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Corrigendum

Corrigendum to "Generalized contractions on partial metric spaces" [Topology Appl. 157 (2010) 2778–2785]

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ABSTRACT

We correct the proof of Theorem 1 in the paper in the title.

The following correction for the paper [1] should be noted.

In the proof of Theorem 1, the authors prove that

$$p^s(x_n, x_{n+1}) \leqslant 4\phi^n\big(p(x_1, x_0)\big)$$

and from this inequality they obtain

$$\lim_{n\to\infty} p^s(x_n, x_{n+1}) = 0. \tag{0.1}$$

In order to prove that $\{x_n\}$ is a Cauchy sequence in the metric space (X, p^s) they use the following inequality

$$\begin{aligned} p^s(x_{n+k}, x_n) &\leqslant p^s(x_{n+k}, x_{n+k-1}) + \cdots + p^s(x_{n+1}, x_n) \\ &\leqslant 4\phi^{n+k-1}\big(p(x_1, x_0)\big) + \cdots + 4\phi^n\big(p(x_1, x_0)\big) \end{aligned} \tag{0.2}$$

and (0.1).

This argument is false as it is proved with the following example. Consider $(\mathbb{R}, d)$, where d is the usual metric in $\mathbb{R}$ and $x_n = \sum_{i=1}^n \frac{1}{i}$. Obviously,

$$\lim_{n\to\infty} d(x_{n+1}, x_n) = \lim_{n\to\infty} |x_{n+1} - x_n| = \lim_{n\to\infty} \frac{1}{n+1} = 0.$$

On the other hand, the sequence $\{x_n\}$ is not a Cauchy sequence because it is not convergent.

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The argument is correct when (X, d) is an ultrametric space.

In order to obtain the conclusion of Theorem 1 we must impose some conditions to the function $\phi : [0, \infty) \to [0, \infty)$.

Suppose that $\phi : [0, \infty) \to [0, \infty)$ is a nondecreasing function and such that $\sum_{n=0}^{\infty} \phi^n(t)$ is a convergent series for any $t > 0$. These functions are known in the literature as (c)-comparison functions.

It is easily proved that if ϕ is a (c)-comparison function then $\phi(t) < t$ for any $t > 0$.

Then Theorem 1 can be replaced by the following theorem.

Theorem 1. *Let (X, p) be a complete partial metric space and $F : X \to X$ be a map such that*

$$p(Fx, Fy) \leqslant \phi\left(\max\left\{p(x, y), p(x, Fx), p(y, Fy), \frac{1}{2}\left[p(x, Fy) + p(y, Fx)\right]\right\}\right)$$

for all $x, y \in X$ where $\phi : [0, \infty) \to [0, \infty)$ is a (c)-comparison function. Then F has a unique fixed point.

Proof. Following the lines of the proof of Theorem 1 of [1], from (0.2) we obtain

$$p^s(x_{n+k}, x_n) \leqslant 4 \sum_{p=n}^{n+k-1} \phi^p\left(p(x_1, x_0)\right)$$

and, as $\sum_{p=0}^{\infty} \phi^p(p(x_1, x_0))$ is convergent, from the last inequality, using Cauchy's criterium for convergent series, we obtain that $\{x_n\}$ is a Cauchy sequence. □

On the other hand, the authors of [1] use the continuity of ϕ in order to prove that $p(x, Fx) = 0$.

More precisely, they obtain the following inequality

$$p(x, Fx) \leqslant p(x, x_{n+1}) + \phi\left(\max\left\{p(x, x_n), p(x, Fx), p(x_n, x_{n+1}), \frac{1}{2}\left[p(x, x_{n+1}) + p(x_n, x) + p(x, Fx)\right]\right\}\right) \tag{0.3}$$

and letting $n \to \infty$ and using the continuity of ϕ

$$p(x, Fx) \leqslant \phi\left(p(x, Fx)\right)$$

and from this fact the authors obtain $p(x, Fx) = 0$.

The following argument proves that the continuity of ϕ is not necessary in order to obtain the same conclusion.

As ϕ is a (c)-comparison function, $\phi(t) < t$ for $t > 0$.

Now, suppose that $p(x, Fx) > 0$, as $\lim_{n\to\infty} p(x_{n+1}, x_n) = 0$ and $\lim_{n\to\infty} p(x_n, x) = 0$, there exists $n_0 \in \mathbb{N}$ such that for $n > n_0$,

$$p(x_{n+1}, x_n) < \frac{1}{3} p(x, Fx) \tag{0.4}$$

and there exists $n_1 \in \mathbb{N}$ such that for $n > n_1$,

$$p(x_n, x) < \frac{1}{3} p(x, Fx). \tag{0.5}$$

If we take $n > \max\{n_0, n_1\}$ then, by (0.4), (0.5) and triangular inequality, we have

$$\begin{aligned}
\frac{1}{2}\left[p(x_n, Fx) + p(x, Fx_n)\right] &\leqslant \frac{1}{2}\left[p(x_n, x) + p(x, Fx) - p(x, x) + p(x, Fx_n)\right] \\
&\leqslant \frac{1}{2}\left[\frac{1}{3} p(x, Fx) + p(x, Fx) + \frac{1}{3} p(x, Fx)\right] \\
&= \frac{5}{6} p(x, Fx).
\end{aligned} \tag{0.6}$$

Now for $n > \max\{n_0, n_1\}$, by (0.4), (0.5) and (0.6), we have

$$\begin{aligned}
p(x_{n+1}, Fx) &= p(Fx_n, Fx) \\
&\leqslant \phi\left(\max\left\{p(x_n, x), p(x_n, Fx_n), p(x, Fx), \frac{1}{2}\left[p(x_n, Fx) + p(x, Fx_n)\right]\right\}\right) \\
&\leqslant \phi\left(p(x, Fx)\right).
\end{aligned}$$

Letting $n \to \infty$ in the last inequality, we have $p(x, Fx) \leqslant \phi(p(x, Fx))$, which is a contradiction. Thus $p(x, Fx) = 0$. Therefore, Theorem 1 is an improvement of Theorem 1 of [1].

Finally, we present an example of a discontinuous (c)-comparison function.

Let $\phi : R_+ \to R_+$ be the function defined by

$$\phi(t) = \begin{cases} \frac{1}{4}t, & 0 \leqslant t < 1, \\ \frac{t}{t+1}, & 1 \leqslant t. \end{cases}$$

It is easily seen that ϕ is a (c)-comparison function and it is not continuous at $t_0 = 1$.

References

[1] I. Altun, F. Sola, H. Simsek, Generalized contractions on partial metric spaces, Topology Appl. 157 (2010) 2778–2785.