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Solving the Optimal Power Flow Quadratic Cost Functions using Vortex Search Algorithm

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Abstract: This study proposes solving the constraint optimal power flow problem (OPF) by using vortex search algorithm (VSA). VSA is inspired by natural vortexes. Piecewise quadratic fuel cost and quadratic cost curve with valve point loadings test cases are solved on IEEE-30 bus test system by taking into consideration the system constraints such as generation limits, voltages at nodes, tap settings. The obtained test results show that VSA gives better results than any other algorithms which are used to solve the OPF problem.

Keywords: Optimal power flow, heuristic programming, electric power systems, algorithms, optimization problems, optimal search techniques

1. INTRODUCTION

In the early years when the electricity use began, the number of subscribers was few, so the number of loads on the power system was limited. Thanks to rapid development of technology, the number of electric appliances and the number of people using these appliances increased. As a result of this increase, loads on power systems began to grow, which made the electricity networks bigger and more complicated. As the networks grew day by day, difficulties arose in planning and operating. Various methods were developed to cope with these difficulties. The most efficient and most widely used of these methods was optimal power flow (OPF) in 1962, published by Carpentier (Carpentier, 1962).

The starting point of OPF is power flow calculations. OPF's difference from the power flow is that the calculations are made while the system limitations are also taken into consideration in these calculations. OPF is trying to optimize the results that will arise in various situations of the system.

Following the development of OPF, various solution techniques and algorithms were used to achieve better results. We can divide these techniques into two main groups: These are classical numerical analysis methods and heuristic algorithms that have emerged in recent years. Examples of classic solution methods in the literature are: Gradient adjustment algorithm (Hermann et. al., 1968), quadratic and linear programming (Nabona et. al., 1973), linear programming based (Srijib et. al., 1992), interior-point (Torres et al, 1998) and quadratic programming (Wibowo et. al., 2013). These methods can be considered successful in obtaining solutions. However, these techniques are inadequate due to their lack of finding optimum results in large scale non-linear problems, excessive dependence on initial values, solving only certain objective functions and solution search takes a lot of time.

Heuristic algorithms have begun to be used to solve the OPF problem in order to overcome these deficiencies of classical methods and achieve more optimal results. Example studies where heuristic algorithms are used to solve OPF problem are: Improved genetic (Lai et. al., 1997), particle swarm optimization (Abido, 2002), simulated annealing (Sepulveda et. al., 2003), ant colony (Bouktir et. al., 2005), chaotic krill herd (Mukherjee et. al., 2005), artificial bee colony (Sumpavakup et. al., 2010), glowworm swarm optimization (Reddy et. al., 2016).

The aim of this study is to solve the piecewise quadratic fuel cost and quadratic cost curve with valve point loadings objective functions using vortex search algorithm (VSA), which is one the heuristic algorithms.

This paper consists of 5 sections. Section 2 provides information on OPF. Section 3 explains the VSA's working logic which is used to solve the OPF problem. In section 4 the obtained results are given and in section 5 results are evaluated.

2. OPTIMAL POWER FLOW

In a few words OPF is a non-linear optimization problem. The solution of the OPF problem is to optimize the chosen objective functions by taking into account the constraints of the system being studied. While optimizing the objective function, the equality and inequality constraints defined in the problem have to be verified (Abido, 2002). The mathematical representation of the OPF is as follows:

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Minimize f(x, u)

Subject to: g(x, u) (1)

h(x, u) ≤ 0

Where, f represents the objective function to be minimized, g(x,u) and h(x,u) represents the constraints that objective function is subject to. Where, x represents dependent variables and can be expressed as:

$$x^T = \left[\, P_{G1},\ V_{L1} \ldots.\ V_{LN_L},\ Q_{G1} \ldots.\ Q_{GN_G},\ S_l \ldots.\ S_{lnl} \,\right] \tag{2}$$

U is called as control variables and can be expressed as:

$$u^T = \left[\, V_{G1} \ldots.\ V_{GN_G},\ P_{G2} \ldots.\ P_{GN_G},\ T_1 \ldots.\ T_{N_T}, Q_{C1} \ldots.\ Q_{CN_c} \,\right] \tag{3}$$

OPF constraints can be divided in to two groups: These are equality constraints and inequality constraints.

2.1 Equality Constraints

OPF equality constraints consist of equations derived from power flow equations. These constraints are divided into active power equations and reactive power equations. Active power equations is as follows:

$$P_{Gi} - P_{Di} - V_i \sum_{j=1}^{N_B} V_j \left[G_{ij} \cos\left(\theta_i - \theta_j\right) + B_{ij} \sin\left(\theta_i - \theta_j\right) \right] = 0 \tag{4}$$

Reactive power equation is as follows:

$$Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{N_B} V_j \left[G_{ij} \sin\left(\theta_i - \theta_j\right) - B_{ij} \cos\left(\theta_i - \theta_j\right) \right] = 0 \tag{5}$$

2.2 Inequality Constraints

Power systems consist of many devices and elements coming together. These device and components have their own physical limits. Inequality constraints consist of these physical minimum and maximum limitations. These constraints are divided into four groups: Generation, shunt VAR compensations, transformers and security.

Generation constraints:

$$V_{Gi}^{min} \le V_{Gi} \le V_{Gi}^{max} \qquad i = 1,2,3, \ldots\ldots N_{PV} \tag{6}$$

$$P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max} \qquad i = 1,2,3, \ldots\ldots N_{PV} \tag{7}$$

$$Q_{Gi}^{min} \le Q_{Gi} \le Q_{Gi}^{max} \qquad i = 1,2,3, \ldots\ldots N_{PV} \tag{8}$$

Transformer constraints:

$$T_i^{min} \le T_i \le T_i^{max} \qquad i = 1,2,3, \ldots\ldots N_T \tag{9}$$

Shunt VAR compensator constraints:

$$Q_{ci}^{min} \le Q_{ci} \le Q_{ci}^{max} \qquad i = 1,2,3, \ldots\ldots N_C \tag{10}$$

Security constraints:

$$V_{Li}^{min} \le V_{Li} \le V_{Li}^{max} \qquad i = 1,2,3, \ldots\ldots N_{PQ} \tag{11}$$

$$S_{li} \le S_{li}^{max} \qquad i = 1,2,3, \ldots\ldots N_{TL} \tag{12}$$

In order to obtain more suitable results, the given inequalities are inserted to the objective function as:

$$f(x,u) = f + \lambda_p (P_{Gi} - P_{Gi}^{\lim})^2 + \lambda_v \sum_{i=1}^{N_L} (V_{Li} - V_{Li}^{\lim})^2 + \lambda_Q \sum_{i=1}^{N_Q} (Q_{Li} - Q_{Li}^{\lim})^2 + \lambda_S \sum_{i=1}^{N_l} (S_{li} - S_{li}^{\lim})^2 \tag{13}$$

λ_p, λ_v, λ_Q and λ_S are penalty coefficients selected by user. Explanations of all abridgments are given in Table 1.

Table 1. Explanations of abridgments

Abbreviation	Explanation
P_{Gi}	Active power generation at bus i
P_{Di}	Active power demand at bus i
Q_{Gi}	Reactive power generation at bus i
Q_{Di}	Reactive power demand at bus i
V_i	Voltage at bus i
V_j	Voltage at bus j
G_{ij}	Conductance between bus i and bus j
B_{ij}	Susceptance between bus i and bus j
V_{Gi}	Generator voltage at ith generation bus
T_i	Tap setting of ith transformer
Q_{ci}	Var injection of ith shunt capacitor
V_{Li}	Load voltage of ith unit
S_{li}	Apparent power flow of ith branch
N_B	Number of bus bars
N_{PV}	Number of PV buses
N_{PQ}	Number of PQ buses
N_T	Number of tap regulating transformers
N_C	Number of shunt var compensators
N_{TL}	Number of transmission lines

3. VORTEX SEARCH ALGORITHM

Vortex search algorithm (VSA) is an artificial intelligence-based optimization technique developed by Dogan and Olmez in 2015 (Dogan et. al., 2015). Developers of VSA are inspired by the natural swirls. The algorithm is based on the random distributed artificial particles which search the two-dimensional solution space to find the optimal solution.

The vortex, consist of nested circles in two-dimensional space. The algorithm tries to decrease the biggest circle's radius to find the optimal solution. Initial circle's center (μ_0) calculated as follows:

$$\mu_0 = \frac{upper\ limit + lower\ limit}{2} \tag{14}$$

Circle's initial radius is equals to the standard deviation and calculation of radius is as same as the calculation of μ_0.

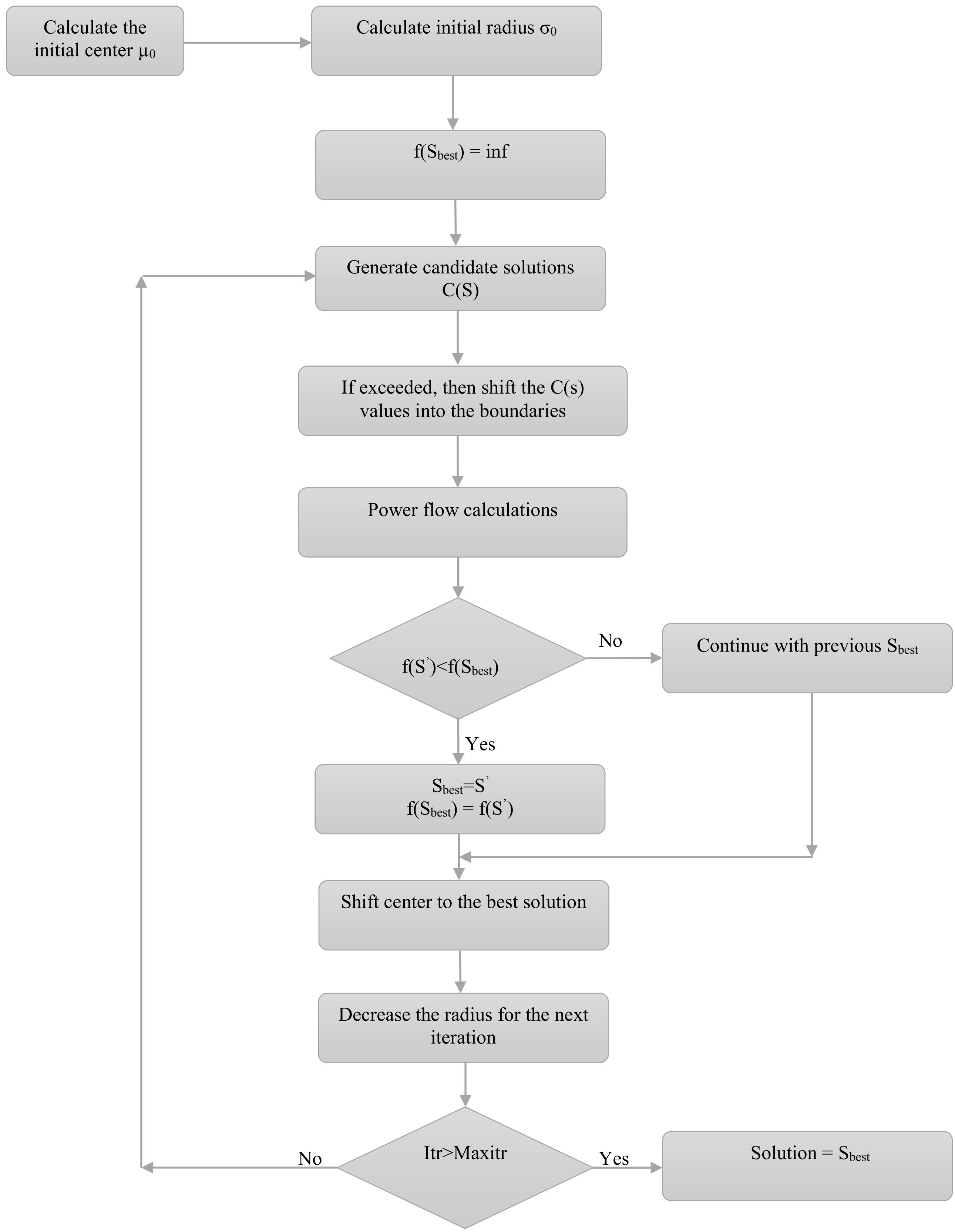

Fig. 1. Flow chart of VSA

$$\sigma_0 = \frac{upper\ limit + lower\ limit}{2} \tag{15}$$

At the beginning of iterations, best solution of objective function is set to the infinity. Thus, the function is provided on the next iterations to get better values. New fitness values possessed by each particle are evaluated and particles that are better positioned than the average values saved by the algorithm in every iteration.

Candidate solutions around the vortex initial center can be generated in the d-dimensional space via gaussian distribution $C_t(s)$, (t represents the iteration index). At the beginning these candidate solutions may sometimes be created outside the limits. If a candidate solution is greater than the upper limit or less than the lower limit, it can be drawn into limits as:

$$C_t(s) = rand\ (upper\ limit^i - lower\ limit^i) + lower\ limit^i \tag{16}$$

In every iteration, solutions are compared to the previous ones. Smaller solutions are saved as best solutions. Correspondingly, the central point of vortex shifted towards the best solution. At the same time, radius of vortex is decreased to find the best solution rapidly. Decreasing the radius can be done by using inverse of gamma function as follows:

$$r_t = \sigma_0 \cdot (1/x) \cdot gammaincinv(x, a_t) \tag{17}$$

a_t can be calculated as follows:

$$a_t = a_o - \frac{t}{MaxItr} \tag{18}$$

t is iteration number, MaxItr is maximum limit of iteration number.

Flow chart of proposed Vortex search algorithm is shown in Fig. 1.

4. RESULTS

The parameters and limits of the test system used in this study are given in Table 2 as given in literature (Yao et. al., 1999). The results are obtained by using MATLAB on a personal computer in the accordance with these parameters. MATPOWER software package is used to calculate the power flow equations.

50 particles are used to search the best value on the solution space. Iteration number is set to 100. Two different cases tested to show the effectiveness of the VSA algorithm. The obtained results are compared to best results reported in the literature.

Table 2. Test system parameters

Parameters and Limits	Information
Test System	IEEE-30 bus test system
Generation Buses	1, 2, 5, 8, 11, 13
Shunt Var Compensator Buses	10, 12, 15, 17, 20, 21, 23, 24, 29
Tap ratio buses	6-9, 6-10, 4-12, 27-28
Active Power Demand	2,834 p.u.
Reactive Power Demand	1,262 p.u.
Base MVA	100
Generator Voltage Limits	V_{min}=0,95 p.u. V_{max}=1,1 p.u.
Load Bus Voltage Limits	V_{min}=0,95 p.u. V_{max}=1,05 p.u.
Tap Setting Limits	T_{min}= 0,9 p.u. T_{max}= 1,1 p.u.
Shunt VAR Compensation Limits	Qc_{min}= 0,0 p.u. Qc_{max}= 5,0 p.u.

4.1 Case1: Piecewise quadratic fuel cost functions

Thermal generation units can be used with different sources such as natural gas, coal and oil. In these cases, different functions should be used when calculating the fuel cost. Cost functions of generators 1 and 2 described as follows.

$$F_i(P_{Gi}) = \begin{cases} a_{i1} + b_{i1}P_{Gi} + c_{i1}P_{Gi1}^2 \\ a_{i2} + b_{i2}P_{Gi} + c_{i2}P_{Gi2}^2 \\ \ldots \\ a_{ik} + b_{ik}P_{Gi} + c_{ik}P_{Gi1}^2 \end{cases} \tag{19}$$

Where, a_{ik}, b_{ik} and c_{ik} are used as coefficients of the ith generator for fuel type 3. These coefficients are given in Table 3. For other generation units, main case coefficients in other words total fuel basic cost coefficients are used. These coefficients are given in Table 4. We used basic coefficients to minimize the total fuel cost and improve the voltage profile in our previous study (Aydin et. al., 2016).

Table 3. Generator cost coefficients for Case 1

Bus	Min MW	Max Mw	a	b	c
1	50	140	55.00	0.70	0.0050
	140	200	82.5	1.05	0.0075
2	20	55	40.00	0.30	0.0100
	55	80	80.00	0.60	0.0200

Table 4. Total fuel cost basic coefficients

Bus	a	b	c
1	0	2	0.00375
2	0	1.75	0.01750
5	0	1	0.06250
8	0	3.25	0.00834
11	0	3	0.0250
13	0	3	0.0250

Objective function of this case is defined as:

$$J = \left(\sum_{i=1}^{2} a_{ik} + b_{ik} P_{Gi} + c_{ik} P_{Gi}^{2} \right) + \left(\sum_{i=3}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^{2} \right) \quad (20)$$

The results are given in Table 5. Results are compared to moth swarm algorithm (MSA) (Mohamed et. al., 2017), gravitational search algorithm (GSA) (Duman et. al. 2012) and artificial bee colony (ABC) (Adaryani et. al.,2013). The convergence of the VSA with minimum fuel cost is shown in Fig. 2. It can be seen that the minimum piecewise quadratic fuel cost is 626.0253 $/h.

Table 5. Results for Case 1

Parameter	MSA	GSA	ABC	VSA
P_{G1} (MW)	139.99	139.99	139.94	124.82
P_{G2} (MW)	54.99	54.92	54.98	51.20
P_{G5} (MW)	24.09	24.99	23.01	43.64
P_{G8} (MW)	35	30.24	33.05	31.86
P_{G11} (MW)	19.50	19.58	17.84	16.28
P_{G13} (MW)	16.61	20.21	21.80	21.38
V_1 (p.u.)	1.075	1.049	1.050	0.998
V_2 (p.u.)	1.056	1.009	1.039	1.047
V_5 (p.u.)	1.026	1.014	1.013	0.969
V_8 (p.u.)	1.035	1.034	1.024	1.033
V_{11} (p.u.)	1.068	0.950	1.100	0.967
V_{13} (p.u.)	1.077	1.003	1.086	0.979
$T_{6,9}$ (p.u.)	0.992	1.100	1.000	0.909
$T_{6,10}$ (p.u.)	0.958	1.099	0.900	0.920
$T_{6,12}$ (p.u.)	1.024	1.099	1.000	0.924
$T_{28,27}$ (p.u.)	0.968	1.079	0.923	1.008
Cost ($/h)	646.83	646.84	649.08	626.02
P_{loss} (MW)	6.80	6.57	7.25	5.81

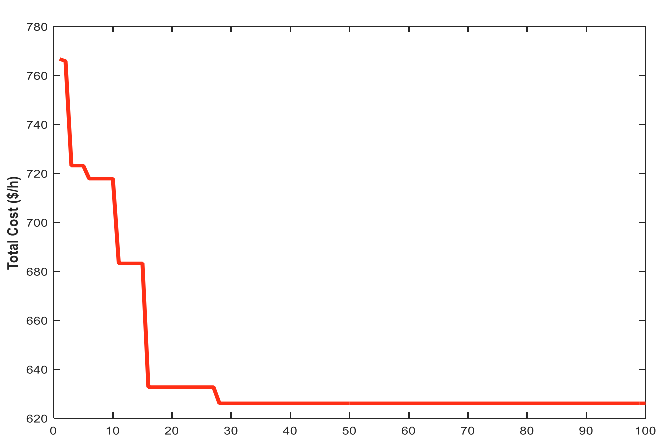

Fig. 2. Convergence characteristic for case 1

4.2 Case 2: Minimization of quadratic fuel cost with valve-point loadings

In this case, first and second generation units considered to have valve-point effects. Coefficients of these two generation units are given in Table 6. Fuel cost coefficients of remaining units are given in Table 4. Generator 1 and 2 have cost characteristic as follows:

$$F_i(P_{Gi}) = \left(\sum_{i=1}^{2} a_i + b_i P_{Gi} + c_i P_{Gi}^{2} + \left| d_i \sin\left(e_i (P_{Gi}^{\min} - P_{Gi}) \right) \right| \right) \quad (21)$$

where, a_i, b_i, c_i, d_i and e_i are fuel coefficients for unit i. Objective function for this case is given as follows:

$$F_i(P_{Gi}) = \left(\sum_{i=1}^{2} a_i + b_i P_{Gi} + c_i P_{Gi}^{2} + \left| d_i \sin\left(e_i (P_{Gi}^{\min} - P_{Gi}) \right) \right| \right) + \left(\sum_{i=3}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^{2} \right) \quad (22)$$

Table 6. Generator cost coefficients for Case 2

Bus no	P_{Gi}^{min}	P_{Gi}^{max}	a	b	c	d	e
1	50	200	150	2	0.0016	50	0.0630
2	20	80	25	2.5	0.0100	40	0.0980

The obtained results are given in Table 7. In addition to these results compared with those results given in literature. Results are compared to moth swarm algorithm (MSA), gravitational search algorithm (GSA) and artificial bee colony (ABC).

Table 7. Results for Case 2

Parameter	MSA	GSA	ABC	VSA
P_{G1} (MW)	197.56	199.59	194.84	199.59
P_{G2} (MW)	51.96	51.94	51.99	51.82
P_{G5} (MW)	15	15	15	14.22
P_{G8} (MW)	10	10	10	9.12
P_{G11} (MW)	10	10	10	9.22
P_{G13} (MW)	12	12	15.65	11.13
V_1 (p.u.)	1.033	1.099	1.022	0.970
V_2 (p.u.)	1.011	1.018	1.003	1.005
V_5 (p.u.)	0.971	1.052	1.025	0.952
V_8 (p.u.)	1.034	0.950	1.007	0.958
V_{11} (p.u.)	1.099	0.963	0.982	1.006
V_{13} (p.u.)	1.099	0.950	1.100	1.003
$T_{6,9}$ (p.u.)	1.1	0.909	1.050	0.987
$T_{6,10}$ (p.u.)	1.053	0.918	1.100	1.049
$T_{6,12}$ (p.u.)	1.069	0.925	0.962	0.962
$T_{28,27}$ (p.u.)	1.065	0.945	0.900	1.071
Cost ($/h)	930.74	929.72	945.44	918.79
P_{loss} (MW)	13.13	15.14	14.09	11.73

As given in Table 7, result for this case is 918.790 $/h which is better than any other results given in literature. Convergence of the algorithm for case 2 is shown in Fig. 3.

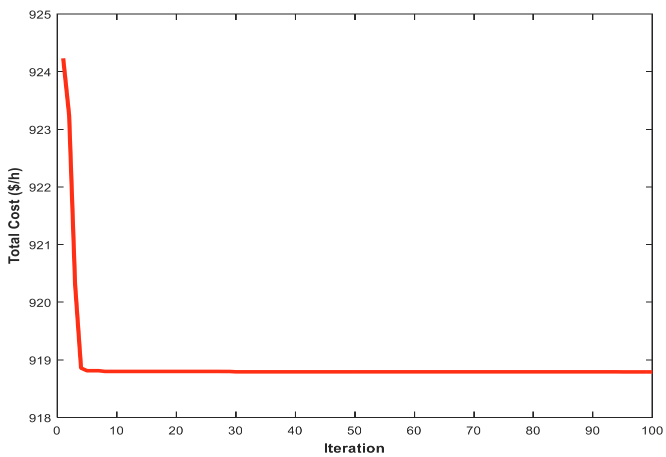

Fig. 3. Convergence characteristic for case 2

5. CONCLUSION

In this paper, VSA is used to solve two quadratic OPF cases. The obtained results are compared to MSA, GSA, ABC algorithms to show the superiority of VSA algorithm on large scale, complex optimization problems. Under favour of VSA's adaptive step size adjustment scheme, all the obtained results are better than other algorithms in the literature.

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