

Investigation of some even-even selenium isotopes within the interacting boson model-2

Research Article

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Abstract: The even-even Selenium isotopes in the $A \sim 80$ mass region and the general features of its structure have been investigated within the framework of the interacting boson model-2. The neutron proton version of the model has been applied to the Se ($A = 74$ to 80) isotopes with emphasis on the description of the 0_1^+ , 2_1^+ , 0_2^+ , 2_2^+ and 4_1^+ states. The energy levels, $B(E2)$ and $B(M1)$ electromagnetic transition probabilities were calculated. The results of these calculations were compared with previous experimental results and were shown to be in good agreement.

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1. Introduction

The light even-even Se isotopes ($Z = 34$, $N = 40 - 46$) lie in transitional neutron-deficient nuclei where strong variation in collective properties has been observed as a function of neutron and proton numbers. These light isotopes exist between the semi-closed shell at the magic, $N = 40$ and $N = 50$, and their low-lying states in the mass region. The Selenium isotopes have been extensively studied by using the interacting boson model (IBM) [1–6] since these isotopes are nearly spherical and their low-lying energy levels structures show vibrational features. During the last decade, the above mentioned in addition to the isotopes of neighboring nuclei of selenium have been widely studied by using the same model [5–12].

Furthermore, IBM has become a useful tool in view of the collective properties of nuclei. The neutron-proton version (IBM-2) of this model in particular has been applied efficiently to the light isotopes of Se, with emphasis primarily on the description of energy levels [1, 3, 4].

In the present paper, the energy levels have been successfully calculated for Se isotopes in $A \sim 80$ mass region. In addition, by comparing these values with experimental results, one can demonstrate reproducibility for the theoretical results in the spectra corresponding to $40 < N < 50$ nuclei, with the Hamiltonian parameters. However, electromagnetic transition probabilities have been also calculated by using the fitted proton and neutron effective boson charges (for $B(E2)$) and the fitted $g_{\pi,\nu}$ proton and neutron boson g -factors (for $B(M1)$). All results calculated were compared with previous experimental findings and were shown to be in good agreement with experimental results. In the following two sections theoretical backgrounds are presented. In Section 4, we discuss the results and finally,

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the conclusions are briefly summarized in Section 5.

2. Interacting boson model (IBM)

The structure of nuclei is generally understood on the basis of the shell model emphasizing independent particle movement and the collective model using a very restricted number of coordinates. However, there are many unanswered questions relating to the interplay of particle and collective degrees of freedom [13]. IBM was introduced in an attempt to describe in a combining way of collective properties of nuclei. This model has been the subject of many investigations and it has been extended to cover most aspects of nuclear structure. IBM originated from early ideas of Feshbach and Iachello [14] to describe the existence of collective excitation states in light nuclei in terms of interacting boson. The latter descriptions were subsequently cast into a different mathematical form to formulate a general framework about limit cases of group theory. Firstly, Arima and Iachello [15] described SU(6) limit to understand collective nuclear states as representations of SU(6), or rather U(6) group. Then, they explored in detail the vibrational [16] and rotational limit [17] and gave algebraic expressions for energies and transition matrix elements. In addition, Scholten et al. [18] presented a phenomenal analysis of the transition from vibrational limit to rotational limit (from SU(5) to SU(3)) of IBM to show this model can be used to calculate energies, electromagnetic transition probabilities etc. This provided a framework for a microscopic description of collective quadrupole states in nuclei and stimulated a large number of theoretical investigations [19]. The major new development was the realization that the bosons could be interpreted as nucleon pairs [20].

A consequence of this investigation is that, since one expected neutron and proton pairs, one is led to consider a model with two types of bosons, proton bosons and neutron bosons [21, 22]. In order to make the distinction between proton and neutron bosons more apparent, the resulting model is called IBM-2 [20]. This model was originally introduced by Otsuka and Ginocchio [23], following earlier ideas of Iachello and Talmi [21]. The IBM [14] provides a unified description of collective nuclear states in terms of a system of interacting bosons. When no distinction between the proton and neutron pairs is made, the IBM is referred to as IBM-1. However, if protons and neutrons are explicitly introduced then the model is known as IBM-2. The separation of the neutron (ν) and proton (π) bosons is the most important advantage of the IBM-2 [24], over the IBM-1, as well as calculating energies, $E2$ and $M1$ transitions between low-lying collective states [2].

The model with a single and simple Hamiltonian [2] is a valuable tool to understand the collective properties of nuclei spanning a large variety of their structures [15, 20]. The simplest version of IBM is assumed that low-lying collective states in medium and heavy even-even nuclei [25] and IBM-1 are defined as a system of N bosons. IBM-2, the neutron proton version of this model, has been applied on some light nuclei ($A \sim 80$ region) [6]. Each of bosons have angular momentum 0 and 2 [13] and proton (neutron) pairs are treated as bosons. Proton (neutron) bosons with angular momentum $L=0$ ($J^P=0^+$) are denoted by s_π (s_ν) and are called s -bosons, while proton (neutron) bosons with angular momentum $L=2$ ($J^P=2^+$) are denoted by d_π (d_ν) and are called d -bosons [13]. Low-lying collective states in medium and heavy even-even nuclei are dominated with the valence protons and the valence neutrons [25]. The shell closures for the even-even Selenium isotopes ($Z = 34$, $N = 40$ to 46) is situated away from both the proton closed shell number at 28 and neutron closed shell at 50.

In this study, two calculations for energy levels of $^{74,76,78,80}\text{Se}$ isotopes have been done by using two different parameters of the IBM-2 Hamiltonian [10, 20]. The main purpose of these processes is to investigate the importance of the Majorana operator in IBM-2 Hamiltonian. The ‘‘Majorana force’’ has some effects on the position of unnatural parity states [8, 9]. Moreover, its $B(E2)$'s for the transitions of the 2_1^+ , 0_2^+ , 2_2^+ and 4_1^+ states and $B(M1)$'s for the transitions of the 2_2^+ state have been calculated. All e_π , e_ν boson effective charges and all g_π , g_ν factors have been determined by normalizing to the experimental values for electromagnetic transition probabilities.

3. IBM-2 Hamiltonian

The IBM-2 Hamiltonian is written as [2]

$$H = H_\pi + H_\nu + V_{\pi\nu}, \quad (1)$$

where H_π , H_ν are the proton and the neutron boson Hamiltonian, and the last term $V_{\pi\nu}$ is the proton–neutron interaction. Calculations for Se isotopes have been performed by using a basic Hamiltonian, which can be written as [3]

$$H = (\varepsilon_d + \varepsilon_{d\pi})n_{d\pi} + (\varepsilon_d + \varepsilon_{d\nu})n_{d\nu} + \kappa Q_\pi \cdot Q_\nu + aM, \quad (2)$$

where ε_d denotes the binding energy for the d bosons and $\varepsilon_{d\pi,\nu}$ is deviations from this value for the proton and neutron d bosons, respectively. The total number of proton and neutron bosons is conserved in the first two terms. The third term $Q_{\pi,\nu}$ quadrupole operators (can be called

as quadrupole force [26]), which contain the deformation parameters $\chi_{\pi,\nu}$, are connected to each other by the parameter, κ . The proton-neutron quadrupole operator is defined as [1]

$$Q_{\mu}^{(2)}(\rho) = (s_{\rho}^{+}\tilde{d}_{\rho} + d_{\rho}^{+}s_{\rho})_{\mu}^{(2)} + \chi_{\rho}(d_{\rho}^{+}\tilde{d}_{\rho})_{\mu}^{(2)}, \quad \rho = \pi, \nu, \quad (3)$$

where the parameter χ_{ρ} determines the ratio of two terms. The last term aM is responsible of the symmetry energy on the nucleon-nucleon interaction [22]. M denotes the Majorana interaction operator with the strength parameter "a". This parameter determines the situation of levels of mixed proton-neutron symmetry [22], this term can also be called the Majorana force and is given by [26, 27]

$$M_{\nu\pi} = \xi_2 \left[(s_{\nu}^{+}d_{\pi}^{+} - d_{\pi}^{+}s_{\nu}^{+})^{(2)} \cdot (s_{\nu}\tilde{d}_{\pi} - \tilde{d}_{\nu}s_{\pi})^{(2)} \right] - 2 \sum_{k=1,3} \xi_k \left[(d_{\nu}^{+}d_{\pi}^{+})^{(k)} \cdot (\tilde{d}_{\nu}\tilde{d}_{\pi})^{(k)} \right]. \quad (4)$$

If this strength parameter equals zero, i.e. $a = 0$ in Eq. (2), Majorana parameters cannot be used and the non-vanishing values for $\varepsilon_{d\pi}$, $\varepsilon_{d\nu}$ are used in the input file of NPBOS [28, 29] which is termed here as Cal. 1 (Table 1 for Cal. 1). When $a \neq 0$, Majorana parameters can be used, but $\varepsilon_{d\pi}$, $\varepsilon_{d\nu}$ cannot be used in the input file of NPBOS [28, 29], as Calculation 2 (Table 2 for Cal. 2). All these calculations have been performed using the computer program NPBOS written by Otsuka and Yoshida [28].

Table 1. IBM-2 Hamiltonian parameters [3] for the first calculation. (The χ_{π} and χ_{ν} parameters have no dimension; all other parameters are given in units of MeV)

N	N_{ν}	ε_d	$\varepsilon_{d\pi}$	$\varepsilon_{d\nu}$	κ	χ_{π}	χ_{ν}
40	5	1.05	0.10	-0.06	-0.13	-0.35	-0.27
42	4	0.96	0.10	-0.075	-0.16	-0.35	-0.055
44	3	0.99	0.10	-0.10	-0.21	-0.35	0.16
46	2	0.98	0.10	-0.15	-0.24	-0.35	0.375

In Table 1, there are two parameters $\varepsilon_{d\pi}$ and $\varepsilon_{d\nu}$, the deviations from ε_d the binding energy value for the proton and neutron d bosons. The binding energy values are nearly equal, but the $\varepsilon_{d\pi}$ values are constant because of the fixed proton number while the $\varepsilon_{d\nu}$ values are linearly decreased when number of neutron boson decrease from 5 to 2. The quadrupole force coupling constant κ values for ^{78}Se , ^{80}Se are approximately double compared to ^{74}Se , ^{76}Se in both of Table 1 and Table 2. The two additional

parameters (χ_{π} and χ_{ν}) determine the quadrupole operator, all χ_{π} values are constant because of the proton number but χ_{ν} values are linearly increased in parallel with neutron numbers for both tables.

The parameter ε is the separating energy between the boson s and d states and first two ε values are roughly equal for ^{74}Se and ^{76}Se isotopes and the other second two ε values are roughly equal for ^{78}Se and ^{80}Se isotopes in Table 2. The other three parameters (κ , χ_{π} and χ_{ν}) exhibit similar attitudes with the Table 1. There exist additional parameters in the Table 2 (like $C_{L\pi}$, $C_{L\nu}$ with $L=0, 2, 4$ and ξ_k with $k=1,2,3$) that are different from the Table 1 but parameters for $\varepsilon_{d\pi}$ and $\varepsilon_{d\nu}$ do not exist in the Table 2. Little has been discussed about this in this paper after Eq. (4). The $C_{L\pi}$ and $C_{L\nu}$ are always called interaction parameters about $V_{\pi\nu}$ proton-neutron interaction term, seen in the Eq. (1). These parameters ($C_{0\pi} = C_{0\nu}$ and $C_{4\pi} = C_{4\nu}$) are totally equal for each neutron (ν) and proton (π) bosons, for angular momenta $L=0$ and $L=4$, respectively. In addition, the three Majorana parameters influence the energy of mixed-symmetric energy levels [2]. The parameters ξ_1 and ξ_3 can be equal because these terms correspond to seniority-conserving matrix elements. However, the parameter ξ_2 is a completely different from the other two parameters (ξ_1 and ξ_3) in the Majorana force. The ξ_2 parameter particularly decreases the number of the neutron d boson (shown in Table 2) because the ξ_2 term corresponds to a matrix element where the seniority in the neutron and proton sector of the space changes [26], as seen Eq. (4).

4. Results and discussion

In this section we discuss two calculations of these low-lying states to analyze Majorana parameters and compare them with the available experimental data (shown in Table 3 and also plotted in Fig. 1). The second calculated energy levels are generally in better agreement with experimental data compared to the initial energy levels calculations. This shows that Majorana parameters have important effects on the IBM-2 studies because of their influence towards improving the position at unnatural parity levels and effects on the symmetry energy of the mixed proton-neutron interaction [8, 9, 22]. As shown in Table 3, all calculated excitation energies (for Cal.-1 and Cal.-2) are almost equal to experimental values for the 2_1^+ level of all Se isotopes. Cal. 1 is more closer to experimental results than Cal. 2 for 0_1^+ , 2_1^+ , 0_2^+ and 2_2^+ levels. Maximum difference from the experimental value for Cal. 1 is obtained as 0.77 at 2_2^+ level of ^{74}Se isotope. However, it is 0.19 for Cal. 2. In addition, all calculated 4_1^+ states by

Table 2. IBM-2 Hamiltonian parameters [2] for the second calculation. (The χ_π and χ_ν parameters have no dimension; all other parameters are given in units of MeV).

N	N_ν	ϵ	κ	χ_π	χ_ν	$C_{0\pi}$	$C_{4\pi}$	$C_{0\nu}$	$C_{4\nu}$	ξ_1	ξ_2	ξ_3
40	5	0.840	-0.055	-1.20	0.14	-0.65	0.22	-0.65	0.22	-0.43	0.32	-0.28
42	4	0.805	-0.086	-1.20	0.38	-0.40	0.18	-0.40	0.18	-0.43	0.30	-0.28
44	3	0.925	-0.135	-1.20	0.65	-0.34	0.14	-0.34	0.14	-0.43	0.22	-0.28
46	2	0.931	-0.139	-1.20	0.80	-0.30	0.12	-0.30	0.12	-0.43	0.20	-0.28

Table 3. The experimental [30, 31] and calculation energy levels of ^{76}Se (MeV).

		$E(0_1^+)$	$E(2_1^+)$	$E(4_1^+)$	$E(2_2^+)$	$E(0_2^+)$
^{74}Se	Cal.-1	0.000	0.642	1.436	1.474	1.624
	Cal.-2	0.000	0.634	1.436	1.343	1.042
	Exp.	0.000	0.635	1.363	1.269	0.854
^{76}Se	Cal.-1	0.000	0.573	1.326	1.342	1.629
	Cal.-2	0.000	0.542	1.278	1.200	1.133
	Exp.	0.000	0.559	1.331	1.261	1.122
^{78}Se	Cal.-1	0.000	0.621	1.468	1.445	1.900
	Cal.-2	0.000	0.613	1.465	1.326	1.446
	Exp.	0.000	0.614	1.503	1.309	1.499
^{80}Se	Cal.-1	0.000	0.669	1.600	1.517	2.004
	Cal.-2	0.000	0.681	1.595	1.390	1.529
	Exp.	0.000	0.666	1.701	1.449	1.479

using parameters in Table 2 are also in very good agreement with experimental values for $^{74,76,78,80}\text{Se}$ isotopes as good as the first calculated results with the parameters in Table 1. One can easily see the differences between Cal. 1 and Cal. 2 for all states demonstrated in Fig. 1 and also compare all calculated values with experimental data. In general, we may say that IBM-2 is a very useful model especially for low-lying states in the $A \sim 80$ mass region.

The calculations of the electromagnetic transitions give a good test of the nuclear structural model wave functions [2]. In this section, firstly we have determined proton and neutron effective charges by normalizing to the experimental values for $B(E2; 2_1^+ \rightarrow 0_1^+)$, $B(E2; 0_2^+ \rightarrow 2_1^+)$, $B(E2; 4_1^+ \rightarrow 2_1^+)$, $B(E2; 2_2^+ \rightarrow 2_1^+)$ and $B(E2; 2_2^+ \rightarrow 0_1^+)$, because effective charges e_π and e_ν are needed in the electric quadrupole transition operator. The $E2$ transition op-

erator employed in this study is defined as [2, 6]

$$T(E2) = e_\pi Q_\pi + e_\nu Q_\nu \quad (5)$$

where Q_π and Q_ν are quadrupole operators. Boson effective charges have been fitted in order to determine the best computational $B(E2)$ transition values. After determining the values of boson effective charges, $B(E2; 2_1^+ \rightarrow 0_1^+)$, $B(E2; 0_2^+ \rightarrow 2_1^+)$, $B(E2; 4_1^+ \rightarrow 2_1^+)$, $B(E2; 2_2^+ \rightarrow 2_1^+)$, $B(E2; 2_2^+ \rightarrow 0_1^+)$ have been calculated with the code NPB-TRN [28, 29].

The calculated $B(E2)$ values for $^{74,76,78,80}\text{Se}$ isotopes are compared with experimental data in Table 4. The calculations have been done according to the second calculated energy levels because it is superior than the first. As seen in Table 4 boson effective charges have been made equal for $B(E2)$ values of each Selenium isotopes in this study. However, the values of boson effective charges can be taken as $e_\pi \approx e_\nu$ or each $B(E2; 2_1^+ \rightarrow 0_1^+)$, $B(E2; 0_2^+ \rightarrow 2_1^+)$, $B(E2; 4_1^+ \rightarrow 2_1^+)$, $B(E2; 2_2^+ \rightarrow 2_1^+)$ and $B(E2; 2_2^+ \rightarrow 0_1^+)$ [3].

IBM-2 including neutron-proton interaction provides a natural mechanism for description of $M1$ properties. The $M1$ and magnetic moment operators [3] are given by

$$T(M1, \mu) = \sqrt{\frac{3}{4\pi}} \left(\frac{1}{2} (g_\pi - g_\nu) L_\mu^{(1)} + \frac{1}{2} (g_\pi - g_\nu) \times (L_{\pi,\mu}^{(1)} - L_{\nu,\mu}^{(1)}) \right), \quad (6)$$

$$L_{\rho,\mu}^{(1)} = \sqrt{10} (d_\rho^+ \tilde{d}_\rho)^{(1)}_\mu, \quad \rho = \pi, \nu,$$

where the first term $T(M1, \mu)$ represents the total angular momentum operator $L_\mu^{(1)} = L_{\pi,\mu}^{(1)} + L_{\nu,\mu}^{(1)}$. $L_{\pi,\nu}$ is the angular momentum operators and $g_{\pi,\nu}$ are the respective boson g-factors for proton and neutron bosons. In this study, the $g_{\pi,\nu}$ values have been determined by fitting and employed throughout in the $M1$ operator. Theoretical investigations of $M1$ properties in the framework of the IBM-2 have been calculated with the code NPBTRN [28, 29] by using the $g_{\pi,\nu}$ parameters in the input file of this code.

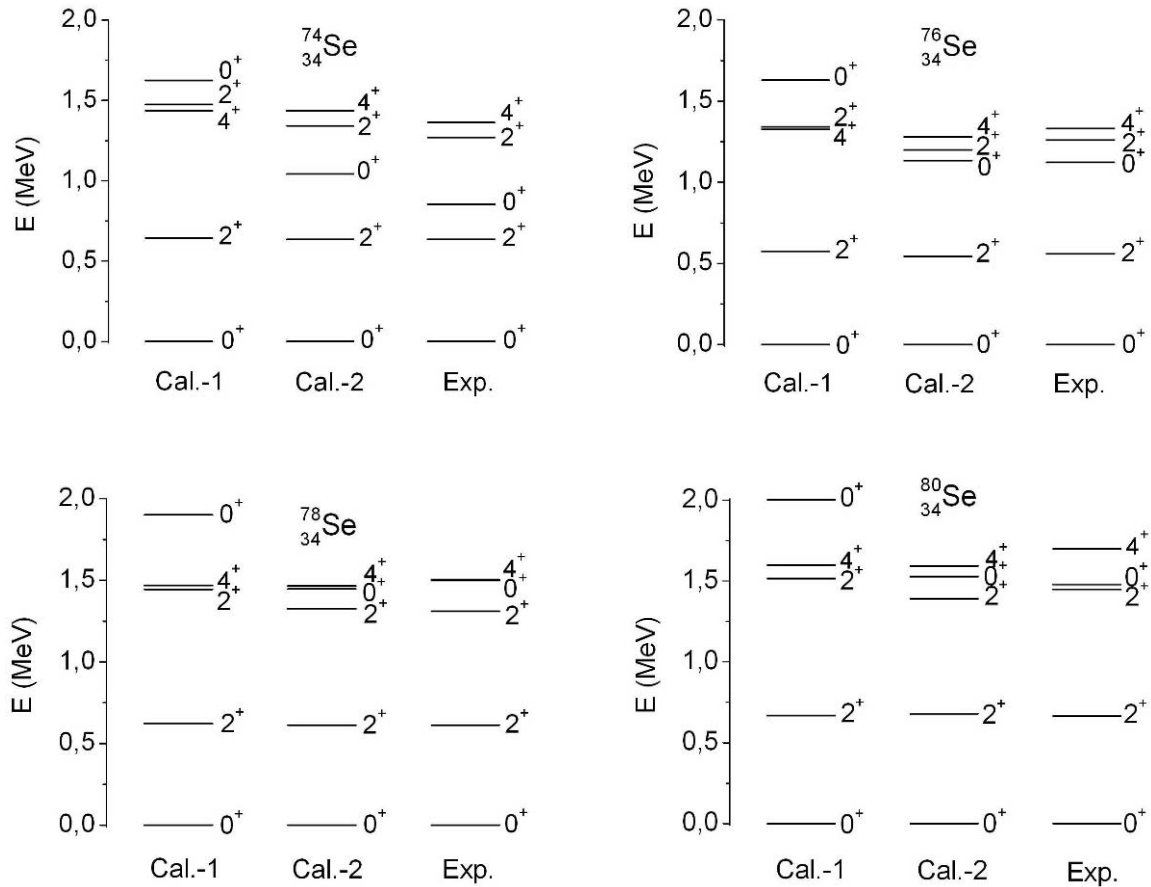


Figure 1. The experimental [30, 31] and calculated low-lying states in $^{74,76,78,80}\text{Se}$ isotopes.

The determined boson g_π and g_ν factors have been listed in Table 5. The calculated $B(M1; 2_2^+ \rightarrow 2_1^+)$ values are in good agreement with the experimental data for ^{74}Se and ^{80}Se isotopes. These values have also been calculated for ^{76}Se and ^{78}Se isotopes. As seen in the table, all calculated $B(M1)$ values are equal and the values of boson g factors are $g_\nu < g_\pi$ for all selenium isotopes.

5. Conclusions

A systematic investigation of the $^{74,76,78,80}\text{Se}$ isotopes has been carried out in the IBM-2 framework. Two sets of calculations were performed with two parameters group. The second includes Majonara parameters (ξ_1 , ξ_2 and ξ_3) and first group does not. We have computed two sets of calculations in order to asses the importance of these parameters in IBM-2 Hamiltonian. The agreement of calculated 0_1^+ , 2_1^+ , 0_2^+ , 2_2^+ and 4_1^+ levels with experimental data is good for $^{74,76,78,80}\text{Se}$ isotopes especially for the

second calculation (the strength parameter $a \neq 0$). As a result we can conclude that Majonara parameters are very important for IBM-2 calculations and investigation of even-even isotopes in the region $A \approx 80$. The calculated $B(E2; 2_1^+ \rightarrow 0_1^+)$, $B(E2; 0_2^+ \rightarrow 2_1^+)$, $B(E2; 4_1^+ \rightarrow 2_1^+)$, $B(E2; 2_2^+ \rightarrow 2_1^+)$, $B(E2; 2_2^+ \rightarrow 0_1^+)$ and $B(M1; 2_2^+ \rightarrow 2_1^+)$ values for the $^{74,76,78,80}\text{Se}$ isotopes are also in good agreement with the experimental data. Our studies indicate that IBM-2 can account for the low-lying band structure of these isotopes.

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Table 4. Effective boson charges, experiment [30, 31] and calculated $B(E2)$'s in $10^3 e^2 b^2$, $10^3 e^2 fm^4$.

		e_π	e_ν	Cal.	Exp.
^{74}Se	$B(E2; 2_1^+ \rightarrow 0_1^+)$	0.265	0.265	782	775
	$B(E2; 0_2^+ \rightarrow 2_1^+)$	0.335	0.335	1504	1500
	$B(E2; 4_1^+ \rightarrow 2_1^+)$	0.285	0.285	1493	1485
	$B(E2; 2_2^+ \rightarrow 2_1^+)$	0.230	0.230	899	884
	$B(E2; 2_2^+ \rightarrow 0_1^+)$	0.340	0.340	5	15
^{76}Se	$B(E2; 2_1^+ \rightarrow 0_1^+)$	0.280	0.280	845	844
	$B(E2; 0_2^+ \rightarrow 2_1^+)$	0.320	0.320	900	906
	$B(E2; 4_1^+ \rightarrow 2_1^+)$	0.285	0.285	1343	1361
	$B(E2; 2_2^+ \rightarrow 2_1^+)$	0.235	0.235	824	823
	$B(E2; 2_2^+ \rightarrow 0_1^+)$	0.340	0.340	9	23
^{78}Se	$B(E2; 2_1^+ \rightarrow 0_1^+)$	0.270	0.270	678	670
	$B(E2; 0_2^+ \rightarrow 2_1^+)$	0.320	0.320	579	578
	$B(E2; 4_1^+ \rightarrow 2_1^+)$	0.270	0.270	986	973
	$B(E2; 2_2^+ \rightarrow 2_1^+)$	0.195	0.195	493	439
	$B(E2; 2_2^+ \rightarrow 0_1^+)$	0.340	0.340	5	15
^{80}Se	$B(E2; 2_1^+ \rightarrow 0_1^+)$	0.270	0.270	508	504
	$B(E2; 0_2^+ \rightarrow 2_1^+)$	0.195	0.195	174	140
	$B(E2; 4_1^+ \rightarrow 2_1^+)$	0.270	0.270	720	722
	$B(E2; 2_2^+ \rightarrow 2_1^+)$	0.195	0.195	384	384
	$B(E2; 2_2^+ \rightarrow 0_1^+)$	0.340	0.340	0.3	27

Table 5. The $g_{\pi,\nu}$ boson g-factors, experiment [30, 31] and calculated $B(M1)$'s in μ_N^2 .

		g_ν	g_π	Cal.	Exp.
^{74}Se	$B(M1; 2_2^+ \rightarrow 2_1^+)$	0.28	0.75	0.0007	0.00072
^{76}Se	$B(M1; 2_2^+ \rightarrow 2_1^+)$	0.36	0.64	0.0007	-
^{78}Se	$B(M1; 2_2^+ \rightarrow 2_1^+)$	0.44	0.56	0.0007	-
^{80}Se	$B(M1; 2_2^+ \rightarrow 2_1^+)$	0.46	0.54	0.0007	0.00071

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