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# E2/M1 Multipole Mixing Ratios of Transitions in Erbium Isotopes

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**Abstract**

Erbium isotopes (Z = 64) lie in the transitional region that occurs at the middle of the range of deformed nuclei. $\gamma$-ray E2/M1 mixing ratios for selected transitions in $^{166-168}$Er are calculated in the proton-neutron interacting boson approximation(IBA-2). The results obtained for $^{166-168}$Er are reasonably in good agreement with the previous experimental and teoretical values.

**Key Words:** IBA-2, Deformation parameter, Quadrupol moment, Mixing ratios.

## 1. Introduction

The neutron-proton version of the interacting boson model (IBA-2) which distinguishes between neutron ($\nu$) and proton ($\pi$) bosons, is used in the present work; a full description of IBA-2 is found in [1–4]. The Hamiltonian operator in IBA-2 has three parts, one for proton bosons, one for neutron bosons and one that describes the interaction between unlike bosons:

$$H = H_\pi + H_\nu + H_{\pi\nu}. \tag{1}$$

The Hamiltonian generally used in phenomenological calculations can be written as

$$H = \varepsilon_d(n_{d_\nu} + n_{d_\pi}) + \kappa(\mathbf{Q}_\nu \cdot \pi) + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi}, \tag{2}$$

where the dot denotes a scalar product. The first term represents the single-boson energies for proton and neutron bosons, $\varepsilon_d$ is the energy difference between s- and d-bosons and $n_{d\rho}$is the number of d-bosons, where $\rho$ corresponds to $\pi$ (proton) or $\nu$ (neutron) bosons. The second term denotes the main part of the boson-boson interaction, i.e the quadrupole-quadrupole interaction between neutron and proton bosons with strength $\kappa$. The quadrupol operator is

$$Q_\rho = [d_\rho^+ s_\rho + s_\rho^+ \tilde{d}_\rho]^{(2)} + \chi_\rho [d_\rho^+ \tilde{d}_\rho]^{(2)} \tag{3}$$

where $\chi_\rho$ determines the structure of the quadrupol operator and is determined empirically. The square brackets in Eq. (3) denotes angular momentum coupling.

The terms $V_{\pi\pi}$ and $V_{\nu\nu}$, which correspond to interactions between like-bosons, are sometimes included in order to improve the fit to experimental energy spectra. They are of the form

$$V_{\rho\rho} = \frac{1}{2} \sum_{L=0,2,4} C_L^\rho \left( \left[d_\rho^+ d_\rho^+\right]^{(L)} . \left[\tilde{d}_\rho \tilde{d}_\rho\right]^{(L)} \right). \tag{4}$$

However, their effects are usually considered minor and often neglected [5].

The Majorana term $M_{\nu\pi}$, which contains three parameters $\xi_1$, $\xi_2$ and $\xi_3$ may be written as

$$M_{\nu\pi} = \frac{1}{2}\xi_2 \left( \left[s_\nu^+ d_\pi^+ - d_\nu^+ s_\pi^+\right]^{(2)} . \left[s_\nu \tilde{d}_\pi - \tilde{d}_\nu s_\pi\right]^{|2|} \right) - \sum_{k=1,3} \xi_k \left( \left[d_\nu^+ d_\pi^+\right]^{(k)} . \left[\tilde{d}_\nu \tilde{d}_\pi\right]^{(k)} \right). \tag{5}$$

By this work we have aimed at two things: first, to give the Hamiltonian of IBA-2 in terms of the formalism; second is to study $^{166-168}$Er by use of this Hamiltonian.

## 2. Theory and Method of calculation

The isotopes $^{166-168}$Er have $N_\pi$= 7 , and $N_\nu$ varies from 8 and 9 , while the parameters $\kappa$, $\chi_\rho$ and $\varepsilon_d$, as well as the Majorana parameters $\xi_k$, with $k$ = 1, 2, 3, were treated as free parameters and their values were estimated by fitting to the measured level energies. This procedure was made by sellecting the "traditional" values of the parameters and then allowing one parameter to vary while keeping the others are held constant until a best fit was obtained. This was carried out iteratively until an overall fit was achieved. The best fit values for the hamiltonian parameters are given in Table 1 and the calculated energy levels are compared with the experimental data and are shown in Table 2 and Table 3 for $^{166-168}$Er.

**Table 1.** Hamiltonian Parameters.

| Isotope | $\varepsilon_d$ | $\kappa$ | $\chi_\nu$ | $\chi_\pi$ | $\xi_{1,2}$ | $\xi_3$ |
|---|---|---|---|---|---|---|
| $^{166}$Er | 0.23 | -0.04 | -0.49 | -0.59 | 0.15 | 0.12 |
| $^{168}$Er | 0.20 | -0.02 | -0.61 | -0.71 | 0.18 | 0.18 |

**Table 2.** Calculated and experimental energy levels of $^{166}$Er.

| Energy Levels | Spin Parity | Energy Levels | Energy Levels [26] |
|---|---|---|---|
| $K^\pi$ | $I^\pi$ | This Work (MeV) | Experimental (MeV) |
| Ground state | $0^+$ | 0.000 | 0.000 |
| Band | $2^+$ | 0.079 | 0.080 |
| $K^\pi$=$0^+$ | $4^+$ | 0.266 | 0.264 |
| | $6^+$ | 0.536 | 0.545 |
| Gamma | $2^+$ | 0.867 | 0.785 |
| Vibrational | $3^+$ | 0.631 | 0.859 |
| Band | $4^+$ | 1.055 | 0.956 |
| $K^\pi$=$2^+$ | $5^+$ | 1.001 | 1.075 |

**Table 3.** Calculated and experimental energy levels of $^{168}$Er.

| Energy Levels | Spin Parity | Energy Levels | Energy Levels [26] |
|---|---|---|---|
| $K^\pi$ | $I^\pi$ | This Work (MeV) | Experimental (MeV) |
| Ground state | $0^+$ | 0.000 | 0.000 |
| Band | $2^+$ | 0.091 | 0.078 |
| $K^\pi$=$0^+$ | $4^+$ | 0.272 | 0.264 |
| | $6^+$ | 0.549 | 0.548 |
| Gamma | $2^+$ | 0.644 | 0.821 |
| Vibrational | $3^+$ | 0.589 | 0.895 |
| Band | $4^+$ | 0.983 | 0.994 |
| $K^\pi$=$2^+$ | $5^+$ | 1.465 | 1.117 |

Arima and Iachello in their original interacting boson approximation (IBA-1) gave the M1 operator in the restricted case of U(5) dynamic symmetry [2] and as well as the general case [6]. However , even when starting with the general operator, they derived the E2/M1 mixing ratio by neglecting the term which break the SU(3) symmetry [7]. It follows that the reduced mixing ratio is given by the same simple formula for both U(5) and SU(3) symmetries. The formula contains only one parameter and the initial and final spins. Warner [8] has developed an IBA description of the E2/M1 mixing ratio whose point of departure is essentially the same as that of Scholten et al [9]. To present time, several systematic studies [10–12] have been performed within the framework of the IBA. The most spectacular difference is that IBA-2 predict collective M1 excitations [13] absent from IBA-1 and they have been observed in both deformed and spherical nuclei [14].

In IBA-2 , the E2 transition operator is given by,

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu, \tag{6}$$

where $e_\pi(e_\nu)$ is the effective charge of proton (neutron) bosons in units of eb. They may be obtained from the B(E2) values of $2^+ \rightarrow 0^+$ transitions [15]. The quadrupole operator $Q_\rho$ has the same definition as in the Hamiltonian Equation (2). The M1 transition operator can be written as

$$T^{(M1)} = \left[\frac{3}{4\pi}\right]^{\frac{1}{2}} (g_\pi L_\pi + g_\nu L_\nu)\,, \tag{7}$$

where $g_\rho$ is the proton (neutron) $g$-factor in units of $\mu_N$ and $L_\rho$ is the angular momentum operator for proton (neutron) and given by

$$L_\rho = \sqrt{10}\left[d_\rho^+ \, \tilde{d}_\rho\right]^{(1)} \tag{8}$$

The values of the effective boson charges, as well as the g-factors for the proton and neutron, are taken from [15]. For the later, Sambataro and Dieperink [16] showed that the experimental $g$ values of $2_1^+$ levels have a simple linear relationship to $g_\pi$ and $g_\nu$ , and they were deduced from relavant experimental data.

The ratio $\Delta$(E2/M1) is defined as the ratio of the reduced E2-matrix element to the reduced M1-matrix element. Rather than attempting to evaluate the E2 and M1 matrix elements for $^{166-168}$Er essential in theoretical mixing ratio calculations, it is possible to obtain these ratios in an analytic form, as the matrix elements have a simple sturucture in the SU(5) and SU(3) limits. This quality is related to the usual $\delta$-mixing ratio by [17]

$$\delta(E2/M1) = 0.835E_\gamma\Delta(E2/M1) \tag{9}$$

where $E_\gamma$ is in MeV and $\Delta$(E2/M1) is $eb/\mu_N$. The $\delta$-mixing ratio were calculated for some selected transitions in $^{166-168}$Er; Table 4 shows the comparisons of our calculations with the experimental results.

**Table 4.** Experimental and theoretical $\delta\,(E2/M1)$ multipole mixing ratios of erbium isotopes.

| Isotope | Spin Parity | Transition Energy | Mixing Ratios $\delta\,(E2/M1)$ | | |
|---|---|---|---|---|---|
| | $I_i \rightarrow I_f$ | (MeV) | This work | Experimental | Previous work |
| $^{166}$Er | $2^+_\gamma \rightarrow 2^+_g$ | 0.7053 | 17.61 | 16.01 (+5:−-13)$^a$ | 16.84$^e$ |
| | $3^+_\gamma \rightarrow 2^+_g$ | 0.7788 | 19.11 | 19.0 (+19:−-9)$^a$ | 18.41$^f$ |
| | $3^+_\gamma \rightarrow 4^+_g$ | 0.5943 | 8.97 | 8.0 (+5:−-3)$^b$ | 17.61$^e$ |
| | $4^+_\gamma \rightarrow 4^+_g$ | 0.6912 | 9.32 | 7.5 (+∞:−-1.5)$^c$ | 9.06$^f$ |
| | $5^+_\gamma \rightarrow 4^+_g$ | 0.8119 | 1.4 | 1.46 (± 0.1)$^d$ | 0.23$^e$ |
| | $5^+_\gamma \rightarrow 6^+_g$ | 0.5298 | 5.38 | 5.0 (± 2.5)$^c$ | 5.4$^c$ |
| $^{168}$Er | $2^+_\gamma \rightarrow 2^+_g$ | 0.7413 | 16.14 | 16 (+12:−-5 )$^g$ | 16.39$^f$ |
| | $3^+_\gamma \rightarrow 2^+_g$ | 0.817 | 1.21 | 1.42 (± 0.4 )$^a$ | 1.76$^h$ |
| | $3^+_\gamma \rightarrow 4^+_g$ | 0.6317 | 3.5 | 9.3 (± 0.6 )$^a$ | 6.6$^e$ |
| | $4^+_\gamma \rightarrow 4^+_g$ | 0.7306 | 11.94 | 5.7 (± 5.7 )$^g$ | 8.42$^f$ |
| | $5^+_\gamma \rightarrow 4^+_g$ | 0.8535 | 2.43 | 3.64(+1.8:−-0.9)$^h$ | 10.13$^f$ |
| | $5^+_\gamma \rightarrow 6^+_g$ | 0.5695 | 4.95 | 25 (± 3 )$^c$ | 5.66$^f$ |

(a) Lange et al [18], (b) Krane et al [19], (c) Baker et al [20], (d) Binarh et al [21]
(e) Lipas et al [22], (f) Warner [23], (g) Domingos et al [24], (h) Schreckenbach et al [25].

## 3. Results and Discussion

Our calculated values of the mixing ratios of $^{166}$Er have reasonable agreements with the experimental data. The mixing ratio found for the 0.7788 MeV transition is 19.11 and this value is in agreement with the experimental values of 19.0 (+190:−-9) of Lange et al [18] and 18:41 of Warner [23]. The obtained results for the transition energies of 0.5943 MeV and 0.1190 MeV are 8.97 and 1.40 are also in good agreement with the experimental values.

The mixing ratio of $^{168}$Er found for the 0.7413 MeV, 0.0747 Mev and 0.8535 MeV transitions are 16.14, 1.21 and 2.43 respectively. These values are in good agreement with the experimental values of 16 (+12:5) of Dominges et al [24], 1.42 (±0.4) of Lange et al [18] and 3.64 (+1.8:−-0.9) of Schreckenbach et al [25].

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