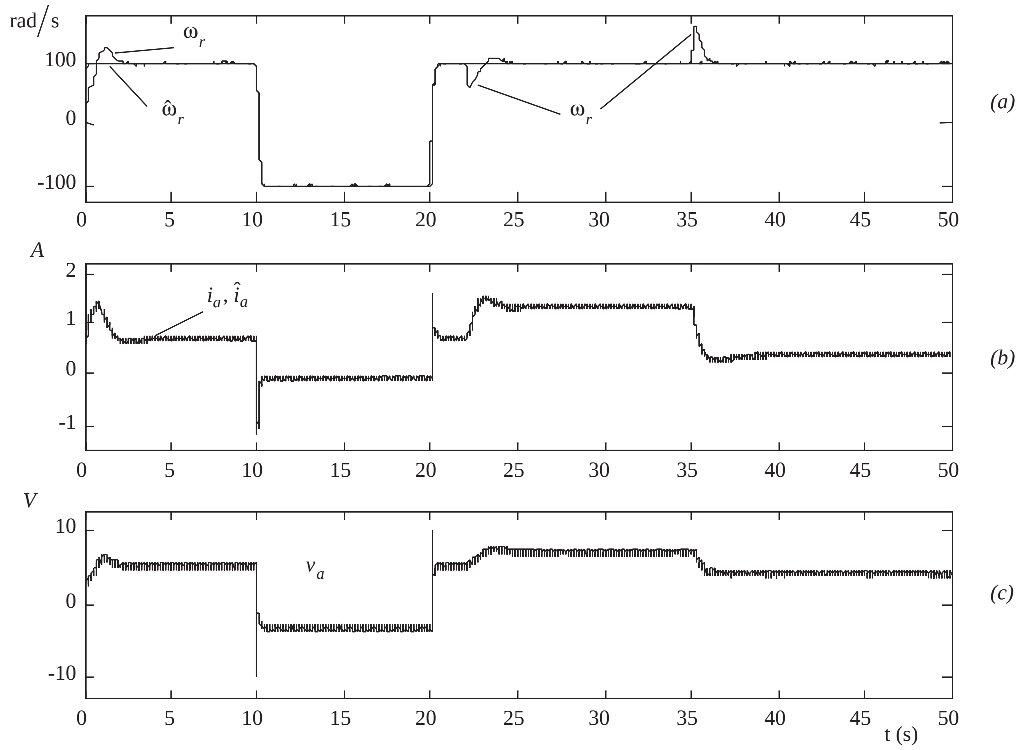

Figure 4. Observer results with the proposed load torque and armature resistance adaptations: **a.** Speed and its estimation, **b.** Armature current and its estimation. **c.** Armature voltage.

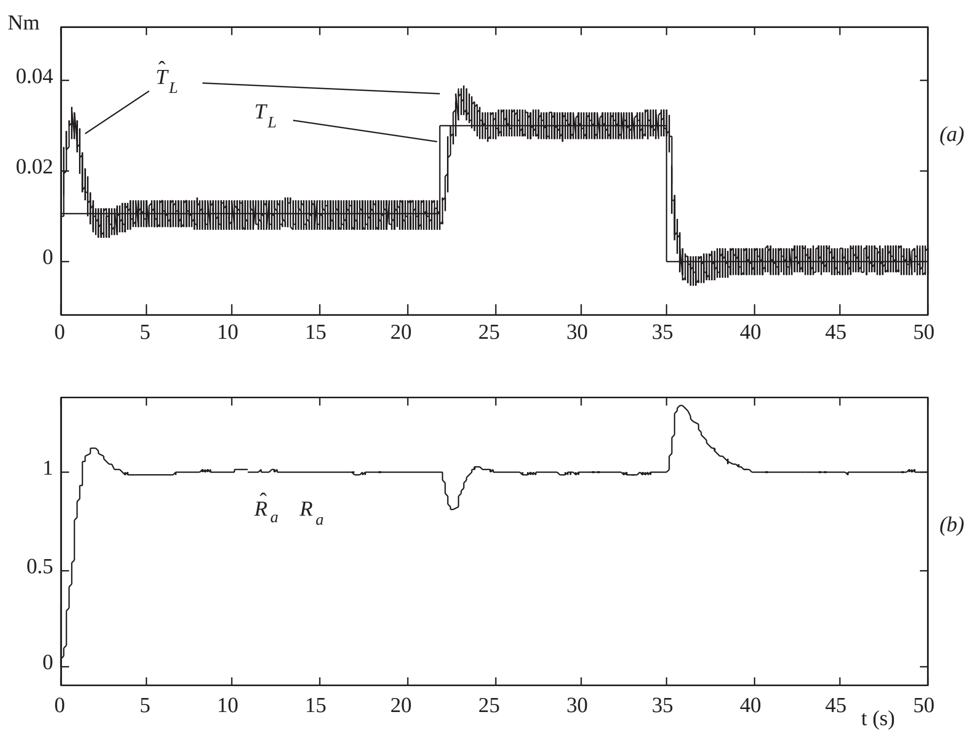

Figure 5a. Load torque and its estimation. **b.** Armature resistance estimation.

very simple for DC servo motors. It is possible to estimate all the motor parameters as well as the states and the load torque simultaneously using the speed and current feedback in adaptations. However, such an application yields satisfactory results under operating conditions containing sufficiently rich frequency components. For operations including constant speeds, only the load torque and armature resistance adaptations are recommended as well as the state estimations. Even though the state feedbacks are already available, the observer without direct feedback filters the noise in the estimated quantities again and allows the designer to use derivative terms in control, e.g., PI**D** control as shown in Section 4, with very low-quality sensors.

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