Some results on the extreme distributions of surplus process with nonhomogeneous claim occurrences

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Abstract

In this paper; survival (non-ruin) probability after a definite time period of an insurance company is studied in a discrete time model based on non-homogenous claim occurrences. Furthermore, distributions of the minimum and maximum levels of surplus in compound binomial risk model with non-homogeneous claim occurrences are obtained and some of its characteristics are given.

1. Introduction

Surplus process (or risk process) is a model of accumulation of insurer's capital and the premium incomes during the periods. So, the surplus process is one of the most important stochastic process for an insurance company which can be defined as discrete or continuous time in actuarial risk theory. Ruin occurs when surplus is zero or negative value which means that the total claim amounts equal or exceed the surplus at a certain time for insurance companies. Furthermore, the estimation of the surplus at a certain time is essential for the insurance companies due to their future investment strategies and actions to be taken just before ruin. In this regard, it is vital importance for controlling the maximum and minimum level of the surplus and its related quantities.

The compound binomial model has been first proposed by Gerber (1988 a). Distributional properties of some actuarial quantities associated with compound binomial model

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have been studied in De Vylder and Goovaerts (1984,1988), Shiu (1989), Willmot (1993), Dickson (1994) and De Vylder and Marceau (1996). The compound binomial model, as a discrete time version of the classical compound Poisson model of risk theory has been widely studied in the recent literature (see, e.g. Yuen and Guo (2001), Cosette and Marceau (2000), Cossette et al. (2003, 2004 and 2006), Liu and Zhao (2007), Tuncel and Tank (2014)).

In classical risk model, the number of the claims is assumed to have a Poisson process $\{N_t : t \geq 0\}$ with parameter λ and the claim amounts $Y_1, Y_2, \ldots$ are non-negative, independent and identically distributed random variables with same distribution function. The total claim amounts process $\{S_t : t \geq 0\}$ is a compound Poisson process with parameter λ where $S_t = \sum_{i=1}^{N_t} Y_i$ designates the total claim amounts up to time t. In this regards, surplus of the insurers at time t can be defined as follows

(1.1) $\qquad U_t = u + ct - S_t$

where $U_0 = u$ is the amount of initial reserve, the premium income is c per each period, and Y_i is the eventual claim amount in period i. For simplicity, throughout the paper we assume that $c = 1$.

Let I_i be a binary random variable representing the claim occurrence. That is $I_i = 1$ if a claim occurs in period i and $I_i = 0$, otherwise. For $i \geq 1$, define $Y_i = I_i X_i$, where the random variable I_i and the individual claim amount random variable X_i are independent in each time period. The random variable X_i is strictly positive and $\{X_i, i \geq 1\}$ forms a sequence of iid random variables with probability mass function (p.m.f.) $f(x) = P\{X = x\}$. Under this assumptions, the process (1.1) can be rewritten as

(1.2) $\qquad U_t = u + t - \sum_{i=1}^{t} I_i X_i,$

where u is non negative integer, N_n is the number of claims up to time n and X_i is the amount of ith claim. It is assumed that X_i random variables are independent and identically distributed (i.i.d.) and independent of the claim number process. Ruin of insurer's occurs when $U_t \leq 0$ for some $t \geq 1$. The random time to ruin is defined as

(1.3) $\qquad T = \inf \{t > 0 : U_t \leq 0\}.$

by Gerber (1988). Thus, ultimate ruin probability and survival probability can be defined as

$\qquad \psi(u) = P(T < \infty | U_o = u)$

$\qquad \phi(u) = 1 - \psi(u)$

respectively. Similarly, ruin probability and survival probability in finite time can be defined as

$\qquad \psi(u, n) = P(T \leq n | U_o = u)$

$\qquad \phi(u, n) = 1 - \psi(u, n)$

respectively.

Let the random indicators $I_1, I_2, \ldots$ be independent with $p = P\{I_i = 1\}$, then the model (1.2) is called the compound binomial model and

$\qquad P\{N_n = k\} = C_n^k p^k (1-p)^{n-k}, k = 0, 1, ..., n.$

In here, distribution of N_n is classical binomial distribution. Tuncel and Tank (2014) suggested a recursive formula when the claim occurrences probabilities are non-homogeneous such as $p_i = P\{I_i = 1\}$.

Let M_n and K_n denote respectively the maximum and minimum levels of the surplus process up to period n,

$$M_n = \max_{1 \le t \le n} U_t \quad, \quad K_n = \min_{1 \le t \le n} U_t.$$

These quantities may be useful tools on possible future investment or borrowing strategies for their consistent financial statement in an insurance company. Recursive equations are given for both marginal and joint distributions of the M_n and K_n values under the condition that insurance company survives at time n for homogenous case by Eryilmaz et.al. (2012).

The remainder of the present paper is organized as follows: Section 2 presents recursive equations to compute marginal and joint distributions of M_n and K_n under the condition $T > n$. Section 3 gives means and variances of M_n and K_n for zero truncated geometric claim size distribution. Finally, discussions are given in Section 4.

2. Distributions of Extremes Surplus Process

For $u = 1, 2, \dots$ and $n \ge 0$, define

$$\phi^{(1,n)}(u) = P_u(T > n)$$

$$\theta^{(1,n)}(u; k) = P_u(M_n \le k, T > n)$$

$$\gamma^{(1,n)}(u; k) = P_u(K_n \ge k, T > n)$$

where

$$P_u(T > n) = \phi^{(1,n)}(u)$$

$$= \begin{cases} 1 & ,n = 0 \\ \sum_{t=1}^{n} p_t \prod_{i=1}^{t-1} q_i \sum_{x=1}^{u+t-1} f(x) P_{u+t-x}(T^{(t+1,n)} > n - t) \\ \quad + \left(1 - \sum_{t=1}^{n} p_t \prod_{i=1}^{t-1} q_i\right) & ,n > 0 \end{cases}$$

and k is a positive threshold which can be also considered as an upper barrier for surviving of the insurance company. In here, $T^{(t+1,n)}$ and $\phi^{(1,n)}(u)$ represents ruin time after the t-th period and non-ruin probability when the claim occurrences have nonhomogeneous probabilities respectively (Tuncel and Tank(2014)).

2.1. Theorem. *For* $u = 1, 2, \dots$

(2.1) $\qquad P_u(M_n \le k \,|T > n) = \dfrac{\theta^{(1,n)}(u; k)}{\phi^{(1,n)}(u)}$

where

  a. *If* $k \ge u + n$ *and* $n \ge 0$ *then*

  $\theta^{(1,n)}(u; k) = \phi^{(1,n)}(u)$

  b. *If* $u \le k < u + n$ *and* $n \ge 0$ *then*

(2.2) $\qquad \theta^{(1,n)}(u; k) = \sum_{t=1}^{k-u+1} p_t \prod_{i=1}^{t-1} q_i \sum_{x=\max(1,u+t-k)}^{u+t-1} f(x) \theta^{(t+1,n-t)}(u + t - x; k)$

Proof. It is clear that $P_u(M_n \le k \mid T > n) = 1$ for $k \ge u + n$. So $\theta^{(1,n)}(u; k) = \phi^{(1,n)}(u)$ is trivial.

By conditioning on W_1, the time of the first claim, for $u \le k < u + n$ and $n \ge 0$ then

$$P_u\left(M_n \le k, T > n\right) = \sum_{t=1}^{\infty} P_u\left(U_1 \le k, ..., U_n \le k \,|W_1 = t\right) P\left(W_1 = t\right)$$

where $P(W_1 = t) = \prod_{i=1}^{t-1} q_i p_t$.

If $t \le n$ then

$$P_u\left(U_1 = u + 1, ..., U_{t-1} = u + t - 1 | W_1 = t\right) = 1$$

and

$$(2.3) \quad P_u\left(U_1 \le k, ..., U_n \le k \,|W_1 = t\right) = P_u\left(U_t \le k, ..., U_n \le k, T^{(t+1,n)} > n - t\right)$$

for $t \le k - u + 1$. Noting that $U_t > 0$ for $t \le n$ since the ruin occurs after period n and than by conditioning on the value of the first claim one obtains

$$P_u\left(U_t \le k, ..., U_n \le k, T > n - t | W_1 = t\right)$$

$$= \sum_{x=1}^{\infty} P_u\left(u + t - X > 0, X = x, M_{n-t}^{(t+1,n)} \le k, T^{(t+1,n)} > n - t\right)$$

$$(2.4) \quad = \sum_{x=\max(1,u+t-k)}^{u+t-1} f(x) P_{u+t-x}\left(M_{n-t}^{(t+1,n)} \le k, T^{(t+1,n)} > n - t\right)$$

for $t \le k - u + 1$. For $t > n$, $P(M_n = u + n) = 1$. Thus $P(M_n \le k, T > n) = 0$, if $k < u + n$ and $t > n$. Thus, for $u \le k < u + n$,

$$\theta^{(1,n)}\left(u; k\right) = \sum_{t=1}^{k-u+1} p_t \prod_{i=1}^{t-1} q_i \sum_{x=\max(1,u+t-k)}^{u+t-1} f(x) \theta^{(t+1,n-t)}\left(u + t - x; k\right)$$

can be obtained by using (2.3) and (2.4). Hence the proof is completed. $\qquad\square$

Expansion of (2.2), which is recursive formula given in Theorem 2.1, as in follows:

- For $n = 1$ and $u \le k < u + 1$

$$\theta^{(1,1)}\left(u; k\right) = \frac{p_1 \sum\limits_{x=u+1-k}^{u} f(x)}{\phi^{(1,1)}\left(u\right)}$$

- For $n = 2$ and $u \le k < u + 2$

$$(2.5) \quad \theta^{(1,2)}\left(u; k\right) = \begin{cases} \frac{1}{\phi^{(1,2)}(u)} \left[A_1\right] & , k = u \\ \frac{1}{\phi^{(1,2)}(u)} \left[A_2\right] & , k = u + 1 \end{cases}$$

where

$$A_1 = p_1 p_2 \sum_{x=\max(1,u+1-k)}^{u} f(x) \sum_{y=\max(1,u+2-k-x)}^{u+1-x} f(y) + p_1 q_2 \sum_{x=\max(1,u+2-k)}^{u} f(x).$$

$$A_2 = p_1 p_2 \sum_{x=\max(1,u+1-k)}^{u} f(x) \sum_{y=\max(1,u+2-k-x)}^{u+1-x} f(y)$$

$$+ p_1 q_2 \sum_{x=\max(1,u+2-k)}^{u} f(x) + q_1 p_2 \sum_{x=\max(1,u+2-k)}^{u+1} f(x).$$

- For $n = 3$ and $u \leq k < u + 3$

$$(2.6) \qquad \theta^{(1,3)}(u;k) = \begin{cases} \frac{1}{\phi^{(1,3)}(u)}\left[A_3\right] & , k = u \\ \frac{1}{\phi^{(1,3)}(u)}\left[A_4\right] & , k = u+1 \\ \frac{1}{\phi^{(1,3)}(u)}\left[A_5\right] & , k = u+2 \end{cases}$$

where

$$A_3 = p_1 p_2 p_3 \sum_{x=\max(1,u+1-k)}^{u} f(x) \sum_{y=\max(1,u+2-k-x)}^{u+1-x} f(y) \sum_{z=\max(1,u+3-k-x-y)}^{u+2-x-y} f(z)$$

$$+ p_1 p_2 q_3 \sum_{x=\max(1,u+1-k)}^{u} f(x) \sum_{y=\max(1,u+3-k-x)}^{u+1-x} f(y)$$

$$+ p_1 q_2 p_3 \sum_{x=\max(1,u+2-k)}^{u} f(x) \sum_{y=\max(1,u+2-k-x)}^{u+2-x} f(y)$$

$$+ p_1 q_2 q_3 \sum_{x=\max(1,u+3-k)}^{u} f(x)$$

$$A_4 = p_1 p_2 p_3 \sum_{x=\max(1,u+1-k)}^{u} f(x) \sum_{y=\max(1,u+2-k-x)}^{u+1-x} f(y) \sum_{z=\max(1,u+3-k-x-y)}^{u+2-x-y} f(z)$$

$$+ p_1 p_2 q_3 \sum_{x=\max(1,u+1-k)}^{u} f(x) \sum_{y=\max(1,u+3-k-x)}^{u+1-x} f(y)$$

$$+ p_1 q_2 q_3 \sum_{x=\max(1,u+3-k)}^{u} f(x) + q_1 p_2 q_3 \sum_{x=\max(1,u+3-k)}^{u+1} f(x)$$

$$+ p_1 q_2 p_3 \sum_{x=\max(1,u+2-k)}^{u} f(x) \sum_{y=\max(1,u+2-k-x)}^{u+2-x} f(y)$$

$$+ q_1 p_2 p_3 \sum_{x=\max(1,u+2-k)}^{u+1} f(x) \sum_{y=\max(1,u+3-k-x)}^{u+2-x} f(y)$$

$$A_5 = p_1 p_2 p_3 \sum_{x=\max(1,u+1-k)}^{u} f(x) \sum_{y=\max(1,u+2-k-x)}^{u+1-x} f(y) \sum_{z=\max(1,u+3-k-x-y)}^{u+2-x-y} f(z)$$

$$+ p_1 q_2 q_3 \sum_{x=\max(1,u+3-k)}^{u} f(x) + q_1 p_2 q_3 \sum_{x=\max(1,u+2-k)}^{u+2} f(x)$$

$$+ p_1 q_2 p_3 \sum_{x=\max(1,u+2-k)}^{u} f(x) \sum_{y=\max(1,u+2-k-x)}^{u+2-x} f(y)$$

$$+ q_1 p_2 p_3 \sum_{x=\max(1,u+2-k)}^{u+1} f(x) \sum_{y=\max(1,u+3-k-x)}^{u+2-x} f(y)$$

$$+ p_1 p_2 q_3 \sum_{x=\max(1,u+1-k)}^{u} f(x) \sum_{y=\max(1,u+3-k-x)}^{u+1-x} f(y).$$

2.2. Theorem. *For $u = 1, 2, \ldots$*

$$(2.7) \qquad P_u\left(K_n \geq k \mid T > n\right) = \frac{\gamma^{(1,n)}\left(u; k\right)}{\phi^{(1,n)}\left(u\right)}$$

where

    a. *If $k \leq n$ and $n = 0$ then*

$$\gamma^{(1,n)}\left(u; k\right) = 1$$

    b. *If $k \leq u + 1$ and $n \geq 0$ then,*

$$\gamma^{(1,n)}\left(u; k\right) = \sum_{t=\max(1,k-u+1)}^{n} p_t \prod_{i=1}^{t-1} q_i \sum_{x=1}^{u+t-k} f(x)\gamma^{(t+1,n-t)}\left(u+t-x; k\right) + \sum_{t=n+1}^{\infty} p_t \prod_{i=1}^{t-1} q_i$$

Proof. The proof is clear for $k \leq n$ and $n = 0$.

By conditioning on the time of first claim, for $k \leq u + 1$

$$(2.8) \qquad P_u\left(K_n \geq k \mid T > n\right) = \sum_{t=1}^{\infty} P_u\left(U_1 \geq k, \ldots, U_n \geq k | W_1 = t\right) P\left(W_1 = t\right)$$

where $P(W_1 = t) = \prod_{i=1}^{t-1} q_i p_t$. If $t \leq n$ and $k \leq u + 1$ then

$$(2.9) \qquad P_u\left(U_1 \geq k, \ldots, U_{t-1} \geq k \mid W_1 = t\right) = 1$$

and

$$P_u\left(U_1 \geq k, \ldots, U_{t-1} \geq k, T > n | W_1 = t\right) = P_u\left(U_t \geq k, \ldots, U_n \geq k, T^{(t+1,n)} > n - t | W_1 = t\right)$$

If $t > n$ and $k \leq u + 1$ then

$$(2.10) \qquad P_u\left(U_1 \geq k, \ldots, U_n \geq k \mid W_1 = t\right) = 1.$$

By conditioning on the time of first claim, for $t \leq n$ and $k \leq u + 1$

(2.11)

$$P_u\left(U_t \geq k, \ldots, U_n \geq k, T > n - t \mid W_1 = t\right) = \sum_{x=1}^{u+t-k} P_{u+t-x}\left(K_{n-t}^{(t+1,n)} \geq k, T^{(t+1,n)} > n - t\right)$$

for $t \leq n$ and $k \leq u + 1$. Hence,

$$\gamma^{(1,n)}\left(u; k\right) = \sum_{t=\max(1,k-u+1)}^{n} p_t \prod_{i=1}^{t-1} q_i \sum_{x=1}^{u+t-k} f(x)\gamma^{(t+1,n-t)}\left(u+t-x; k\right) + \sum_{t=n+1}^{\infty} p_t \prod_{i=1}^{t-1} q_i$$

can be obtained by using (2.9),(2.10) and (2.11). Thus the proof is completed.   □

Expansion of (2.7) for $n = 1, 2, 3$, which is also recursive formula given in Theorem 2.2, as in follows:

    • For $n = 1$

$$(2.12) \qquad \gamma^{(1,1)}\left(u; k\right) = \begin{cases} 1 & , k = u + 1 \\ \frac{1}{\phi^{(1,1)}(u)}\left[a_1\right] & , k < u + 1 \end{cases}$$

    where

$$a_1 = p_1 \sum_{x=1}^{u+1-k} f(x) + q_1$$

- For $n = 2$

(2.13) $\quad \gamma^{(1,2)}(u;k) = \begin{cases} \frac{1}{\phi^{(1,2)}(u)}\left[a_2\right] & , k = u+1 \\ \frac{1}{\phi^{(1,2)}(u)}\left[a_3\right] & , k < u+1 \end{cases}$

where

$$a_2 = q_1 p_2 \sum_{x=1}^{u+2-k} f(x) + q_1 q_2$$

and

$$a_3 = p_1 p_2 \sum_{x=1}^{u+1-k} f(x) \sum_{y=1}^{u+2-x-k} f(y) + p_1 q_2 \sum_{x=1}^{u+1-k} f(x) + q_1 p_2 \sum_{x=1}^{u+2-k} f(x) + q_1 q_2$$

- For $n = 3$

(2.14) $\quad \gamma^{(1,3)}(u;k) = \begin{cases} \frac{1}{\phi^{(1,3)}(u)}\left[a_4\right] & , k = u+1 \\ \frac{1}{\phi^{(1,3)}(u)}\left[a_5\right] & , k < u+1 \end{cases}$

where

$$a_4 = q_1 q_2 p_3 \sum_{y=1}^{u+3-k-x} f(y) + q_1 p_2 p_3 \sum_{x=1}^{u+2-k} f(x) \sum_{y=1}^{u+3-k-x} f(y) + q_1 q_2 q_3 + q_1 p_2 q_3 \sum_{y=1}^{u+2-k} f(y)$$

and

$$a_5 = p_1 p_2 p_3 \sum_{x=1}^{u+1-k} f(x) \sum_{y=1}^{u+2-k-x} f(y) \sum_{z=1}^{u+3-k-x-y} f(z)$$

$$+ p_1 q_2 q_3 \sum_{x=1}^{u+1-k} f(x) + p_1 q_2 p_3 \sum_{x=1}^{u+1-k} f(x) \sum_{y=1}^{u+3-k-x} f(y)$$

$$+ p_1 p_2 q_3 \sum_{x=1}^{u+1-k} f(x) \sum_{y=1}^{u+2-k-x} f(y) + q_1 q_2 p_3 \sum_{y=1}^{u+3-k} f(y)$$

$$+ q_1 p_2 q_3 \sum_{x=1}^{u+2-k} f(x) + q_1 p_2 p_3 \sum_{x=1}^{u+2-k} f(x) \sum_{y=1}^{u+3-k-x} f(y)$$

$$+ q_1 q_2 q_3$$

3. Case study

As mentioned before, insurance company may face nonhomogeneous claim occurrences probabilities in different periods (e.g. month). For this reason, in this section four different cases are considered for different values of α and u in finite time model and given in Table 1 where $P(I_i = 1) = p_i$ for $i = 1, ..., 12$.

Table 1. Claim occurrence probabilities

Case1	Case2
$p_i = 0.01 * i$, $i = 1, ..., 12$	$p_i = 0.02 * i$, $i = 1, ..., 12$
Case3	Case4
$p_i = 0.03 * i$, $i = 1, ..., 12$	$p_i = 0.04 * i$, $i = 1, ..., 12$

Let claim size distribution be geometric with the following cdf and pmf

(3.1) $F(x) = 1 - \alpha^x$, $x = 1, 2, ...$

(3.2) $f(x) = (1 - \alpha)\alpha^{x-1}$, $x = 1, 2, ...$

respectively. It is clear that

(3.3) $E(X) = \dfrac{1}{1 - \alpha}$, $0 < \alpha < 1$.

According to the cases which are given in Table 1, we obtained expected values and variances of M_n and K_n for the cases, which are given in Table 2 where the claim amount distribution as in (3.2) and $\mu_1 = E(M_n \mid T > n)$, $\sigma_1^2 = Var(M_n \mid T > n)$, $\mu_2 = E(K_n \mid T > n)$ and $\sigma_2^2 = Var(K_n \mid T > n)$.

Table 2. Expected values and variances of M_n and K_n for $\alpha = 9/10$ in cases

u	Cases	μ_1	σ_1^2	μ_2	σ_2^2
4	Case1	14.7363	3.6557	4.9425	0.1632
	Case2	13.7613	5.0934	4.8865	0.3170
	Case3	12.9846	5.4676	4.8333	0.4561
	Case4	12.3376	5.3540	4.7801	0.5920
8	Case1	18.3975	5.2155	8.8418	0.8073
	Case2	17.2249	6.9138	8.6875	1.5438
	Case3	16.3182	7.1493	8.5432	2.2048
	Case4	15.6258	6.7602	8.4036	2.8110

We sketch the graphics of cumulative distribution function of M_n and K_n for cases in Figure 1 and Figure 2 respectively. In the Figures 1 and 2 solid line represents for $u = 4$ and dashed line represents $u = 8$.
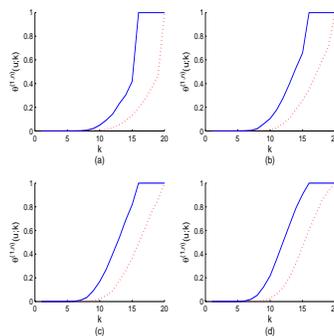
Figure 1. Cumulative Distribution function of M_n given $T > n$
a) Case 1   b) Case 2
c) Case 3   d) Case 4
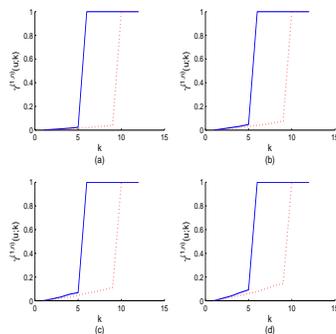
Figure 2. Cumulative Distribution function of K_n given $T > n$
a) Case 1   b) Case 2
c) Case 3   d) Case 4

4. Conclusions

This study presents some characteristical results and distributions of maximum and minimum levels of surplus in compound binomial risk model with nonhomogeneous claim occurrences by different cases which have critical importance for an insurance company. This study may also lead to future studies with stochastic premium income in continuous time model.

Acknowledgments

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