Hindawi
Mathematical Problems in Engineering
Volume 2017, Article ID 4389064, 29 pages
https://doi.org/10.1155/2017/4389064

Research Article

Optimization of Production-Distribution Problem in Supply Chain Management under Stochastic and Fuzzy Uncertainties

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Received 4 April 2017; Revised 4 July 2017; Accepted 12 July 2017; Published 17 October 2017

Academic Editor: Anna M. Gil-Lafuente

Production-Distribution Problem (PDP) in Supply Chain Management (SCM) is an important tactical decision. One of the challenges in this decision is the size and complexity of supply chain system (SCS). On the other side, a tactical operation is a mid-term plan for 6–12 months; therefore, it includes different types of uncertainties, which is the second challenge. In the literature, the uncertain parameters were modeled as stochastic or fuzzy. However, there are a few studies in the literature that handle stochastic and fuzzy uncertainties simultaneously in PDP. In this paper, the modeling and solution approaches of PDP which contain stochastic and fuzzy uncertainties simultaneously are investigated for a SCS that includes multiple suppliers, multiple products, multiple plants, multiple warehouses, multiple retailers, multiple transport paths, and multiple time periods, which, to the best of the author's knowledge, is not handled in the literature. The PDP contains deterministic, fuzzy, fuzzy random, and random fuzzy parameters. To the best of the author's knowledge, there is no study in the literature which considers all of them simultaneously in PDP. An analytic solution approach has been developed by using possibilistic programming and chance-constrained programming approaches. The proposed modeling and solution approaches are implemented in a numerical example. The solution has shown that the proposed approaches successfully handled uncertainties and produce robust solutions for PDP.

1. Introduction

The global competition enforces the firms to manage their facilities more effectively and to make right decisions in the market. Supply Chain Management (SCM), which is defined as the integration of key business processes from end user through original suppliers which provides products, services, and information that add value for customers and other stakeholders by the Global Supply Chain Forum (GSCF) [1], is a useful management approach to survive in the global market. SCM includes several processes such as supplier relation management, product development and commercialization, procurement, order fulfillment, manufacturing flow management, demand management, customer relationship management, returns management, and information management [1]. Production-Distribution Problem (PDP) in SCM is an important planning operation that affects several processes such as procurement, order fulfillment, and manufacturing flow management.

PDP starts to plan by determining raw materials provided by suppliers and makes decisions about the production planning and the distribution of final products to customers. The researchers and practitioners have been interested in PDP over the past years. Fahimnia et al. [2] indicated that there might be two main reasons increasing the number of studies on PDP: (1) affecting the profitability and (2) responding to the market changes quickly. The studies on the PDP can be classified into the different clusters according to the different criteria such as complexity of the supply chain system (SCS), decision levels, solution approaches, and structure of parameters.

PDP can be handled at different decision levels such as operational, tactical, and strategical levels. The strategic decisions are long-term plans that have vital effects on surviving in the market. The papers in this cluster focus on supply chain network design. They also consider opening plants, warehouses, and so forth [3–5]. In tactical perspective, PDP can be used to determine the production and transportation quantities for aggregate production planning and distribution planning. Besides, it is useful for capacity and resources planning decisions [6–9]. The PDP in operational level seeks to optimize the SCS by adding operational decisions to the aggregate models such as scheduling problem and routing problem [10–13].

There are several differences between operational, tactical, and strategical decisions such as time period, detail of information, responsibility, and the cost of a wrong decision. One of them is uncertainty which depends on the length of time period. The precision and exactness of information about problem parameters decrease when the time period of decision increases. Therefore, uncertainty is a challenge in PDP. PDP can be classified into four groups according to the structure of parameters. The first group is *deterministic parameters*: these models do not include any uncertainty in their parameters. All of the parameters are exact and are known at the beginning of solution process [12, 14]. The second group is *stochastic parameters*: the parameters include stochastic uncertainty. The probability theory models these parameters [3, 15–17]. The third group is *fuzzy parameters*: the fuzzy set theory is an effective modeling approach when the information on parameters is imprecise or inexact. It enables reflecting the decision maker's judgements into the problem [18–21]. The fourth group is *fuzzy and stochastic parameters*: in some situations, both fuzzy and stochastic uncertainties can occur in parameters simultaneously such as fuzzy random or random fuzzy parameters [22–24].

On the other hand, the size and complexity of supply chain system (SCS) are big challenges in PDP like uncertainty. Fahimnia et al. [2] classified the studies into seven clusters according to the SCS complexity. These clusters are given as follows: *Cluster 1*: single-product models [25–28]; *Cluster 2*: multi-product, single-plant models [29–31]; *Cluster 3*: multiple-products, multiple-plants, single- or no-warehouse models [32–34]; *Cluster 4*: multiple-products, multiple-plants, multiple-warehouses, single-/no-end-user models [35–37]; *Cluster 5*: multiple-products, multiple-plants, multiple-warehouses, multiple end users, single-transport-path models [20, 38, 39]; *Cluster 6*: multiple-products, multiple-plants, multiple-warehouses, multiple-end-users, multiple-transport-paths, no time period models [40, 41]; *Cluster 7*: multiple-products, multiple-plants, multiple-warehouses, multiple-end-users, multiple-transport-paths, multible-period-models [2]. To the best of the author's knowledge, a new cluster (Cluster 8) can be added to these classifications: *Cluster 8*: multiple-products, multiple-plants, multiple-warehouses, multiple-end-users, single-transport-path, multiple-periods models [42, 43].

The PDP requires using various techniques for solving this problem because of the properties of the PDP which are discussed above. Fahimnia et al. [2] classified these techniques into four clusters: analytic techniques, heuristic techniques, simulation, and genetic algorithms. For *analytic techniques*, the studies in this cluster use mathematical programming to solve PDP, that is, linear programming, nonlinear programming, mixed integer programming, and Lagrangian relaxation [44–46]. For *heuristic techniques*, since analytic techniques have a limitation on solving large-scale PDP, the researchers developed heuristic techniques that obtain feasible solution close to an optimal solution [16, 35, 47]. For *simulation modeling*, simulation is a very useful tool to analyze the system's behavior and performance criteria when the considered system is very complex to solve analytically [30, 48]. For *genetic algorithms (GA)*, they are effective algorithms that use direct and stochastic search methods to solve large-scale problems [49, 50].

In this paper, the PDP has been handled from a tactical perspective for a SCS. The SCS includes multiple suppliers, multiple products, multiple plants, multiple warehouses, multiple retailers, multiple transport paths, and multiple time periods, which, to the best of the author's knowledge, is not handled in the literature. A, 0-1 mixed-integer programming model has been developed for the PDP which includes deterministic, fuzzy, fuzzy random, and random fuzzy parameters. To the best of the author's knowledge, there is no study in the literature which considers deterministic, fuzzy, fuzzy random, and random fuzzy parameters simultaneously in PDP. An analytic solution approach has been developed for 0-1 mixed-integer programming model by using possibilistic programming and chance-constrained programming approaches.

The paper is organized as follows: the modeling uncertainty is given in Section 2. Section 3 represents mathematical model and uncertain parameters for PDP. The proposed solution approach is given in Section 4. The implementation of the proposed solution approach for a real-life industry case is presented in Section 5. The paper is finalized with concluding remarks in Section 6.

2. Modeling Uncertainty

Let us give the definitions of some uncertainty types such as random, fuzzy, random fuzzy, and fuzzy random variables.

Definition 1. If ξ is an experiment having sample space $\wp$ and X is a function that assigns a real number $X(e)$ to every outcome $e \in \wp$, then $X(e)$ is called a *random variable* [51].

Definition 2. Let Ω be a set of all outcomes of a random experiment. A (nonempty) collection A of subsets (called events) of Ω is assumed to have the following properties: (a) $\Omega \in A$; (b) if $\mathrm{A} \in A$, then $\mathrm{A}^c \in A$; and (c) if $\mathrm{A}_n \in A$ is a countable sequence of events, then $\bigcup_n A_n \in A$. Such a collection A is called a σ-algebra. For each random event A, there is a nonnegative number $\Pr\{A\}$, called its probability, such that (i) $\Pr\{\varnothing\} = 0$ and $\Pr\{\Omega\} = 1$ and (ii) $\Pr\{\bigcup_n A_n\} = \sum_n \Pr\{A_n\}$ for every countable sequence of mutually disjoint events A_n. The triplet $(\Omega, A, \Pr)$ is called a probability space and the function Pr is referred to as a probability measure. A *random variable* on the probability space $(\Omega, A, \Pr)$ is a function ξ from Ω to the real line $\mathfrak{R}$ for any Borel set O of $\mathfrak{R}$ [52].

Normal distribution is very important in both theory and application of statistics. The notation $X \sim N(\mu, \sigma^2)$ is often used to indicate that the random variable X is normally distributed with mean μ and variance σ^2.

After random variable definition, now we can get fuzzy variable definition and properties. Fuzzy set theory was proposed by L. Zadeh and applications of his theory can be found, for example, in artificial intelligent, computer science, control engineering, operation research, and decision theory [53].

Definition 3. Let U denote a universal set. Then a fuzzy subset $\widetilde{A}$ of U is defined by its membership function $\mu_{\widetilde{A}} : U \to [0, 1]$ which assigns to each element $x \in U$ a real number $\mu_{\widetilde{A}}(x)$ in the interval $[0, 1]$, where the value of $\mu_{\widetilde{A}}(x)$ at x represents the grade of membership of x in $\widetilde{A}$. *A fuzzy variable* is defined as a function from the possibility space $(\Theta, P(\Theta), \mathrm{Pos})$ to the real line $\mathfrak{R}$ [52].

Triangular fuzzy variable is the most known and used fuzzy variable which is denoted by the triplet (a_l, a, a_u) and has the shape of a triangle.

The concept of the random fuzzy variable was initialized by Liu and defined as a fuzzy variable taking "random values."

Definition 4. A *random fuzzy variable* is defined as a function from the possibility space $(\Theta, P(\Theta), \mathrm{Pos})$ to the set of random variables [54].

Assume that $\eta_1, \eta_2, \ldots, \eta_m$ are random variables and that $u_1, u_2, \ldots, u_m$ are real numbers in $[0, 1]$ such that $u_1 \vee u_2 \vee \cdots \vee u_m = 1$. Then

$$\zeta = \begin{cases} \eta_1, & \text{with possibility } u_1, \\ \eta_2, & \text{with possibility } u_2, \\ \eta_m, & \text{with possibility } u_m \end{cases} \tag{1}$$

is a clearly discrete random fuzzy variable [54].

Definition 5. Assume that ζ is *a random fuzzy variable*. Then *the probability* $\Pr\{\zeta(\theta) \in B\}$ is a *fuzzy variable* for any Borel set B of $\mathfrak{R}$ [54].

Definition 6. A random fuzzy variable ζ is said to be normal if, for each θ, $\zeta(\theta)$ is a normally distributed random variable; that is, $\zeta(\theta) \sim N(X(\theta), Y(\theta))$, with X and Y being fuzzy variables defined on the space Θ such that $Y > 0$. A normally distributed random fuzzy variable is usually denoted as $\zeta(\theta) \sim N(X, Y)$, and the fuzziness of random fuzzy variable ζ is said to be characterized by fuzzy vector (X, Y) [55].

Roughly speaking, a fuzzy random variable is a measurable function from a probability space to the set of fuzzy variables. In other words, a fuzzy random variable is a random variable taking fuzzy values.

Definition 7. *A fuzzy random variable* is a function ξ from a probability space $(\Omega, A, \Pr)$ to the set of fuzzy variables such that $\mathrm{Pos}(\xi(w) \in B)$ is a measurable function of w for any Borel set B of $\mathfrak{R}$.

A random fuzzy variable is defined as a function that assigns a random value to each fuzzy subset. On the other hand, a fuzzy random variable is defined as a function that assigns a fuzzy subset to each possible output of a random experiment.

3. Mathematical Model and Uncertain Parameters for PDP

3.1. Mathematical Model. The SCS includes multiple suppliers, multiple products, multiple plants, multiple warehouses, multiple retailers, multiple transport paths, and multiple time periods. In the PDP, multiple raw materials are supplied from multiple suppliers and transported to the multiple plants by using multiple transport paths. In plants, multiple products are manufactured by using regular time and overtime and the final products are transported to the multiple warehouses by using multiple transport paths. Multiple warehouses deliver multiple products to the multiple retailers by using multiple transport paths. The customers pick up their products from multiple retailers.

Several assumptions have been made to construct a 0-1 mixed-integer programming model which are given as follows:

(i) The quantities of raw materials in suppliers are restricted.

(ii) The number and capacity of transport paths between all the components in SCS are restricted.

(iii) The starting and ending inventories of product and raw materials in plants, warehouses, and retailers are zero.

(iv) The plants have ability to produce several products.

(v) The plants have ability to store raw materials and products.

(vi) The storage capacities of plants are restricted.

(vii) The plants have regular-time and overtime production.

(viii) The warehouses have ability to store products.

(ix) The storage capacities of warehouses are restricted.

(x) The retailers have ability to store products.

(xi) The storage capacities of retailers are restricted.

(xii) The unsatisfied demands are lost.

In this paper, the PDP in SCM has been considered in tactical level. Since it is a mid-term plan, it includes a lot of uncertainties. The uncertainties make the problem more complex compared to the deterministic ones because there are challenges in modeling parameters and obtaining robust solutions. In the literature, the researchers have tried to overcome these challenges by using fuzzy set theory or probability theory.

The fuzzy set theory provides a highly effective means of handling imprecise data. It enables incorporating the decision-maker's expertise and judgements into the problem. However, it is not a powerful theory like probability theory for modeling and solution. On the other side, the probability theory is an effective tool for modeling uncertainties in the stochastic process. It acts with the past data analysis for the forecasting of future events and does not include decision-makers into the decision-making process.

However, the decision-makers have an impact on the future events by the way of their decisions. In PDP, the unit cost of raw materials may change based on the purchased quantity. The unit production cost directly depends on lot size. Producing in overtime or producing and holding in regular time at previous periods are based on planning manager's decision. The unit transportation cost is related to path type and transported quantity. The unit price of a product may be changed by making discount, giving an advertisement. The capacity of raw materials supplied from the market is based on the contracts made by the decision-makers. The decision-maker can change the workforce level by hiring and firing; therefore, the production capacity can be changed. The parameters in the model related to the above discussion can be modeled by using triangular fuzzy numbers. The triangular fuzzy numbers are well known and are commonly used in many applications because the decision-maker has opinions about pessimistic, optimistic, and most possible values by using his/her expertise and expectation.

The transportation capacities of all echelons in the SCS are related to the number of the transporters in the portfolio of the decision-makers, transportation quantities, and vehicle routing decisions that make the transportation capacity uncertain. Therefore, transportation capacities can be modeled as triangular fuzzy numbers by using the decision-maker's expertise and judgements. However, the available transportation capacity can occur in different situations based on the suitability of the transporter in the market which are defined as discrete events. These discrete events are determined as high, medium, and low capacities. It is possible to increase the number of situations; however, it will cause confusion in the categorization process. By analyzing the past data, probability levels can be determined for occurrences of each of the situations. Therefore, the transportation capacities can be modeled as fuzzy random parameters.

The demand of product can be modeled as a probability distribution by analyzing the past data. Since the PDP is a mid-term plan, a sum of identically distributed independent demand variables has a normal distribution according to the central limit theorem. However, the decision-maker can affect the demand quantity by making discount, advertisement, or other strategies. These marketing strategies are based on management decisions. Therefore, the demand quantity is modeled as a random fuzzy variable.

The mathematical model is given in Notations.

$$
\begin{aligned}
\max \quad Z &= \left[\sum_{t=1}^{T}\sum_{j=1}^{J}\left(\widetilde{\mathrm{POP}}_{jt} * \sum_{c=1}^{C}\sum_{r=1}^{R}\mathrm{SPQ}_{jrtc}\right)\right] \\
&\quad - \left[\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{i=1}^{I}\left(\widetilde{\mathrm{RUP}}_{ist} * \sum_{p=1}^{P}\sum_{k=1}^{K}\mathrm{TRQ}_{ispkt}\right)\right] \\
&\quad - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{s=1}^{S}\left(\mathrm{FTCS}_{spkt} * \mathrm{UKS}_{spkt}\right)\right] \\
&\quad - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{i=1}^{I}\left(\widetilde{\mathrm{VTCS}}_{ispkt} * \mathrm{TRQ}_{ispkt}\right)\right] \\
&\quad - \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{I}\left(\widetilde{\mathrm{SRC}}_{ipt} * \mathrm{SRP}_{ipt}\right)\right] \\
&\quad - \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\widetilde{\mathrm{RPC}}_{jpt} * \mathrm{RPQ}_{jpt}\right)\right] \\
&\quad - \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\widetilde{\mathrm{OPC}}_{jpt} * \mathrm{OPQ}_{jpt}\right)\right] \\
&\quad - \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\widetilde{\mathrm{PHC}}_{jpt} * \mathrm{SLP}_{jpt}\right)\right] \\
&\quad - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{w=1}^{W}\sum_{p=1}^{P}\left(\mathrm{FTCP}_{pwkt} * \mathrm{UKP}_{pwkt}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&-\left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{w=1}^{W}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\widetilde{\mathrm{VTCP}}_{jpwkt} * \mathrm{TPQP}_{jpwkt}\right)\right] \\
&-\left[\sum_{t=1}^{T}\sum_{w=1}^{W}\sum_{j=1}^{J}\left(\widetilde{\mathrm{WHC}}_{jwt} * \mathrm{SLW}_{jwt}\right)\right] \\
&-\left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{w=1}^{W}\left(\mathrm{FTCW}_{wrkt} * \mathrm{UKW}_{wrkt}\right)\right] \\
&-\left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{w=1}^{W}\sum_{j=1}^{J}\left(\widetilde{\mathrm{VTCW}}_{jwrkt} * \mathrm{TPQW}_{jwrkt}\right)\right] \\
&-\left[\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{j=1}^{J}\left(\widetilde{\mathrm{HCR}}_{jrt} * \mathrm{SLR}_{jrt}\right)\right] \\
&-\left[\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{j=1}^{J}\left(\widetilde{\mathrm{BCR}}_{jrt} * \left(\sum_{c=1}^{C}\mathrm{BLR}_{jrtc}\right)\right)\right]
\end{aligned} \tag{2}
$$

$$
\text{s.t.} \quad \sum_{k=1}^{K}\sum_{p=1}^{P}\mathrm{TRQ}_{ispkt} \le \widetilde{\mathrm{RC}}_{ist} \tag{3}
$$

$\forall i,\ \forall s,\ \forall t,$

$$
\sum_{i=1}^{I}\left(\mathrm{RRC}_{i} * \mathrm{TRQ}_{ispkt}\right) \le \overline{\mathrm{TCSP}}_{spkt} + \left(1 - \mathrm{UKS}_{spkt}\right) * M \tag{4}
$$

$\forall s,\ \forall p,\ \forall k,\ \forall t,$

$$
\sum_{i=1}^{I}\left(\mathrm{RRC}_{i} * \mathrm{TRQ}_{ispkt}\right) \le \mathrm{UKS}_{spkt} * M \tag{5}
$$

$\forall s,\ \forall p,\ \forall k,\ \forall t,$

$$
\begin{aligned}
&\left(\sum_{k=1}^{K}\sum_{s=1}^{S}\mathrm{TRQ}_{ispkt} + \mathrm{SRP}_{ip(t-1)}\right) \\
&\quad - \sum_{j=1}^{J}\left(\mathrm{RRM}_{ij} * \left(\mathrm{RPQ}_{jpt} + \mathrm{OPQ}_{jpt}\right)\right) = \mathrm{SRP}_{ipt}
\end{aligned} \tag{6}
$$

$\forall i,\ \forall p,\ \forall t,$

$$
\sum_{j=1}^{J}\left(\mathrm{UPT}_{jpt} * \mathrm{RPQ}_{jpt}\right) \le \widetilde{\mathrm{ARC}}_{pt} \tag{7}
$$

$\forall p,\ \forall t,$

$$
\sum_{j=1}^{J}\left(\mathrm{UPT}_{jpt} * \mathrm{OPQ}_{jpt}\right) \le \widetilde{\mathrm{AOC}}_{pt} \tag{8}
$$

$\forall p,\ \forall t,$

$$\sum_{j=1}^{J}\left(\mathrm{RHC}_j * \mathrm{SLP}_{jpt}\right) \le \mathrm{PIC}_p \quad \forall p,\ \forall t, \tag{9}$$

$$\mathrm{RPQ}_{jpt} + \mathrm{OPQ}_{jpt} + \mathrm{SLP}_{jp(t-1)} - \sum_{k=1}^{K}\sum_{w=1}^{W}\mathrm{TPQP}_{jpwkt} = \mathrm{SLP}_{jpt} \quad \forall j,\ \forall p,\ \forall t, \tag{10}$$

$$\sum_{j=1}^{J}\left(\mathrm{RTC}_j * \mathrm{TPQP}_{jpwkt}\right) \le \overline{\mathrm{TCPW}}_{pwkt} + \left(1 - \mathrm{UKP}_{pwkt}\right) * M \quad \forall p,\ \forall w,\ \forall k,\ \forall t, \tag{11}$$

$$\sum_{j=1}^{J}\left(\mathrm{RTC}_j * \mathrm{TPQP}_{jpwkt}\right) \le \mathrm{UKP}_{pwkt} * M \quad \forall p,\ \forall w,\ \forall k,\ \forall t, \tag{12}$$

$$\sum_{k=1}^{K}\sum_{p=1}^{P}\mathrm{TPQP}_{jpwkt} + \mathrm{SLW}_{jw(t-1)} - \sum_{k=1}^{K}\sum_{r=1}^{R}\mathrm{TPQW}_{jwrkt} = \mathrm{SLW}_{jwt} \quad \forall j,\ \forall w,\ \forall t, \tag{13}$$

$$\sum_{j=1}^{J}\left(\mathrm{RHC}_j * \mathrm{SLW}_{jwt}\right) \le \mathrm{WIC}_w \quad \forall w,\ \forall t, \tag{14}$$

$$\sum_{j=1}^{J}\left(\mathrm{RTC}_j * \mathrm{TPQW}_{jwrkt}\right) \le \overline{\mathrm{TCWR}}_{wrkt} + \left(1 - \mathrm{UKW}_{wrkt}\right) * M \quad \forall w,\ \forall r,\ \forall k,\ \forall t, \tag{15}$$

$$\sum_{j=1}^{J}\left(\mathrm{RTC}_j * \mathrm{TPQW}_{jwrkt}\right) \le \mathrm{UKW}_{wrkt} * M \quad \forall w,\ \forall r,\ \forall k,\ \forall t, \tag{16}$$

$$\sum_{k=1}^{K}\sum_{w=1}^{W}\mathrm{TPQW}_{jwrkt} + \mathrm{SLR}_{jr(t-1)} - \sum_{c=1}^{C}\mathrm{SPQ}_{jrtc} = \mathrm{SLR}_{jrt} - \sum_{c=1}^{C}\mathrm{BLR}_{jrtc} \quad \forall j,\ \forall r,\ \forall t, \tag{17}$$

$$\sum_{j=1}^{J}\left(\mathrm{RHC}_j * \mathrm{SLR}_{jrt}\right) \leq \mathrm{RIC}_r \quad \forall r,\ \forall t, \tag{18}$$

$$\mathrm{SPQ}_{jrtc} \leq \overline{\overline{\mathrm{CDP}}}_{jrtc} \quad \forall j,\ \forall r,\ \forall t,\ \forall c, \tag{19}$$

$$\begin{aligned} &\mathrm{TRQ}_{ispkt}, \mathrm{SRP}_{ipt}, \mathrm{RPQ}_{jpt}, \mathrm{OPQ}_{jpt}, \mathrm{SLP}_{jpt}, \mathrm{TPQP}_{jpwkt} \\ &\geq 0, \\ &\mathrm{SLR}_{jrt}, \mathrm{BLR}_{jrtc}, \mathrm{SPQ}_{jrtc}, \mathrm{SLW}_{jwt}, \mathrm{TPQW}_{jwrkt} \geq 0, \\ &\mathrm{UKS}_{spkt}, \mathrm{UKP}_{pwkt}, \mathrm{UKW}_{wrkt} = 0, 1. \end{aligned} \tag{20}$$

The objective function, given in (2), maximizes the total profit. Total profit is obtained by total revenue, which is gained from total sales, plus total cost. Total cost includes raw material purchasing cost, fixed costs of using path k for transportations between all components of SCS, unit transportation costs between all components of SCS, unit production costs in regular time and overtime, holding costs in plants, warehouses, and retailers, and backorder cost. Equation (3) is a capacity constraint for the supplier that ensures that the total transported quantity from supplier s for material i at each period will be less than or equal to total capacity. Equations (4) and (5) are constructed to select the transportation path from supplier to plant and not to exceed its capacity where M is a big number. Equation (6) is an inventory balance constraint for the raw material in a plant. Equations (7), (8), and (9) are capacity constraints for regular production, overtime production, and inventory level in plants, respectively. Equation (10) is an inventory balance constraint for the product in a plant. Equations (11) and (12) are designed to select the transportation path from a plant to the warehouse and not to exceed its capacity. Equation (13) is an inventory balance constraint in a warehouse. Equation (14) is an inventory capacity constraint for a warehouse. Equations (15) and (16) are designed to select the transportation path from a warehouse to the retailer and not to exceed its capacity. Equation (17) is a balance constraint for inventory and backorder level. Equation (18) is an inventory capacity constraint for a retailer. Equation (19) ensures meeting customer demand. Equation (20) gives the definitions of the decisions variables.

3.2. Uncertain Parameters in PDP. The 0-1 mixed-integer mathematical model given in (2)–(20) includes uncertain parameters in both objective function and constraints which are fuzzy, fuzzy random, and random fuzzy.

3.2.1. Fuzzy Parameters. Fuzzy parameters, which are symbolized by "~," are $\widetilde{\mathrm{RUP}}_{ist}$, $\widetilde{\mathrm{VTCS}}_{ispkt}$, $\widetilde{\mathrm{SRC}}_{ipt}$, $\widetilde{\mathrm{RPC}}_{jpt}$, $\widetilde{\mathrm{OPC}}_{jpt}$, $\widetilde{\mathrm{PHC}}_{jpt}$, $\widetilde{\mathrm{VTCP}}_{jpwkt}$, $\widetilde{\mathrm{VTCW}}_{jwrkt}$, $\widetilde{\mathrm{WHC}}_{jwt}$, $\widetilde{\mathrm{HCR}}_{jrt}$, $\widetilde{\mathrm{BCR}}_{jrt}$, and $\widetilde{\mathrm{POP}}_{jt}$ in objective function and $\widetilde{\mathrm{RC}}_{ist}$, $\widetilde{\mathrm{ARC}}_{pt}$, and $\widetilde{\mathrm{AOC}}_{pt}$ in constraints. All of the fuzzy parameters in objective function and constraints are modeled by using triangular fuzzy numbers. As mentioned above, triangular fuzzy number is denoted by the triplet (a_l, a, a_u):

(1) The most possible value (a) that definitely belongs to the set of available values (membership degree = 1 if normalized)

(2) The most optimistic value (a_u) that has a very low likelihood of belonging to the set of available values (membership degree = 0 if normalized)

(3) The most pessimistic value (a_l) that has a very low likelihood of belonging to the set of available values (membership = 0 if normalized)

3.2.2. Fuzzy Random Parameters. Fuzzy random parameters, which are symbolized by "–," are $\overline{\mathrm{TCSP}}_{spkt}$, $\overline{\mathrm{TCPW}}_{pwkt}$, and $\overline{\mathrm{TCWR}}_{wrkt}$ in constraints. Fuzzy random parameters are related to the capacities of transportation paths. They can be modeled by using triangular fuzzy number as follows:

$$\xi = \begin{cases} (a_l, a, a_u), & \text{with probabilty } \mathrm{Pr}_a, \\ (b_l, b, b_u), & \text{with probabilty } \mathrm{Pr}_b, \\ (c_l, c, c_u), & \text{with probabilty } \mathrm{Pr}_c, \end{cases} \tag{21}$$

where Pr_a, Pr_b, and Pr_c represent the probabilities of transportation capacity situations such as high, medium, and low capacities. On the other hand, triangular fuzzy numbers (a_l, a, a_u), (b_l, b, b_u), and (c_l, c, c_u) represent the amounts of each of the transportation capacities for probability levels.

3.2.3. Random Fuzzy Parameters. There is one random fuzzy parameter, $\overline{\overline{\mathrm{CDP}}}_{jrtc}$, which represents the customer demand. The customer demand includes two main parameters: the probability and quantity. Therefore, the demand can be calculated as the sum of multiplication of probability value and quantity which can be referred to as expected value of

Table 1: Random fuzzy demand probability.

Situation	Pol. dev.	Comp. str.	Sec. exp.	Possibility	Probability of demand status		
					High	Medium	Low
(1)	Good	Medium	Good	1	0.8	0.15	0.05
(2)	Good	Strong	Good	0.8	0.6	0.3	0.1
(3)	Medium	Weak	Medium	0.7	0.15	0.7	0.15
(4)	Medium	Medium	Bad	0.6	0.1	0.5	0.4
(5)	Bad	Medium	Medium	0.5	0.1	0.4	0.5
(6)	Bad	Strong	Bad	0.4	0.05	0.15	0.8

Pol. dev.: political development. Comp. str.: competitors' strategy. Sec. exp.: sectoral expectation.

discrete random variable. The probability and quantity of demand are random fuzzy variables.

The probability of demand is modeled as follows: there are three states about the demand; it may be high, medium, or low. Assume that these three probabilities are represented as $\Pr(D)_{\text{high}}$, $\Pr(D)_{\text{medium}}$, and $\Pr(D)_{\text{low}}$. The probability of demand state is affected by three indicators which are (1) political developments, (2) competitors' strategies, and (3) sectoral expectation. For example, if the competitors perform a strong strategy in the market, the demand quantity will be affected by this situation; most likely it will decrease. These indicators are related to the expertise and expectations. Therefore, it is a very difficult task to model the demand states in deterministic or stochastic case. However, random fuzzy variables enable modeling these situations easier than the remaining ones. It is possible to reflect the decision-maker's judgements and expectations into the demand state by using random fuzzy variables. The modeling of demand states by using random fuzzy variables can be explained with an example for situations (1) and (2) in Table 1.

The first situation assumes that political development will be good, competitors' strategy will be medium, and sectoral expectation will be good. According to the decision-maker's judgements, expertise, and expectation, the possibility of occurrence of situation (1) is one. It means that situation (1) is an event that can absolutely occur. The analysis of historical data shows that when situation (1) occurs, the probabilities of demand which may be high, medium, and low are 0.8, 0.15, and 0.05, respectively. Situation (2) can be interpreted like situation (1). Consequently, the only way to model $\Pr(D)_{\text{high}}$, $\Pr(D)_{\text{medium}}$, and $\Pr(D)_{\text{low}}$ is using random fuzzy variables. According to Definition 4, $\Pr(D)_{\text{high}}$ can be modeled as follows:

$$\zeta\left(\Pr(D)_{\text{high}}\right) = \begin{cases} 0.8, & \text{with possibility } 1.0, \\ 0.6, & \text{with possibility } 0.8. \end{cases} \tag{22}$$

After modeling the probability of demand, modeling demand quantity is needed. Assume that the demand quantity follows normal distribution for each demand situation. According to Definition 6, the demand quantities are represented as $\zeta(\theta_{\text{high}}) \sim N(X(\theta_{\text{high}}), Y(\theta_{\text{high}}))$, $\zeta(\theta_{\text{medium}}) \sim N(X(\theta_{\text{medium}}), Y(\theta_{\text{medium}}))$ and $\zeta(\theta_{\text{low}}) \sim N(X(\theta_{\text{low}}), Y(\theta_{\text{low}}))$, where mean parameters $X(\theta_{\text{high/medium/low}})$ are fuzzy variables defined on the space Θ for high, medium, and low demand situations, respectively. The reason of modeling mean parameter as fuzzy variable is the ability to manage demand by using advertisements, discounts, and dynamic pricing which depend on decision makers' actions.

All of the remaining parameters in the model are deterministic.

4. Solution Approach

The idea of uncertain programming is to convert the uncertain nature of a model into an equivalent deterministic one [56]. Therefore, the uncertain parameters in PDP will be transformed into some equivalent deterministic ones by using properties of fuzzy, fuzzy random, and random fuzzy variables.

4.1. Transforming Uncertain Parameters into Deterministic Equivalents. The uncertain parameters have occurred in both objective function and constraints. Therefore, transforming operations of uncertain parameters are considered based on the location of uncertain parameters in the mathematical model.

4.1.1. Uncertain Parameters in Constraints. Let transformation operation of fuzzy parameter start and that operation is called "defuzzification" in the literature [57].

Definition 8. The α-cut of a fuzzy set $\widetilde{A}$ is a crisp subset of X and is denoted by $\widetilde{A}_\alpha = \{x \mid \mu_{\widetilde{A}}(x) \geq \alpha \text{ and } x \in X\}$.

The α-cut of the triangular fuzzy variable $\widetilde{A} = (a_l, a, a_u)$ is the closed interval $\widetilde{A}_\alpha = [a_l{}^\alpha, a_u{}^\alpha] = [(a - a_l)\alpha + a_l, -(a_u - a)\alpha + a_u]$, $\alpha \in (0, 1]$.

Definition 9. The multiplication of a fuzzy variable $\widetilde{A}$ by a real number $k > 0$ can be defined [58]: $(k\widetilde{A})_\alpha = k\widetilde{A}_\alpha = [ka_l{}^\alpha, ka_u{}^\alpha]$. A real number m can be defined as a triangular fuzzy number by the triplet (m_l, m, m_u), where $m_l = m$, $m = m$, and $m_u = m$.

Definition 10. Assume that $\widetilde{X}$ and $\widetilde{Y}$ are two fuzzy numbers. The result Z of the addition of the fuzzy numbers $\widetilde{X}$ and $\widetilde{Y}$ can be defined by the α-cut sets [59]. That is, $\widetilde{Z}_\alpha = \widetilde{X}_\alpha + \widetilde{Y}_\alpha = [x_l{}^\alpha + y_l{}^\alpha, x_u{}^\alpha + y_u{}^\alpha]$.

All of the fuzzy parameters in the right-hand sides of (3), (7), and (8) can be transformed into deterministic close interval by using α-cut approach.

Now let consider fuzzy random parameters.

Definition 11. Let X be a discrete random variable taking values $x_1, x_2, \ldots$ with probabilities $\mathrm{Pr}_1, \mathrm{Pr}_2, \ldots$, respectively. Then the expected value of this random variable is the infinite sum

$$E(X) = \sum_{i=1}^{\infty} \mathrm{Pr}_i * x_i. \tag{23}$$

Corollary 12. *If the capacities of transportation paths ($\overline{TCSP}_{spkt}$, $\overline{TCPW}_{pwkt}$, or $\overline{TCWR}_{wrkt}$) are discrete fuzzy random variables, the expected value of transportation capacity can be calculated by using Definitions 8, 9, and 11 as follows (for $\overline{TCSP}_{spkt}$):*

$$\begin{aligned} E\left(\overline{TCSP}_{spkt}\right) &= \mathrm{Pr}_a * \left(a_l, a, a_u\right) + \mathrm{Pr}_b * \left(b_l, b, b_u\right) \\ &\quad + \mathrm{Pr}_c * \left(c_l, c, c_u\right), \\ E\left(\overline{TCSP}_{spkt}\right) &= \mathrm{Pr}_a * \left[a_l^{\alpha}, a_u^{\alpha}\right] + \mathrm{Pr}_b * \left[b_l^{\alpha}, b_u^{\alpha}\right] \\ &\quad + \mathrm{Pr}_c * \left[c_l^{\alpha}, c_u^{\alpha}\right], \\ E\left(\overline{TCSP}_{spkt}\right) &= \left[\mathrm{Pr}_a * a_l^{\alpha} + \mathrm{Pr}_b * b_l^{\alpha} + \mathrm{Pr}_c * c_l^{\alpha}, \mathrm{Pr}_a \right. \\ &\quad \left. * a_u^{\alpha} + \mathrm{Pr}_b * b_u^{\alpha} + \mathrm{Pr}_c * c_u^{\alpha}\right], \\ E\left(\overline{TCSP}_{spkt}\right) &= \left[TCSP_L, TCSP_U\right], \end{aligned} \tag{24}$$

where $TCSP_L = [\mathrm{Pr}_a * a_l^{\alpha} + \mathrm{Pr}_b * b_l^{\alpha} + \mathrm{Pr}_c * c_l^{\alpha}]$ *and* $TCSP_U = [\mathrm{Pr}_a * a_u^{\alpha} + \mathrm{Pr}_b * b_u^{\alpha} + \mathrm{Pr}_c * c_u^{\alpha}]$.

All of the fuzzy random parameters in the right-hand sides of (4), (11), and (15) are transformed into deterministic close interval according to Corollary 12.

Now let consider random fuzzy parameters. As defined in Section 3.2, the customer demand has three discrete events; it may be high, medium, or low with probability values $\mathrm{Pr}(D)_{\text{high}}$, $\mathrm{Pr}(D)_{\text{medium}}$, and $\mathrm{Pr}(D)_{\text{low}}$, respectively. On the other side, the demand quantity for each event follows normal distribution with different fuzzy mean parameters and different variances which are $N(\widetilde{X}(\theta_{\text{high/medium/low}}), Y(\theta_{\text{high/medium/low}}))$. According to Definition 11, total customer demand can be written as follows:

$$\begin{aligned} &\text{Total customer demand} \\ &\quad = \mathrm{Pr}(D)_{\text{high}} * N\left(\widetilde{X}\left(\theta_{\text{high}}\right), Y\left(\theta_{\text{high}}\right)\right) \\ &\qquad + \mathrm{Pr}(D)_{\text{medium}} * N\left(\widetilde{X}\left(\theta_{\text{medium}}\right), Y\left(\theta_{\text{medium}}\right)\right) \\ &\qquad + \mathrm{Pr}(D)_{\text{low}} * N\left(\widetilde{X}\left(\theta_{\text{low}}\right), Y\left(\theta_{\text{low}}\right)\right), \end{aligned} \tag{25}$$

where $\mathrm{Pr}(D)_{\text{high/medium/low}}$ and $N(\widetilde{X}(\theta_{\text{high/medium/low}}), Y(\theta_{\text{high/medium/low}}))$ are random fuzzy parameters; therefore, total customer demand is a random fuzzy parameter.

In order to transform total customer demand into its deterministic equivalent, it is required to transform the probabilities and quantities into deterministic cases.

The following definition and corollary have been made for transforming the probabilities.

Definition 13. Let ξ be a normalized discrete fuzzy variable whose possibility distribution function is defined by $\mu(r) = \{\mu_1,\ \text{if } r = b_1;\ \mu_2,\ \text{if } r = b_2;\ \mu_n,\ \text{if } r = b_n\}$.

The expected value of ξ is as follows:

$$E[\xi] = \sum_{i=1}^{n} b_i p_i, \tag{26}$$

where the weights are given by

$$\begin{aligned} p_i = {} & \frac{1}{2}\left(\max_{1\le j\le i} \mu_j - \max_{0\le j\le i-1} \mu_j\right) \\ & + \frac{1}{2}\left(\max_{i\le j\le n} \mu_j - \max_{i+1\le j\le n+1} \mu_j\right). \end{aligned} \tag{27}$$

($\mu_0 = 0,\ \mu_{n+1} = 0$) for $i = 1, 2, \ldots, n$ and satisfy the following constraints: $p_i \ge 0$ and $\sum_{i=1}^{n} p_i = \max_{1\le i\le n}\mu_i = 1$, since ξ is a normalized fuzzy variable [60].

Corollary 14. *If the demand state is a discrete random fuzzy variable, the probabilities of demand states are fuzzy variables according to Definition 5 and then by using expected value of the fuzzy variable (Definition 13), crisp expected probability values can be calculated for high, medium, and low demand states.*

According to Corollary 14, the expected probability values for high, medium, and low demand states, which are represented by T_{high}, T_{medium}, and T_{low} to prevent confusions in next formulations, are written as follows:

$$\begin{aligned} E\left[\mathrm{Pr}\left(D_{\text{high}}\right)\right] &= \sum_{i=1}^{n} \mathrm{Pr}\left(D_{\text{high}}\right)_i p_i = T_{\text{high}}, \\ E\left[\mathrm{Pr}\left(D_{\text{medium}}\right)\right] &= \sum_{i=1}^{n} \mathrm{Pr}\left(D_{\text{medium}}\right)_i p_i = T_{\text{medium}}, \\ E\left[\mathrm{Pr}\left(D_{\text{low}}\right)\right] &= \sum_{i=1}^{n} \mathrm{Pr}\left(D_{\text{low}}\right)_i p_i = T_{\text{low}}. \end{aligned} \tag{28}$$

Let us consider demand quantity.

Definition 15. If X and Y are identical independent normally distributed parameters, $X \sim N(\mu_x, {\sigma_x}^2)$ and $Y \sim N(\mu_y, {\sigma_y}^2)$; their sum of $Z = X + Y$ is normally distributed with mean $\mu_x + \mu_y$ and variance ${\sigma_x}^2 + {\sigma_y}^2$: $Z \sim N(\mu_x + \mu_y, {\sigma_x}^2 + {\sigma_y}^2)$. Multiplication of a normal distribution, $X \sim N(\mu_x, {\sigma_x}^2)$, by a scalar ($k$) is a normal distribution: $kX \sim N(k\mu_x, k^2{\sigma_x}^2)$ [51].

Corollary 16. *Total customer demand, given in (25), can be transformed into a random variable by using Definitions (8), (9), (10), and (11) as follows:*

$$\begin{aligned} TCD &= T_{high} * N\left(\widetilde{X}\left(\theta_{high}\right), Y\left(\theta_{high}\right)\right) + T_{medium} \\ &* N\left(\widetilde{X}\left(\theta_{medium}\right), Y\left(\theta_{medium}\right)\right) + T_{low} \\ &* N\left(\widetilde{X}\left(\theta_{low}\right), Y\left(\theta_{low}\right)\right) = T_{high} \\ &* N\left(\left[\mu(high)_l^{\alpha}, \mu(high)_u^{\alpha}\right], Y\left(\theta_{high}\right)\right) \\ &+ T_{medium} * N\left(\left[\mu(medium)_l^{\alpha}, \mu(medium)_u^{\alpha}\right],\right. \\ &\left. Y\left(\theta_{medium}\right)\right) + T_{low} * N\left(\left[\mu(low)_l^{\alpha}, \mu(low)_u^{\alpha}\right],\right. \\ &\left. Y\left(\theta_{low}\right)\right) = N\left(\left[T_{high} * \mu(high)_l^{\alpha}, T_{high}\right.\right. \\ &\left.\left. * \mu(high)_u^{\alpha}\right], Y\left(\theta_{high}\right)\right) + N\left(\left[T_{medium}\right.\right. \\ &\left. * \mu(medium)_l^{\alpha}, T_{medium} * \mu(medium)_u^{\alpha}\right], \\ &\left. Y\left(\theta_{medium}\right)\right) + N\left(\left[T_{low} * \mu(low)_l^{\alpha}, T_{low}\right.\right. \\ &\left.\left. * \mu(low)_u^{\alpha}\right], Y\left(\theta_{low}\right)\right) = N\left(\left[T_{high} * \mu(high)_l^{\alpha}\right.\right. \\ &+ T_{medium} * \mu(medium)_l^{\alpha} + T_{low} * \mu(low)_l^{\alpha}, T_{high} \\ &* \mu(high)_u^{\alpha} + T_{medium} * \mu(medium)_u^{\alpha} + T_{low} \\ &\left. * \mu(low)_u^{\alpha}\right], T_{high}^2 * Y\left(\theta_{high}\right) + T_{medium}^2 \\ &* Y\left(\theta_{medium}\right) + T_{low}^2 * Y\left(\theta_{low}\right)\Big) = N\left(\left[\mu_{lo}, \mu_{up}\right],\right. \\ &\left.\sigma^2\right), \end{aligned} \tag{29}$$

where

$$\begin{aligned} &T_{high} * \mu(high)_l^{\alpha} + T_{medium} * \mu(medium)_l^{\alpha} + T_{low} \\ &\quad * \mu(low)_l^{\alpha} = \mu_{lo}, \\ &T_{high} * \mu(high)_u^{\alpha} + T_{medium} * \mu(medium)_u^{\alpha} + T_{low} \\ &\quad * \mu(low)_u^{\alpha} = \mu_{up}, \\ &T_{high}^2 * Y\left(\theta_{high}\right) + T_{medium}^2 * Y\left(\theta_{medium}\right) + T_{low}^2 \\ &\quad * Y\left(\theta_{low}\right) = \sigma^2. \end{aligned} \tag{30}$$

According to Corollary 16, the right-hand side of (19) is transformed into a random parameter; however, it is still uncertain. The chance-constrained programming can be used to obtain its deterministic equivalent.

The structure of a chance-constraint is as follows [56]:

$$P\left\{\sum_{i=1}^{N} a_i x_i \le b\right\} \ge 1 - \beta, \quad x_i \ge 0 \ \forall i, \ 0 < \beta < 1. \tag{31}$$

It means that the constraint is realized with a minimum probability of $1 - \beta$. If b is normally distributed parameter, $b \sim N(\mu_b, \sigma_b^2)$, the constraint is converted as follows:

$$P\left\{\frac{\sum_{i=1}^{N} a_i x_i - \mu_b}{\sqrt{\sigma_b^2}} \le \frac{b - \mu_b}{\sqrt{\sigma_b^2}}\right\} \ge 1 - \beta, \tag{32}$$

where $(b - \mu_b)/\sqrt{\sigma_b^2}$ represents a standard normal variate with a mean of zero and a variance of one. Then, the stochastic chance-constraint is transformed into the following inequality:

$$\Phi\left(\frac{\sum_{i=1}^{N} a_i x_i - \mu_b}{\sqrt{\sigma_b^2}}\right) \le \Phi\left(K_{1-\beta}\right), \tag{33}$$

where $\Phi(K_{1-\beta}) = 1 - \beta$ and $\Phi(\ )$ represents the standard normal cumulative distribution function. This yields the following linear deterministic constraint:

$$\sum_{i=1}^{N} a_i x_i \le \mu_b + K_{1-\beta}\sqrt{\sigma_b^2}. \tag{34}$$

4.1.2. Uncertain Parameters in Objective Function. The uncertainties in the constraints are converted into their deterministic equivalents. However, objective function still includes fuzzy parameters. Therefore, Lai and Hwang's [59] approach has been used to obtain deterministic equivalent of the objective function.

Lai and Hwang [59] had handled a mathematical model as given in the following equation:

$$\begin{aligned} \max \quad & \tilde{c}x, \\ \text{s.t.} \quad & \widetilde{A}x \le \tilde{b}, \\ & x \ge 0, \end{aligned} \tag{35}$$

where $\widetilde{A}, \tilde{b}$, and $\tilde{c}$ are triangular fuzzy numbers.

The fuzzy objective function is fully defined by three corner points $(C^p, 0)$, $(C^m, 1)$, and $(C^o, 0)$ geometrically. Lai and Hwang [59] suggested that maximizing the fuzzy objective can be obtained by pushing these three critical points in the direction of the right-hand side. The vertical coordinates of the critical points are fixed at 1 or 0. The only considerations then are the three horizontal coordinates. Therefore, the objective function is translated to the form given in the following equation:

$$\begin{aligned} \max \quad & \left(C^m x, C^p x, C^o x\right), \\ & x \in X. \end{aligned} \tag{36}$$

Instead of maximizing these three objectives simultaneously, Lai and Hwang [59] proposed maximizing $C^m x$, minimizing $[C^m x - C^p x]$, and maximizing $[C^o x - C^m x]$. The proposed approach involves maximizing the most possible value of the profit, minimizing the risk of obtaining lower profit, and maximizing the possibility of obtaining higher profit. The last two objectives actually are relative measures from $C^m x$. This leads us to the auxiliary multiobjective linear programming model given in the following equation:

$$\begin{aligned} \min \quad & z_1 = \left(C^m - C^p\right) x, \\ \max \quad & z_2 = C^m x, \\ \max \quad & z_3 = \left(C^o - C^m\right) x, \\ \text{s.t.} \quad & x \in X. \end{aligned} \tag{37}$$

Lai and Hwang suggested using Zimmermann's [61] fuzzy programming method to convert the auxiliary multiobjective linear programming model into an equivalent single-goal LP problem. First, the positive ideal solutions (PIS) and negative ideal solutions (NIS) of the objective functions can be specified as follows [59]:

$$\begin{aligned} {z_1}^{\text{PIS}} = \min \quad & \left(C^m - C^p\right) x, \\ & x \in X, \\ {z_1}^{\text{NIS}} = \max \quad & \left(C^m - C^p\right) x, \\ & x \in X, \\ {z_2}^{\text{PIS}} = \max \quad & \left(C^m\right) x, \\ & x \in X, \\ {z_2}^{\text{NIS}} = \min \quad & \left(C^m\right) x, \\ & x \in X, \\ {z_3}^{\text{PIS}} = \max \quad & \left(C^o - C^m\right) x, \\ & x \in X, \\ {z_3}^{\text{NIS}} = \min \quad & \left(C^o - C^m\right) x, \\ & x \in X. \end{aligned} \tag{38}$$

The linear membership function of each objective function is defined as follows:

$$\begin{aligned} f_1\left(z_1\right) &= \begin{cases} 1, & z_1 < {z_1}^{\text{PIS}}, \\ \dfrac{\left({z_1}^{\text{NIS}} - z_1\right)}{\left({z_1}^{\text{NIS}} - {z_1}^{\text{PIS}}\right)}, & {z_1}^{\text{PIS}} \le z_1 \le {z_1}^{\text{NIS}}, \\ 0, & z_1 > {z_1}^{\text{NIS}}, \end{cases} \\ f_2\left(z_2\right) &= \begin{cases} 1, & z_2 > {z_2}^{\text{PIS}}, \\ \dfrac{\left({z_2}^{\text{NIS}} - z_2\right)}{\left({z_2}^{\text{NIS}} - {z_2}^{\text{PIS}}\right)}, & {z_2}^{\text{NIS}} \le z_2 \le {z_2}^{\text{PIS}}, \\ 0, & z_2 < {z_2}^{\text{NIS}}; \end{cases} \end{aligned} \tag{39}$$

$f_3(z_3)$ is similar to $f_2(z_2)$.

Lai and Hwang used fuzzy ranking concepts for the constraints and combined them with their strategy for imprecise objective function. The constraints can be modeled by using α-cut approach as follows:

$$\begin{aligned} A^m_\alpha x &\le b^m_\alpha, \\ A^p_\alpha x &\le b^p_\alpha, \\ A^o_\alpha x &\le b^o_\alpha, \\ x &\ge 0. \end{aligned} \tag{40}$$

If only the right-hand sides include fuzzy parameters, Lai and Hwang propose the weighted average method to obtain crisp right-hand side values. Assume that only the right-hand side of the constraint in (35) ($\tilde{b}$) is fuzzy. For a given minimum acceptable possibility, α, the crisp equality constraints can be constructed as follows:

$$\sum_{i=1}^{n} A_i X_i \le w_1 * b^p_\alpha + w_2 * b^m_\alpha + w_3 * b^o_\alpha, \tag{41}$$

where $w_1 + w_2 + w_3 = 1$; w_1, w_2, and w_3 represent the weights of the most pessimistic, most possible, and most optimistic values of the imprecise right-hand side, respectively.

Finally, Zimmermann's following equivalent single-objective linear programming model is used to solve the model [60].

$$\begin{aligned} \max \quad & \lambda, \\ \text{s.t.} \quad & f_i\left(z_i\right) \ge \lambda, \\ & i = 1, 2, 3, \\ & A^m_\alpha x \le b^m_\alpha, \\ & A^p_\alpha x \le b^p_\alpha, \\ & A^o_\alpha x \le b^o_\alpha, \\ & x \ge 0. \end{aligned} \tag{42}$$

4.2. Proposed Solution Approach. The proposed solution approach has been developed by integrating both fuzzy programming and stochastic programming.

The objective function which is fully fuzzy has been handled by using Zimmermann's [60] fuzzy programming method. Therefore, there is no different technique in the proposed approach to convert the objective function. However, different techniques are used in constraints.

The goals of determining positive and negative ideal solutions are to calculate the minimum and maximum values of objective functions. Therefore, the positive and negative ideal solutions are determined according to the pessimistic and optimistic scenarios in uncertain models to obtain robust solutions. However, Lai and Hwang proposed a weighted average method in constraints that only includes fuzziness on right-hand side for obtaining positive and negative ideal solutions. In weighted average method, $w_1 = w_3 = 1/6$ and

$w_2 = 4/6$. This method produces a crisp value that is very close to the most possible value. Therefore, the weighted average method prevents obtaining lower and higher ideal solutions. Naturally, the weighted average method may produce unfeasible solution.

In the proposed approach, the most pessimistic and optimistic values of the right-hand side are used in the fuzzy constraints for the minimum and maximum values of z_1, z_2, and z_3, respectively, instead of weighted average method. The proposed ranking concepts are given as follows (for (3)):

$$\begin{aligned}&\sum_{k=1}^{K}\sum_{p=1}^{P}\mathrm{TRQ}_{ispkt} \le \mathrm{RC}^{p}_{\alpha,ist}\\ &\qquad \forall i,\ \forall s,\ \forall t \text{ for minimum value of } z_1, z_2, z_3,\\ &\sum_{k=1}^{K}\sum_{p=1}^{P}\mathrm{TRQ}_{ispkt} \le \mathrm{RC}^{o}_{\alpha,ist}\\ &\qquad \forall i,\ \forall s,\ \forall t \text{ for minimum value of } z_1, z_2, z_3.\end{aligned} \tag{43}$$

On the other side, the right-hand side of (19) $(\overline{\overline{\mathrm{CDP}}}_{jrtc})$ is converted into a random variable by using Corollary 16 and defined as $N([\mu_{lo}, \mu_{up}], \sigma^2)$. It represents a family of normal distributions whose mean parameter differs in close interval $[\mu_{lo}, \mu_{up}]$ with same variance (σ^2). It follows a normal distribution with $N(\mu_{lo}, \sigma^2)$ and $N(\mu_{up}, \sigma^2)$ parameters in pessimistic and optimistic scenarios, respectively. Therefore, the structure of the chance-constraint for (19) can be written for the minimum and maximum values of z_1, z_2, and z_3 as follows:

$$\begin{aligned}&\mathrm{SPQ}_{jrtc} \le (\mu_{lo})_{jrtc} + K_{1-\beta}\sqrt{\sigma^2}\\ &\qquad \forall j,\ \forall r,\ \forall t,\ \forall c \text{ for minimum value of } z_1, z_2, z_3,\\ &\mathrm{SPQ}_{jrtc} \le (\mu_{up})_{jrtc} + K_{1-\beta}\sqrt{\sigma^2}\\ &\qquad \forall j,\ \forall r,\ \forall t,\ \forall c \text{ for minimum value of } z_1, z_2, z_3.\end{aligned} \tag{44}$$

The PDP model given in (2)–(20) is converted into a deterministic multiobjective 0-1 mixed-integer linear programming model (MOMILP) as follows:

$$\begin{aligned}Z_1 &= \left[\sum_{t=1}^{T}\sum_{j=1}^{J}\left(\left(\mathrm{POP}^{m}_{jt} - \mathrm{POP}^{p}_{jt}\right) * \sum_{c=1}^{C}\sum_{r=1}^{R}\mathrm{SPQ}_{jrtc}\right)\right] - \left[\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{i=1}^{I}\left(\left(\mathrm{RUP}^{m}_{ist} - \mathrm{RUP}^{p}_{ist}\right) * \sum_{p=1}^{P}\sum_{k=1}^{K}\mathrm{TRQ}_{ispkt}\right)\right]\\
&- \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{s=1}^{S}\left(\mathrm{FTCS}_{spkt} * \mathrm{UKS}_{spkt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{i=1}^{I}\left(\left(\mathrm{VTCS}^{m}_{ispkt} - \mathrm{VTCS}^{p}_{ispkt}\right) * \mathrm{TRQ}_{ispkt}\right)\right]\\
&- \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{I}\left(\left(\mathrm{SRC}^{m}_{ipt} - \mathrm{SRC}^{p}_{ipt}\right) * \mathrm{SRP}_{ipt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\left(\mathrm{RPC}^{m}_{jpt} - \mathrm{RPC}^{p}_{jpt}\right) * \mathrm{RPQ}_{jpt}\right)\right]\\
&- \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\left(\mathrm{OPC}^{m}_{jpt} - \mathrm{OPC}^{p}_{jpt}\right) * \mathrm{OPQ}_{jpt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\left(\mathrm{PHC}^{m}_{jpt} - \mathrm{PHC}^{p}_{jpt}\right) * \mathrm{SLP}_{jpt}\right)\right]\\
&- \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{w=1}^{W}\sum_{p=1}^{P}\left(\mathrm{FTCP}_{pwkt} * \mathrm{UKP}_{pwkt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{w=1}^{W}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\left(\mathrm{VTCP}^{m}_{jpwkt} - \mathrm{VTCP}^{p}_{jpwkt}\right) * \mathrm{TPQP}_{jpwkt}\right)\right]\\
&- \left[\sum_{t=1}^{T}\sum_{w=1}^{W}\sum_{j=1}^{J}\left(\left(\mathrm{WHC}^{m}_{jwt} - \mathrm{WHC}^{p}_{jwt}\right) * \mathrm{SLW}_{jwt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{w=1}^{W}\left(\mathrm{FTCW}_{wrkt} * \mathrm{UKW}_{wrkt}\right)\right]\\
&- \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{w=1}^{W}\sum_{j=1}^{J}\left(\left(\mathrm{VTCW}^{m}_{jwrkt} - \mathrm{VTCW}^{p}_{jwrkt}\right) * \mathrm{TPQW}_{jwrkt}\right)\right]\\
&- \left[\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{j=1}^{J}\left(\left(\mathrm{HCR}^{m}_{jrt} - \mathrm{HCR}^{p}_{jrt}\right) * \mathrm{SLR}_{jrt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{j=1}^{J}\left(\left(\mathrm{BCR}^{m}_{jrt} - \mathrm{BCR}^{p}_{jrt}\right) * \left(\sum_{c=1}^{C}\mathrm{BLR}_{jrtc}\right)\right)\right],\end{aligned} \tag{45}$$

$$\begin{aligned}
Z_2 &= \left[\sum_{t=1}^{T}\sum_{j=1}^{J}\left(\mathrm{POP}_{jt}^{m} * \sum_{c=1}^{C}\sum_{r=1}^{R}\mathrm{SPQ}_{jrtc}\right)\right] - \left[\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{i=1}^{I}\left(\mathrm{RUP}_{ist}^{m} * \sum_{p=1}^{P}\sum_{k=1}^{K}\mathrm{TRQ}_{ispkt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{s=1}^{S}\left(\mathrm{FTCS}_{spkt} * \mathrm{UKS}_{spkt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{i=1}^{I}\left(\mathrm{VTCS}_{ispkt}^{m} * \mathrm{TRQ}_{ispkt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{I}\left(\mathrm{SRC}_{ipt}^{m} * \mathrm{SRP}_{ipt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\mathrm{RPC}_{jpt}^{m} * \mathrm{RPQ}_{jpt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\mathrm{OPC}_{jpt}^{m} * \mathrm{OPQ}_{jpt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\mathrm{PHC}_{jpt}^{m} * \mathrm{SLP}_{jpt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{w=1}^{W}\sum_{p=1}^{P}\left(\mathrm{FTCP}_{pwkt} * \mathrm{UKP}_{pwkt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{w=1}^{W}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\mathrm{VTCP}_{jpwkt}^{m} * \mathrm{TPQP}_{jpwkt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{w=1}^{W}\sum_{j=1}^{J}\left(\mathrm{WHC}_{jwt}^{m} * \mathrm{SLW}_{jwt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{w=1}^{W}\left(\mathrm{FTCW}_{wrkt} * \mathrm{UKW}_{wrkt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{w=1}^{W}\sum_{j=1}^{J}\left(\mathrm{VTCW}_{jwrkt}^{m} * \mathrm{TPQW}_{jwrkt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{j=1}^{J}\left(\mathrm{HCR}_{jrt}^{m} * \mathrm{SLR}_{jrt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{j=1}^{J}\left(\mathrm{BCR}_{jrt}^{m} * \left(\sum_{c=1}^{C}\mathrm{BLR}_{jrtc}\right)\right)\right],
\end{aligned} \tag{46}$$

$$\begin{aligned}
Z_3 &= \left[\sum_{t=1}^{T}\sum_{j=1}^{J}\left(\left(\mathrm{POP}_{jt}^{o} - \mathrm{POP}_{jt}^{m}\right) * \sum_{c=1}^{C}\sum_{r=1}^{R}\mathrm{SPQ}_{jrtc}\right)\right] - \left[\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{i=1}^{I}\left(\left(\mathrm{RUP}_{ist}^{o} - \mathrm{RUP}_{ist}^{m}\right) * \sum_{p=1}^{P}\sum_{k=1}^{K}\mathrm{TRQ}_{ispkt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{s=1}^{S}\left(\mathrm{FTCS}_{spkt} * \mathrm{UKS}_{spkt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{i=1}^{I}\left(\left(\mathrm{VTCS}_{ispkt}^{o} - \mathrm{VTCS}_{ispkt}^{m}\right) * \mathrm{TRQ}_{ispkt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{i=1}^{I}\left(\left(\mathrm{SRC}_{ipt}^{o} - \mathrm{SRC}_{ipt}^{m}\right) * \mathrm{SRP}_{ipt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\left(\mathrm{RPC}_{jpt}^{o} - \mathrm{RPC}_{jpt}^{m}\right) * \mathrm{RPQ}_{jpt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\left(\mathrm{OPC}_{jpt}^{o} - \mathrm{OPC}_{jpt}^{m}\right) * \mathrm{OPQ}_{jpt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\left(\mathrm{PHC}_{jpt}^{o} - \mathrm{PHC}_{jpt}^{m}\right) * \mathrm{SLP}_{jpt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{w=1}^{W}\sum_{p=1}^{P}\left(\mathrm{FTCP}_{pwkt} * \mathrm{UKP}_{pwkt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{w=1}^{W}\sum_{p=1}^{P}\sum_{j=1}^{J}\left(\left(\mathrm{VTCP}_{jpwkt}^{o} - \mathrm{VTCP}_{jpwkt}^{m}\right) * \mathrm{TPQP}_{jpwkt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{w=1}^{W}\sum_{j=1}^{J}\left(\left(\mathrm{WHC}_{jwt}^{o} - \mathrm{WHC}_{jwt}^{m}\right) * \mathrm{SLW}_{jwt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{w=1}^{W}\left(\mathrm{FTCW}_{wrkt} * \mathrm{UKW}_{wrkt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{w=1}^{W}\sum_{j=1}^{J}\left(\left(\mathrm{VTCW}_{jwrkt}^{o} - \mathrm{VTCW}_{jwrkt}^{m}\right) * \mathrm{TPQW}_{jwrkt}\right)\right] \\
&- \left[\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{j=1}^{J}\left(\left(\mathrm{HCR}_{jrt}^{o} - \mathrm{HCR}_{jrt}^{m}\right) * \mathrm{SLR}_{jrt}\right)\right] - \left[\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{j=1}^{J}\left(\left(\mathrm{BCR}_{jrt}^{o} - \mathrm{BCR}_{jrt}^{m}\right) * \left(\sum_{c=1}^{C}\mathrm{BLR}_{jrtc}\right)\right)\right],
\end{aligned} \tag{47}$$

$$\text{s.t.}\quad \sum_{k=1}^{K}\sum_{p=1}^{P}\mathrm{TRQ}_{ispkt} \le \mathrm{RC}_{\alpha,ist}^{p} \tag{48}$$

$$\forall i,\ \forall s,\ \forall t,$$

$$\sum_{k=1}^{K}\sum_{p=1}^{P}\mathrm{TRQ}_{ispkt} \le \mathrm{RC}_{\alpha,ist}^{o} \tag{49}$$

$$\forall i,\ \forall s,\ \forall t,$$

$$\sum_{i=1}^{I}\left(\mathrm{RRC}_i * \mathrm{TRQ}_{ispkt}\right) \le \left(\mathrm{TCSP}_L\right)_{spkt} + \left(1-\mathrm{UKS}_{spkt}\right) * M \tag{50}$$

$$\forall s,\ \forall p,\ \forall k,\ \forall t,$$

$$\sum_{i=1}^{I}\left(\mathrm{RRC}_i * \mathrm{TRQ}_{ispkt}\right) \le \left(\mathrm{TCSP}_U\right)_{spkt} + \left(1-\mathrm{UKS}_{spkt}\right) * M \tag{51}$$

$$\forall s,\ \forall p,\ \forall k,\ \forall t,$$

$$\sum_{i=1}^{I}\left(\mathrm{RRC}_i * \mathrm{TRQ}_{ispkt}\right) \le \mathrm{UKS}_{spkt} * M \tag{52}$$

$$\forall s,\ \forall p,\ \forall k,\ \forall t,$$

$$\left(\sum_{k=1}^{K}\sum_{s=1}^{S}\mathrm{TRQ}_{ispkt} + \mathrm{SRP}_{ip(t-1)}\right) - \sum_{j=1}^{J}\left(\mathrm{RRM}_{ij} * \left(\mathrm{RPQ}_{jpt} + \mathrm{OPQ}_{jpt}\right)\right) = \mathrm{SRP}_{ipt} \tag{53}$$

$$\forall i,\ \forall p,\ \forall t,$$

$$\sum_{j=1}^{J}\left(\mathrm{UPT}_{jpt} * \mathrm{RPQ}_{jpt}\right) \le \mathrm{ARC}_{\alpha,pt}^{p} \tag{54}$$

$$\forall p,\ \forall t,$$

$$\sum_{j=1}^{J}\left(\mathrm{UPT}_{jpt} * \mathrm{RPQ}_{jpt}\right) \le \mathrm{ARC}_{\alpha,pt}^{o} \tag{55}$$

$$\forall p,\ \forall t,$$

$$\sum_{j=1}^{J}\left(\mathrm{UPT}_{jpt} * \mathrm{OPQ}_{jpt}\right) \le \mathrm{AOC}_{\alpha,pt}^{p} \tag{56}$$

$$\forall p,\ \forall t,$$

$$\sum_{j=1}^{J}\left(\mathrm{UPT}_{jpt} * \mathrm{OPQ}_{jpt}\right) \le \mathrm{AOC}_{\alpha,pt}^{o} \tag{57}$$

$$\forall p,\ \forall t,$$

$$\sum_{j=1}^{J}\left(\mathrm{RHC}_j * \mathrm{SLP}_{jpt}\right) \le \mathrm{PIC}_p \tag{58}$$

$$\forall p,\ \forall t,$$

$$\mathrm{RPQ}_{jpt} + \mathrm{OPQ}_{jpt} + \mathrm{SLP}_{jp(t-1)} - \sum_{k=1}^{K}\sum_{w=1}^{W}\mathrm{TPQP}_{jpwkt} = \mathrm{SLP}_{jpt} \quad (59)$$

$\forall j,\ \forall p,\ \forall t,$

$$\sum_{j=1}^{J}\left(\mathrm{RTC}_j * \mathrm{TPQP}_{jpwkt}\right) \leq \left(\mathrm{TCPW}_L\right)_{pwkt} + \left(1 - \mathrm{UKP}_{pwkt}\right) * M \quad (60)$$

$\forall p,\ \forall w,\ \forall k,\ \forall t,$

$$\sum_{j=1}^{J}\left(\mathrm{RTC}_j * \mathrm{TPQP}_{jpwkt}\right) \leq \left(\mathrm{TCPW}_U\right)_{pwkt} + \left(1 - \mathrm{UKP}_{pwkt}\right) * M \quad (61)$$

$\forall p,\ \forall w,\ \forall k,\ \forall t,$

$$\sum_{j=1}^{J}\left(\mathrm{RTC}_j * \mathrm{TPQP}_{jpwkt}\right) \leq \mathrm{UKP}_{pwkt} * M \quad (62)$$

$\forall p,\ \forall w,\ \forall k,\ \forall t,$

$$\sum_{k=1}^{K}\sum_{p=1}^{P}\mathrm{TPQP}_{jpwkt} + \mathrm{SLW}_{jw(t-1)} - \sum_{k=1}^{K}\sum_{r=1}^{R}\mathrm{TPQW}_{jwrkt} = \mathrm{SLW}_{jwt} \quad (63)$$

$\forall j,\ \forall w,\ \forall t,$

$$\sum_{j=1}^{J}\left(\mathrm{RHC}_j * \mathrm{SLW}_{jwt}\right) \leq \mathrm{WIC}_w \quad (64)$$

$\forall w,\ \forall t,$

$$\sum_{j=1}^{J}\left(\mathrm{RTC}_j * \mathrm{TPQW}_{jwrkt}\right) \leq \left(\mathrm{TCWR}_L\right)_{wrkt} + \left(1 - \mathrm{UKW}_{wrkt}\right) * M \quad (65)$$

$\forall w,\ \forall r,\ \forall k,\ \forall t,$

$$\sum_{j=1}^{J}\left(\mathrm{RTC}_j * \mathrm{TPQW}_{jwrkt}\right) \leq \left(\mathrm{TCWR}_U\right)_{wrkt} + \left(1 - \mathrm{UKW}_{wrkt}\right) * M \quad (66)$$

$\forall w,\ \forall r,\ \forall k,\ \forall t,$

$$\sum_{j=1}^{J}\left(\mathrm{RTC}_j * \mathrm{TPQW}_{jwrkt}\right) \leq \mathrm{UKW}_{wrkt} * M \quad (67)$$

$\forall w,\ \forall r,\ \forall k,\ \forall t,$

$$\sum_{k=1}^{K}\sum_{w=1}^{W}\mathrm{TPQW}_{jwrkt} + \mathrm{SLR}_{jr(t-1)} - \sum_{c=1}^{C}\mathrm{SPQ}_{jrtc} = \mathrm{SLR}_{jrt} - \sum_{c=1}^{C}\mathrm{BLR}_{jrtc} \quad (68)$$

$\forall j,\ \forall r,\ \forall t,$

$$\sum_{j=1}^{J}\left(\mathrm{RHC}_j * \mathrm{SLR}_{jrt}\right) \leq \mathrm{RIC}_r \quad (69)$$

$\forall r,\ \forall t,$

$$\begin{aligned}&\mathrm{SPQ}_{jrtc} \le \left(\mu_{lo}\right)_{jrtc} + K_{1-\beta}\sqrt{\sigma^2}\\&\forall j,\ \forall r,\ \forall t,\ \forall c,\end{aligned} \tag{70}$$

$$\begin{aligned}&\mathrm{SPQ}_{jrtc} \le \left(\mu_{up}\right)_{jrtc} + K_{1-\beta}\sqrt{\sigma^2}\\&\forall j,\ \forall r,\ \forall t,\ \forall c,\end{aligned} \tag{71}$$

$$\begin{aligned}&\mathrm{TRQ}_{ispkt}, \mathrm{SRP}_{ipt}, \mathrm{RPQ}_{jpt}, \mathrm{OPQ}_{jpt}, \mathrm{SLP}_{jpt}, \mathrm{TPQP}_{jpwkt} \ge 0,\\&\mathrm{SLR}_{jrt}, \mathrm{BLR}_{jrtc}, \mathrm{SPQ}_{jrtc}, \mathrm{SLW}_{jwt}, \mathrm{TPQW}_{jwrkt} \ge 0,\\&\mathrm{UKS}_{spkt}, \mathrm{UKP}_{pwkt}, \mathrm{UKW}_{wrkt} = 0, 1.\end{aligned} \tag{72}$$

In order to obtain PIS and NIS values for z_1, z_2, and z_3, the mathematical models given below are solved:

For ${z_1}^{\mathrm{PIS}}$: min Z_1

s.t. (48), (50), (52), (53), (54), (56), (58), (59), (60), (62), (63), (64), (65), (67), (68), (69), (70), and (72)

For ${z_1}^{\mathrm{NIS}}$: max Z_1

s.t. (49), (51), (52), (53), (55), (57), (58), (59), (61), (62), (63), (64), (66), (67), (68), (69), (71), and (72)

For ${z_2}^{\mathrm{PIS}}$: max Z_2

s.t. (49), (51), (52), (53), (55), (57), (58), (59), (61), (62), (63), (64), (66), (67), (68), (69), (71), and (72)

For ${z_2}^{\mathrm{NIS}}$: min Z_2

s.t. (48), (50), (52), (53), (54), (56), (58), (59), (60), (62), (63), (64), (65), (67), (68), (69), (70), and (72)

For ${z_3}^{\mathrm{PIS}}$: max Z_3

s.t. (49), (51), (52), (53), (55), (57), (58), (59), (61), (62), (63), (64), (66), (67), (68), (69), (71), and (72)

For ${z_3}^{\mathrm{NIS}}$: min Z_3

s.t. (48), (50), (52), (53), (54), (56), (58), (59), (60), (62), (63), (64), (65), (67), (68), (69), (70), and (72)

Finally, Zimmermann's following equivalent single-objective 0-1 mixed-integer programming model is used to solve the model.

$$\begin{aligned}\max\quad & \lambda,\\ \text{s.t.}\quad & f_i\left(z_i\right) \ge \lambda,\\ & i = 1, 2, 3,\\ & \text{Eq. } (48), (50), (51), (52), (53), (55), (57), (58), (59), (60), (61), (62), (63), (64), (65), (66), (68), (69).\end{aligned} \tag{73}$$

The algorithm of the solution methodology for practical PDP decisions is as follows.

Step 1. Formulate the PDP model.

Step 2. Model the fuzzy parameters as triangular fuzzy numbers, model the discrete fuzzy random parameters (the capacities of transportation paths) as triangular fuzzy numbers with probability values, and model the random fuzzy parameter (demand) as normal distributions with triangular fuzzy mean parameters.

Step 3. Develop three new crisp objective functions of the auxiliary MOMILP problem from the fully fuzzy objective function which are equivalent simultaneously maximizing the most possible total profit, minimizing the risk of obtaining lower profit, and maximizing the of possibility of obtaining higher profit.

Step 4. Determine an α value and transform the fuzzy parameters in the right-hand side of the constraints into deterministic close intervals by using α-cut approach. Produce two separate constraints from these constraints that one of them uses lower bound of close interval and the other one uses upper bound of close interval.

Step 5. Transform the discrete fuzzy random parameters in the right-hand side of the constraints into deterministic close intervals according to Corollary 12. Produce two separate constraints from these constraints that one of them uses lower bound of close interval and the other one uses upper bound of close interval.

Step 6. Transform the random fuzzy parameters in the right-hand side of the constraints into normally distributed random parameters with deterministic close interval mean parameters according to Corollaries 14 and 16.

Step 7. Determine an acceptable probability value (β) and model the constraint obtained in Step 6 as deterministic linear constraint by using chance-constraint approach. Produce two separate constraints from this constraint that one of them uses lower bound of close interval and the other one uses upper bound of close interval.

Step 8. Solve z_1^{PIS}, z_1^{NIS}, z_2^{PIS}, z_2^{NIS}, z_3^{PIS}, and z_3^{NIS}. Obtain maximum and minimum values for z_1, z_2, and z_3. Specify the linear membership functions for each of them, and then convert the auxiliary MOMILP problem into a single-objective 0-1 mixed-integer programming model.

Step 9. Solve the single-objective 0-1 mixed-integer programming model and obtain the solution.

Step 10. If the DM is not satisfied with the initial solution, return to Step 4 and modify α and β values and repeat the remaining steps until a satisfactory solution is found.

5. Implementation of the Proposed Solution Approach

The proposed modeling and solution approaches have been implemented for furniture manufacturer in Turkey. The firm has two plants ($p = 1, 2$) in Ankara and Bursa. It produces three different types of products ($j = 1, 2, 3$) by using six different types of raw materials ($i = 1, 2, \ldots, 6$) which are supplied from three suppliers ($s = 1, 2, 3$). The firm uses three warehouses ($w = 1, 2, 3$) and five retailers ($r = 1, 2, \ldots, 5$) to deliver the products to the costumers by using three different types of transportation paths ($k = 1, 2, 3$) which are trucks, trains, and planes. The demands of the products significantly increase in summer; therefore, time period is three months, that is, June, July, and August ($t = 1, 2, 3$), for planning horizon. The costumers were not categorized ($c = 1$) because of tactical planning decision. The proposed modeling and solution approaches were performed at February 2017 for summer season in 2017. The assumptions given in Section 3.1 are held for this real-life application.

The implementation of the algorithm for real-life application is given as follows.

Step 1. PDP for furniture manufacturer has been modeled as (2)–(20).

Step 2. Unit prices of raw materials ($\widetilde{\mathrm{RUP}}_{ist}$) which are given in Table 8, capacities of raw materials ($\widetilde{\mathrm{RC}}_{ist}$) which are given in Table 9, unit transportation costs of raw material from suppliers to plants ($\widetilde{\mathrm{VTCS}}_{ispkt}$) which are given in Table 12, unit holding costs of raw materials in plants ($\widetilde{\mathrm{SRC}}_{ipt}$) which are given in Table 15, unit production costs of products in regular time ($\widetilde{\mathrm{RPC}}_{jpt}$) which are given in Table 17, available regular time capacities of plants ($\widetilde{\mathrm{ARC}}_{pt}$) which are given in Table 18, unit production costs of products in overtime ($\widetilde{\mathrm{OPC}}_{jpt}$) which are given in Table 19, available overtime capacities of plants ($\widetilde{\mathrm{AOC}}_{pt}$) which are given in Table 20, unit holding costs of products in plants ($\widetilde{\mathrm{PHC}}_{jpt}$) which are given in Table 22, unit transportation costs of products from plants to warehouses ($\widetilde{\mathrm{VTCP}}_{jpwkt}$) which are given in Table 26, unit transportation costs of products from warehouses to retailers ($\widetilde{\mathrm{VTCW}}_{jwrkt}$) which are given in Table 30, unit holding costs of products in warehouses ($\widetilde{\mathrm{WHC}}_{jwt}$) which are given in Table 31, unit holding costs of products in retailers ($\widetilde{\mathrm{HCR}}_{jrt}$) which are given in Table 33, backorder costs of products ($\widetilde{\mathrm{BCR}}_{jrt}$) which are given in Table 35, and price of products ($\widetilde{\mathrm{POP}}_{jt}$) which is given in Table 37 have been modeled as triangular fuzzy numbers by making a discussion and analysis with a group of staff which contains procurement, production planning, marketing, and warehouse and retailers managers.

It has been observed that the capacities of transportation paths between echelons of SCS can occur in three different situations, high, medium, and low capacities, when the historical transportation data was analyzed. The probabilities of obtaining high, medium, and low capacities have been determined as 0.5, 0.35, and 0.15, respectively. However, for each situation, the quantities of capacities can change based on the availability of transporters. Therefore, total capacities of transportation from suppliers to plants ($\overline{\mathrm{TCSP}}_{spkt}$) which are given in Table 10, total capacities of transportation from plants to warehouses ($\overline{\mathrm{TCPW}}_{pwkt}$) which are given in Table 24, and total capacities of transportation from warehouses to retailers ($\overline{\mathrm{TCWR}}_{wrkt}$) which are given in Table 28 have been modeled as triangular fuzzy numbers by making a discussion and analysis with a group of staff.

It has been observed that the demands of products can occur in three different states, high, medium, and low demands, by analyzing the historical demand data with bubble charts and the quantity of demand follows a probability distribution for each state. Three indicators have been identified which affect the demand state, that is, political development, competitors' strategy, and sectoral expectation. Six alternative situations have been generated by using these three indicators according to the expertise of management and a possibility value of occurrence has been assigned to each situation by the managers. The probability values of demand that can be high, medium, or low at each situation have been calculated according to the frequency analysis. Therefore, the probability of demand has been modeled as random fuzzy number which is given in Table 1. On the other side, the quantities of the demand for high, medium, and low states follow a normal distribution with a mean and variance parameter according to the results of Anderson-Darling test. However, the managers mentioned that they can affect the demand by using advertisements and discounts. Therefore the mean parameters of the normal distributions have been modeled as triangular fuzzy numbers which are given in Table 36. Fixed costs of using transport paths from suppliers to plants, required transportation capacities of raw materials, required amounts of raw materials for unit products, unit production times of products in plants at each period, inventory capacities in plants, required capacities to store products, fixed cost of using transport paths from plants to warehouses, required capacities to transport unit products, fixed cost of using paths from warehouses to retailers, and inventory capacities in warehouses are given in Tables 11, 13, 14, 16, 21, 23, 25, 27, 29, and 32, respectively.

Step 3. Three new crisp objective functions of the auxiliary MOMILP problem have been developed from the fully fuzzy objective function.

Step 4. The α value has been determined as 0.4 and the constraints that contain fuzzy parameters in the right-hand side have been transformed into deterministic close intervals by using α-cut approach. Two separate constraints have been produced from these constraints that one of them uses lower bound of close interval and the other one uses upper bound of close interval.

Step 5. The constraints that contain the discrete fuzzy random parameters in the right-hand side have been transformed into deterministic close intervals according to Corollary 12. Two separate constraints have been produced from these constraints that one of them uses lower bound of close interval and the other one uses upper bound of close interval.

Step 6. The constraints which contain the random fuzzy parameters in the right-hand sides have been transformed into normally distributed random parameters with deterministic close interval mean parameters according to Corollaries 14 and 16.

Step 7. An acceptable probability value (β) has been determined as 0.95 and the constraint obtained in Step 6 has been modeled as deterministic linear constraint by using chance-constraint approach. Two separate constraints have been produced from this constraint that one of them uses lower bound of close interval and the other one uses upper bound of close interval.

Step 8. ${z_1}^{\text{PIS}}$, ${z_1}^{\text{NIS}}$, ${z_2}^{\text{PIS}}$, ${z_2}^{\text{NIS}}$, ${z_3}^{\text{PIS}}$, and ${z_3}^{\text{NIS}}$ have been solved globally optimally by using GAMS/CPLEX solver. Maximum and minimum values for z_1, z_2, and z_3 have been obtained as follows:

$$\begin{aligned}
{z_1}^{\text{PIS}} &= \min\left(C^m - C^p\right)x = 1462617.078084, \quad x \in X, \\
{z_1}^{\text{NIS}} &= \max\left(C^m - C^p\right)x = 2286767.846530, \quad x \in X, \\
{z_2}^{\text{PIS}} &= \max\left(C^m\right)x = 10726634.165484, \quad x \in X, \\
{z_2}^{\text{NIS}} &= \min\left(C^m\right)x = 4993515.423244, \quad x \in X, \\
{z_3}^{\text{PIS}} &= \max\left(C^o - C^m\right)x = 2142661.24258, \quad x \in X, \\
{z_3}^{\text{NIS}} &= \min\left(C^o - C^m\right)x = 1392563.360486, \quad x \in X.
\end{aligned} \tag{74}$$

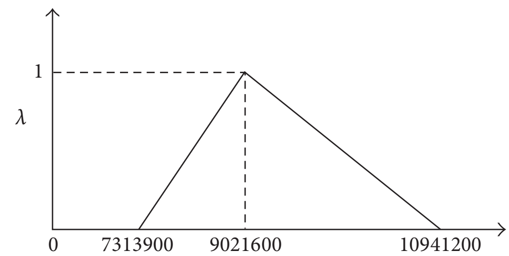

Figure 1

The linear membership functions for each of them have been specified and then the auxiliary MOMILP problem has been converted into a single-objective 0-1 mixed-integer programming model.

Step 9. When the equivalent single-objective 0-1 mixed-integer programming model has been solved, the total profit is obtained as a triangular fuzzy variable with 7313900, 9021600, and 10941200, which is given in Figure 1. The overall degree of DM satisfaction with multiple goal values λ is achieved at 0.703. The solutions of the model are represented in Tables 2–6.

Step 10. DM has been satisfied with the initial solution and the algorithm has been terminated.

Summary of the results during the three time periods is given in Tables 2–6. Total sales quantities are represented in Table 2. According to these solutions, capacities of the retailers, given in Table 34, are adequate to meet the demands. Therefore, the managers do not focus on the retailers.

Total quantities of products transported from warehouses to retailers are given in Table 3. It is seen that transportation capacities between warehouses and retailers, given in Table 28, are adequate when compared with the results in Table 3. Therefore, the managers do not seek additional transportation capacities.

In Table 4, it is observed that plant 2 has not sent any products to warehouse 3 based on high transformation costs. Therefore, the managers search decreasing transformation costs or any other transformation alternatives from plant 2 to warehouse 3.

On the other hand, plant 1 does not manufacture product 3 according to Table 5. The high production cost may be the reason. The managers seek the way of decreasing production costs.

Table 6 shows that the SCS has not satisfied the demands exactly; therefore, there is a bottleneck in the SCS system. When the SCS system is analyzed in detail, it is identified that the reason of the bottleneck in the system is the limited transportation capacity of products from plants to warehouses. Therefore, there is abundance in the capacities of the remaining components of SCS.

It is obvious that the decision-makers can decrease the backorders costs and consequently increase the total profit by increasing the transportation capacity from plants to warehouses supplied by new transporters. However, they

Table 2: Total sale quantities of product j in retailer r.

	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$j = 1$	1761.29	1729.05	1742.92	1604.47	1682.56
$j = 2$	1264.76	1366.29	1023.31	766.33	1026.82
$j = 3$	1288.14	1072.73	1096.14	1100	1050.32

Table 3: Total quantities of product j transported from warehouse w to retailer r.

		$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$w = 1$	$j = 1$	1325.01	936.95	1742.92	0	1568.58
	$j = 2$	745.11	0	0	530.38	275.02
	$j = 3$	1288.14	456.61	1096.14	391.64	402.92
$w = 2$	$j = 1$	0	792.10	0	1604.47	0
	$j = 2$	384.04	1366.29	1023.31	235.94	284.61
	$j = 3$	0	616.12	0	708.35	647.40
$w = 3$	$j = 1$	436.28	0	0	0	113.97
	$j = 2$	135.60	0	0	0	467.17
	$j = 3$	0	0	0	0	0

Table 4: Total quantities of product j transported from plant p to warehouse w.

		$w = 1$	$w = 2$	$w = 3$
$p = 1$	$j = 1$	5573.47	1810.07	550.25
	$j = 2$	107.83	2412.32	602.78
	$j = 3$	0	0	0
$p = 2$	$j = 1$	0	586.50	0
	$j = 2$	1442.68	881.89	0
	$j = 3$	3635.47	1971.88	0

Table 5: Total quantities of product j manufactured in plant p.

Product	Plant	
	$p = 1$	$p = 2$
$j = 1$	7933.80	586.50
$j = 2$	3122.95	2324.58
$j = 3$	0	5607.36

Table 6: Total backorder quantities of product j in retailer r.

	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$j = 1$	2.38	2.55	2.54	162.89	2.46
$j = 2$	491.59	327.63	628.57	979.30	715.75
$j = 3$	343.09	538.79	550.52	552.48	527.60

predict that the cooperation with new transporters may cause harmonization problems and extra costs. Therefore, the decision-makers plan to find new transporters and cooperate with them for next planning periods after performing a transporter evaluation system.

The real-life application has been performed at different α and β parameter values. Total profits in triangular fuzzy number and defuzzified form for different parameter values are presented in Table 7. The weighted average method was used in defuzzification with $w_1 = w_3 = 1/6$ and $w_2 = 4/6$. In the first column of Table 7, the effects of changes in β parameter are examined when α is fixed. The change in β directly affects the demand quantity. Total unsatisfied demand quantity increases when β increases; therefore, total profit decreases for $\beta = 0.98$ and $\beta = 0.99$. On the other side, it is expected to obtain lower profit when β decreases because of lower demand. However, it is observed that total profit is increased for $\beta = 0.90$ based on decreasing unsatisfied demand quantity.

The change in α parameter directly/only affects the SCS capacity (raw material, transportation, and production).

Table 7: Total profits at different parameter levels.

	Total profit ($\alpha = 0.4$) Fuzzy	Defuzzified		Total profit ($\beta = 0.95$) Fuzzy	Defuzzified
$\beta = 0.85$	(7283661; 8891223; 10140210)	8831461	$\alpha = 0.30$	(7503750; 9223326; 11283544)	9280100
$\beta = 0.90$	(7401500; 9123400; 11032550)	9154608	$\alpha = 0.35$	(7410700; 9150245; 11111550)	9187205
$\beta = 0.98$	(7113572; 8724770; 9794213)	8634478	$\alpha = 0.45$	(7265180; 8824735; 10771893)	8889336
$\beta = 0.99$	(7052392; 8671245; 9731677)	8578175	$\alpha = 0.50$	(7092630; 8622100; 10480112)	8676857

Table 8: Unit price of raw material i in supplier s at period t.

Raw material	Supplier $s = 1$	$s = 2$	$s = 3$
$i = 1$	(40; 42; 43)	(39; 40; 41)	(37; 38; 39)
$i = 2$	(35; 36; 37)	(37; 38; 39)	(38; 40; 41)
$i = 3$	(39; 40; 41)	(37; 38; 39)	(41; 42; 44)
$i = 4$	(28; 30; 32)	(29; 30; 33)	(29; 30; 31)
$i = 5$	(31; 32; 33)	(32; 33; 34)	(30; 31; 32)
$i = 6$	(16; 18; 19)	(20; 21; 22)	(18; 19; 21)

Table 9: Capacity of raw material i supplied by s at period t.

Raw material	Supplier $s = 1$	$s = 2$	$s = 3$
$i = 1$	(10500; 12000; 13000)	(11000; 12900; 14000)	(13000; 14500; 16000)
$i = 2$	(10000; 10600; 11500)	(8500; 9600; 11000)	(15000; 16800; 17500)
$i = 3$	(10000; 11300; 12000)	(12000; 12900; 13800)	(13000; 15000; 16000)
$i = 4$	(11000; 13900; 15000)	(10500; 11800; 13200)	(15000; 17900; 19000)
$i = 5$	(12000; 13200; 14000)	(9400; 10300; 11500)	(15000; 16700; 18000)
$i = 6$	(10500; 11000; 13500)	(9000; 9400; 10800)	(16500; 18200; 19000)

Table 10: Capacity of transport path k from supplier s to plant p at period t.

Path	Supplier	High probability (0.5) $p = 1$	$p = 2$	Medium probability (0.35) $p = 1$	$p = 2$	Low probability (0.15) $p = 1$	$p = 2$
$k1$	$S1$	(20000; 20800; 21600)	(21500; 22125; 23000)	(15800; 16300; 17000)	(17000; 17850; 18500)	(15000; 16000; 16800)	(16500; 17400; 18000)
	$S2$	(20000; 20550; 21300)	(19700; 20600; 21200)	(15500; 16250; 17000)	(17000; 17600; 18000)	(14000; 14500; 15250)	(16500; 17000; 18000)
	$S3$	(15800; 16800; 18000)	(23800; 24400; 25000)	(15500; 16100; 16700)	(17000; 17200; 18000)	(14000; 14500; 15000)	(16000; 16600; 17200)
$k2$	$S1$	(22750; 23500; 25000)	0	(17000; 18000; 19000)	0	(16500; 17300; 18000)	0
	$S2$	(23800; 24500; 25000)	0	(17000; 17850; 19000)	0	(16000; 17200; 18000)	0
	$S3$	(21600; 22300; 23500)	0	(16000; 16500; 17000)	0	(15600; 16300; 17100)	0
$k3$	$S1$	(15900; 16200; 16500)	(14900; 15200; 16000)	(14000; 14600; 15000)	(14000; 14800; 15500)	(12500; 13700; 14300)	(11200; 12000; 12500)
	$S2$	(16500; 17300; 18000)	(16900; 17300; 17800)	(14250; 15000; 15500)	(16500; 17200; 17600)	(11800; 12500; 14000)	(12700; 13300; 14000)
	$S3$	(17500; 18000; 18500)	(16000; 16400; 17000)	(15000; 15500; 16000)	(16000; 16700; 17000)	(11800; 12500; 13000)	(12500; 13000; 13700)

Table 11: Fixed cost of transport path k from supplier s to plant p at period t.

Path	Supplier	Plant	
		$p = 1$	$p = 2$
$k1$	$s = 1$	690	735
	$s = 2$	570	700
	$s = 3$	500	540
$k2$	$s = 1$	810	0
	$s = 2$	965	0
	$s = 3$	990	0
$k3$	$s = 1$	1400	1300
	$s = 2$	1250	1300
	$s = 3$	1120	1200

Table 12: Unit transportation cost of raw material i from supplier s to plant p by using path k at period t.

Path	Supplier	$p = 1$	$p = 2$
$k1$	$s = 1$	(0.4; 0.5; 0.6)	(0.45; 0.5; 0.6)
	$s = 2$	(0.5; 0.6; 0.7)	(0.4; 0.5; 0.65)
	$s = 3$	(0.4; 0.5; 0.6)	(0.45; 0.6; 0.7)
$k2$	$s = 1$	(0.8; 1; 1.2)	0
	$s = 2$	(0.9; 1.1; 1.2)	0
	$s = 3$	(0.9; 1; 1.1)	0
$k3$	$s = 1$	(2.1; 2.4; 2.5)	(2.5; 2.6; 2.8)
	$s = 2$	(2; 2.2; 2.4)	(2.2; 2.4; 2.6)
	$s = 3$	(1.8; 2; 2.1)	(2; 2.2; 2.5)

Table 13: Required transportation capacity of raw material i.

$i = 1$	1.35
$i = 2$	1.12
$i = 3$	1.71
$i = 4$	2.52
$i = 5$	1.63
$i = 6$	1.92

Table 14: Required amount of raw material i for unit product j.

	Product		
	$j = 1$	$j = 2$	$j = 3$
$i = 1$	2	3	0
$i = 2$	1	1	3
$i = 3$	0	1	3
$i = 4$	1	3	0
$i = 5$	1	0	2
$i = 6$	3	0	2

Therefore, total profit decreases or increases when α increases or decreases, respectively.

The different α and β parameter values represent different scenarios; therefore, it is not possible to determine the best or worst solution in Table 7.

Table 15: Unit holding cost of raw material i in plant p at period t.

Raw material	Plant	
	$p = 1$	$p = 2$
$i = 1$	(0.15; 0.2; 0.22)	(0.15; 0.2; 0.24)
$i = 2$	(0.14; 0.16; 0.19)	(0.15; 0.18; 0.22)
$i = 3$	(0.16; 0.18; 0.21)	(0.13; 0.16; 0.2)
$i = 4$	(0.15; 0.18; 0.2)	(0.14; 0.17; 0.2)
$i = 5$	(0.15; 0.18; 0.2)	(0.14; 0.16; 0.18)
$i = 6$	(0.12; 0.14; 0.16)	(0.13; 0.17; 0.2)

Table 16: Unit production time of product j in plant p at period t.

Product	Plant	
	$p = 1$	$p = 2$
$j = 1$	6.15	6.02
$j = 2$	6.92	6.94
$j = 3$	8.54	6.74

Table 17: Unit production cost of product j at regular time in plant p at period t.

Product	Plant	
	$p = 1$	$p = 2$
$j = 1$	(62.12; 64.58; 65.81)	(66.22; 72.24; 77.06)
$j = 2$	(69.89; 72.66; 74.04)	(76.34; 83.28; 88.83)
$j = 3$	(86.25; 89.67; 91.38)	(74.14; 80.88; 86.27)

Table 18: Available regular capacity (time) in plant p at period t.

Plant	
$p = 1$	$p = 2$
(21000; 23000; 25000)	(18000; 19000; 20000)

6. Conclusion

In this paper, a PDP for a SCS which includes multiple suppliers, multiple products, multiple plants, multiple warehouses, multiple retailers, multiple transport paths, and multiple time periods has been considered in uncertain environment at

Table 19: Unit production cost of product j at overtime in plant p at period t.

Product	Plant	
	$p = 1$	$p = 2$
$j = 1$	(77.64; 80.72; 82.26)	(82.78; 90.3; 96.32)
$j = 2$	(87.37; 90.83; 92.56)	(95.43; 104.1; 111.04)
$j = 3$	(107.82; 112.09; 114.22)	(92.68; 101.1; 107.84)

Table 20: Available overtime capacity (time) in plant p at period t.

Plant	
$p = 1$	$p = 2$
(4000; 5000; 5500)	(3000; 4000; 5000)

Table 21: Inventory capacity in plant p.

$p = 1$	$p = 2$	$p = 3$
8000	10000	6000

Table 22: Unit holding cost of product j in plant p at period t.

Product	Plant	
	$p = 1$	$p = 2$
$j = 1$	(6.21; 6.46; 6.58)	(6.62; 7.22; 7.71)
$j = 2$	(6.99; 7.27; 7.4)	(7.63; 8.33; 8.88)
$j = 3$	(8.63; 8.97; 9.14)	(7.41; 8.09; 8.63)

Table 23: Required capacity to store unit product j.

$j = 1$	$j = 2$	$j = 3$
2	3	2.5

tactical level. A modeling approach and a solution approach have been proposed for PDP.

In modeling approach, a 0-1 mixed-integer mathematical model has been developed for PDP. The uncertain parameters in the model have been handled as fuzzy, random, fuzzy random, and random fuzzy variables. A solution approach is developed for 0-1 mixed-integer mathematical model by integrating possibilistic programming and chance-constrained programming techniques.

The proposed modeling and solution approaches have been implemented on a PDP of furniture firm. The PDP has been solved globally optimally by using GAMS optimization package. The solutions have been represented to the firm managers and managers have been satisfied with these solutions. The main problem in SCS is the lower transportation capacity between plants and warehouses. Therefore, managers decide to seek new transporters for the next planning periods.

The proposed modeling and solution approaches can be compared with the other PDP approaches such as deterministic models, stochastic models, or fully fuzzy models. The solution of the deterministic PDP can be easily infeasible in dynamic structure of the system at tactical level because of ignoring uncertainty. It is an alternative way to perform the deterministic model several times with different parameter values to analyze the changes of the parameters. However, hundreds of solutions are obtained for a SCS which includes a lot of uncertain parameters. Consequently, it causes another uncertainty called abundance of the solutions. Therefore, it is not an effective way to make decision at tactical level.

On the other hand, stochastic models only contain probabilistic uncertainties in SCS and do not consider the other uncertainties that are based on decision-maker's expertise and judgements. Probability theory successfully handles the variation in problem nature by assuming that the conditions of the experiment will not change. However, the conditions such as political developments, competitors' strategy, or manager's decision may change in a dynamic system and that creates a situation which has not been observed in the past. Therefore, the solutions of the stochastic models can be infeasible.

Contrary to stochastic models, fully fuzzy models for PDP do not consider the variations in the problem nature. The fully fuzzy models produce rough and subjective solutions.

The main advantage of the proposed modeling and solution approaches is including the decision-maker into the problem formulation and solving processes with probabilistic uncertainties. Decision-makers directly affect the problem parameters by their decisions. Fuzzy, fuzzy random, and random fuzzy variables enable including the decision-maker's judgements and expertise into the probabilistic model. In this way, the mathematical model produces robust solutions.

There are some limitations of the proposed modeling and solution approaches. One of them is the determination of the α value. The α value represents the minimum acceptable possibility degree, in other words, satisfaction level of decision-maker. It may not be meaningful in decision-maker's mind like probability level. Therefore, it should be explained clearly to the decision-maker in implementation process. Another one is the limitation of the optimization packages. The optimization packages cannot solve the PDP globally optimally for big-size SCS because of the number of the binary variables; therefore, it is required to develop a metaheuristic algorithm.

In this study, single-objective function, maximization of the total profit, has been considered for the PDP in uncertain environment. In future studies, it can be modeled as biobjective functions by considering maximization of the customer satisfaction level or minimization of the total transportation time. The proposed modeling approach can be used in different optimization problems that have fuzzy and random parameters in their nature.

Notations

Indices

i: Raw materials
j: Products
s: Suppliers
p: Plant
w: Warehouses
r: Retailers

Table 24: Transportation capacity of path k from plant p to warehouse w at period t.

	Plants	High probability (0.5)			Medium probability (0.35)			Low probability (0.15)		
		$w = 1$	$w = 2$	$w = 3$	$w = 1$	$w = 2$	$w = 3$	$w = 1$	$w = 2$	$w = 3$
$k1$	$p = 1$	(881; 1030; 1101)	(440; 487; 556)	(80; 88; 96)	(837; 960; 1013)	(408; 454; 510)	(68; 80; 91)	(705; 775; 837)	(348; 380; 440)	(57; 68; 80)
	$p = 2$	(837; 969; 1101)	(440; 487; 533)	(68; 84; 102)	(749; 863; 969)	(417; 464; 510)	(63; 77; 91)	(617; 731; 837)	(315; 371; 417)	(51; 68; 80)
$k2$	$p = 1$	(881; 1022; 1057)	(394; 464; 556)	(0; 0; 0)	(793; 960; 1013)	(325; 371; 440)	(0; 0; 0)	(661; 749; 837)	(302; 348; 394)	(0; 0; 0)
	$p = 2$	(0; 0; 0)	(0; 0; 0)	(0; 0; 0)	(0; 0; 0)	(0; 0; 0)	(0; 0; 0)	(0; 0; 0)	(0; 0; 0)	(0; 0; 0)
$k3$	$p = 1$	(0; 0; 0)	(417; 487; 533)	(74; 86; 102)	(0; 0; 0)	(371; 431; 487)	(68; 82; 93)	(0; 0; 0)	(325; 371; 417)	(51; 63; 80)
	$p = 2$	(0; 0; 0)	(371; 461; 510)	(74; 83; 96)	(0; 0; 0)	(302; 348; 417)	(63; 74; 91)	(0; 0; 0)	(255; 325; 371)	(46; 57; 74)

Table 25: Fixed cost of using path k from plant p to warehouse w at period t.

Path	Plants	Warehouses		
		$w = 1$	$w = 2$	$w = 3$
$k1$	$p = 1$	500	550	675
	$p = 2$	650	600	550
$k2$	$p = 1$	875	925	0
	$p = 2$	0	0	0
$k3$	$p = 1$	0	1100	130
	$p = 2$	0	1400	1200

Table 26: Unit transportation cost of product j from plant p to warehouse w by using path k at period t.

Path	Plants	$j = 1$ Warehouses			$j = 2$ Warehouses			$j = 3$ Warehouses		
		$w = 1$	$w = 2$	$w = 3$	$w = 1$	$w = 2$	$w = 3$	$w = 1$	$w = 2$	$w = 3$
$k1$	$p = 1$	(15; 15.89; 16.5)	(15.34; 16.11; 17.15)	(15.6; 16.37; 17.2)	(16.98; 17.99; 18.68)	(17.36; 18.24; 19.41)	(15.6; 16.37; 17.2)	(18.05; 19.12; 19.85)	(18.46; 19.38; 20.63)	(18.77; 19.7; 20.69)
	$p = 2$	(16; 16.71; 17.4)	(16.25; 17.03; 18.3)	(17.68; 18.14; 19.1)	(18.11; 18.91; 19.7)	(17.68; 18.14; 19.1)	(18.39; 19.28; 20.71)	(19.25; 20.11; 20.94)	(19.55; 20.49; 22.02)	(21.27; 21.83; 22.98)
$k2$	$p = 1$	(17.5; 19.76; 20.8)	(18; 18.72; 19.5)	0	(19.81; 22.37; 23.54)	(20.37; 21.19; 22.07)	0	(21.06; 23.77; 25.03)	(21.66; 22.52; 23.46)	0
	$p = 2$	0	0	0	0	0	0	0	0	0
$k3$	$p = 1$	0	(20.1; 21.74; 23.1)	(19.71; 21.09; 23.22)	0	(22.75; 24.61; 26.15)	(19.71; 21.09; 23.22)	0	(24.18; 26.16; 27.79)	(23.71; 25.38; 27.94)
	$p = 2$	0	(20.3; 21.37; 23.4)	(20.06; 21.92; 23.83)	0	(20.06; 21.92; 23.83)	(22.98; 24.87; 26.49)	0	(24.42; 26.43; 28.15)	(24.14; 26.37; 28.67)

Table 27: Required capacity to transport unit product j.

$j = 1$	$j = 2$	$j = 3$
0.7	1.1	0.9

k: Transportation path
c: Customers
t: Time period.

Parameters

$\widetilde{\mathrm{RUP}}_{ist}$: Unit price of raw material i in supplier s at period t

$\widetilde{\mathrm{RC}}_{ist}$: Capacity of raw material i supplied by s at period t

$\overline{\mathrm{TCSP}}_{spkt}$: Total capacity of transport path k from supplier s to plant p at period t

TABLE 28: Transportation capacity of path k from warehouse w to retailer r at period t.

Path	Warehouses	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
		High probability (0.5)				
$k1$	$w = 1$	(310; 328; 340)	(320; 343; 360)	(305; 325; 340)	(310; 323; 340)	(330; 359; 375)
	$w = 2$	(340; 357; 370)	(310; 331; 350)	(335; 360; 375)	(340; 356; 370)	(325; 355; 370)
	$w = 3$	(300; 324; 340)	(320; 339; 350)	(320; 346; 360)	(305; 326; 340)	(335; 350; 370)
$k2$	$w = 1$	(520; 545; 570)	(500; 523; 550)	(500; 522; 590)	0	(530; 569; 610)
	$w = 2$	(500; 528; 560)	(545; 576; 595)	(500; 513; 580)	0	(490; 515; 570)
	$w = 3$	0	0	0	0	0
$k3$	$w = 1$	0	0	0	0	0
	$w = 2$	(240; 272; 300)	0	(215; 246; 280)	(240; 268; 300)	0
	$w = 3$	(250; 284; 310)	0	(210; 235; 250)	(200; 232; 270)	0
		Medium probability (0.3)				
$k1$	$w = 1$	(217; 230; 238)	(224; 241; 252)	(214; 228; 238)	(217; 227; 238)	(231; 252; 263)
	$w = 2$	(238; 250; 259)	(217; 232; 245)	(235; 252; 263)	(238; 250; 259)	(228; 249; 259)
	$w = 3$	(210; 227; 238)	(224; 238; 245)	(224; 243; 252)	(214; 229; 238)	(235; 245; 259)
$k2$	$w = 1$	(364; 382; 399)	(350; 367; 385)	(350; 366; 413)	0	(371; 399; 427)
	$w = 2$	(350; 370; 392)	(382; 404; 417)	(350; 360; 406)	0	(343; 361; 399)
	$w = 3$	0	0	0	0	0
$k3$	$w = 1$	0	0	0	0	0
	$w = 2$	(168; 191; 210)	0	(151; 173; 196)	(168; 188; 210)	0
	$w = 3$	(175; 199; 217)	0	(147; 165; 175)	(140; 163; 189)	0
		Low probability (0.2)				
$k1$	$w = 1$	(155; 164; 170)	(160; 172; 180)	(153; 163; 170)	(155; 162; 170)	(165; 180; 188)
	$w = 2$	(170; 179; 185)	(155; 166; 175)	(168; 180; 188)	(170; 178; 185)	(163; 178; 185)
	$w = 3$	(150; 162; 170)	(160; 170; 175)	(160; 173; 180)	(153; 163; 170)	(168; 175; 185)
$k2$	$w = 1$	(260; 273; 285)	(250; 262; 275)	(250; 261; 295)	0	(265; 285; 305)
	$w = 2$	(250; 264; 280)	(273; 288; 298)	(250; 257; 290)	0	(245; 258; 285)
	$w = 3$	0	0	0	0	0
$k3$	$w = 1$	0	0	0	0	0
	$w = 2$	(120; 136; 150)	0	(108; 123; 140)	(120; 134; 150)	0
	$w = 3$	(125; 142; 155)	0	(105; 118; 125)	(100; 116; 135)	0

TABLE 29: Fixed cost of using path k from warehouse w to retailer r at period t.

Paths	Warehouses	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$k1$	$w = 1$	449	556	414	481	749
	$w = 2$	563	636	572	422	474
	$w = 3$	507	639	422	685	474
$k2$	$w = 1$	721	741	797	0	801
	$w = 2$	860	765	863	0	858
	$w = 3$	0	0	0	0	0
$k3$	$w = 1$	0	0	0	0	0
	$w = 2$	912	0	1059	923	0
	$w = 3$	999	0	820	1031	0

FTCS_{spkt}: Fixed cost of using path k from supplier s to plant p at period t

$\widetilde{\mathrm{VTCS}}_{ispkt}$: Unit transportation cost of raw material i from supplier s to plant p by using path k at period t

RRC_i: Required capacity to transport a unit raw material i

RRM_{ij}: Required amount of raw material i for unit product j

$\widetilde{\mathrm{SRC}}_{ipt}$: Unit holding cost of raw material i in plant p at period t

Table 30: Unit transportation cost of product j from warehouse w to retailer r by using path k at period t.

	Warehouses	Retailer $r=1$	$r=2$	$r=3$	$r=4$	$r=5$
$j=1$						
$k1$	$w=1$	(5.89; 6.79; 7.5)	(5.62; 6.47; 7.2)	(6.11; 7.03; 7.83)	(6.67; 7.17; 7.88)	(5.82; 6.55; 7.18)
	$w=2$	(5.95; 6.85; 7.65)	(5.73; 6.71; 7.33)	(6.22; 7.08; 7.94)	(5.78; 6.34; 7.24)	(5.97; 6.85; 7.47)
	$w=3$	(6.15; 7.17; 7.8)	(5.81; 6.53; 7.25)	(6.36; 7.07; 7.91)	(5.92; 6.56; 7.4)	(5.44; 6.39; 7)
$k2$	$w=1$	(7.12; 8; 8.68)	(7.03; 7.53; 8.16)	(7.03; 7.73; 8.5)	0	(7.11; 7.92; 8.44)
	$w=2$	(6.98; 7.62; 8.24)	(6.73; 7.33; 7.98)	(6.88; 7.56; 8.4)	0	(6.82; 7.3; 8)
	$w=3$	0	0	0	0	0
$k3$	$w=1$	0	0	0	0	0
	$w=2$	(7.58; 8.34; 8.88)	0	(7.43; 8.21; 8.92)	(8.1; 8.78; 9.28)	0
	$w=3$	(7.71; 8.51; 9.05)	0	(7.55; 8.32; 9)	(8.15; 8.81; 9.34)	0
$J=2$						
$k1$	$w=1$	(6.6; 7.6; 8.4)	(6.29; 7.25; 8.06)	(6.84; 7.87; 8.77)	(7.47; 8.03; 8.83)	(6.52; 7.34; 8.04)
	$w=2$	(6.66; 7.67; 8.57)	(6.42; 7.52; 8.21)	(6.97; 7.93; 8.89)	(6.47; 7.1; 8.11)	(6.69; 7.67; 8.37)
	$w=3$	(6.89; 8.03; 8.74)	(6.51; 7.31; 8.12)	(7.12; 7.92; 8.86)	(6.63; 7.35; 8.29)	(6.09; 7.16; 7.84)
$k2$	$w=1$	(7.97; 8.96; 9.72)	(7.87; 8.43; 9.14)	(7.87; 8.66; 9.52)	0	(7.96; 8.87; 9.45)
	$w=2$	(7.82; 8.53; 9.23)	(7.54; 8.21; 8.94)	(7.71; 8.47; 9.41)	0	(7.64; 8.18; 8.96)
	$w=3$	0	0	0	0	0
$k3$	$w=1$	0	0	0	0	0
	$w=2$	(8.49; 9.34; 9.95)	0	(8.32; 9.2; 9.99)	(9.07; 9.83; 10.39)	0
	$w=3$	(8.64; 9.53; 10.14)	0	(8.46; 9.32; 10.08)	(9.13; 9.87; 10.46)	0
$J=3$						
$k1$	$w=1$	(7.01; 8.08; 8.93)	(6.69; 7.7; 8.57)	(7.27; 8.37; 9.32)	(7.94; 8.53; 9.38)	(6.93; 7.79; 8.54)
	$w=2$	(7.08; 8.15; 9.1)	(6.82; 7.98; 8.72)	(7.4; 8.43; 9.45)	(6.88; 7.54; 8.62)	(7.1; 8.15; 8.89)
	$w=3$	(7.32; 8.53; 9.28)	(6.91; 7.77; 8.63)	(7.57; 8.41; 9.41)	(7.04; 7.81; 8.81)	(6.47; 7.6; 8.33)
$k2$	$w=1$	(8.47; 9.52; 10.33)	(8.37; 8.96; 9.71)	(8.37; 9.2; 10.12)	0	(8.46; 9.42; 10.04)
	$w=2$	(8.31; 9.07; 9.81)	(8.01; 8.72; 9.5)	(8.19; 9; 10)	0	(8.12; 8.69; 9.52)
	$w=3$	0	0	0	0	0
$k3$	$w=1$	0	0	0	0	0
	$w=2$	(9.02; 9.92; 10.57)	0	(8.84; 9.77; 10.61)	(9.64; 10.45; 11.04)	0
	$w=3$	(9.17; 10.13; 10.77)	0	(8.98; 9.9; 10.71)	(9.7; 10.48; 11.11)	0

Table 31: Unit holding cost of product j in warehouse w at period t.

Products	Warehouses $w=1$	$w=2$	$w=3$
$j=1$	(6.83; 7.10; 7.23)	(7.28; 7.94; 8.47)	(8.86; 9.37; 7.45)
$j=2$	(7.68; 7.99; 8.14)	(8.39; 9.16; 9.77)	(9.92; 10.50; 8.38)
$j=3$	(9.48; 9.86; 10.05)	(8.15; 8.89; 9.48)	(9.79; 10.35; 10.35)

Table 32: Inventory capacity in warehouse w.

Warehouses $w=1$	$w=2$	$w=3$	$w=4$	$w=5$
8000	8500	8000	9000	8500

UPT_{jpt}: Unit production time of product j in plant p at period t

$\widetilde{\mathrm{RPC}}_{jpt}$: Unit production cost of product j in regular time in plant p at period t

$\widetilde{\mathrm{ARC}}_{pt}$: Available regular capacity (time) in plant p at period t

$\widetilde{\mathrm{OPC}}_{jpt}$: Unit production cost of product j in overtime in plant p at period t

$\widetilde{\mathrm{AOC}}_{pt}$: Available overtime capacity (time) in plant p at period t

PIC_p: Inventory capacity in plant p

$\widetilde{\mathrm{PHC}}_{jpt}$: Unit holding cost of product j in plant p at period t

RHC_j: Required capacity to store a unit product j

TABLE 33: Unit holding cost of product j in retailer r at period t.

Products	Retailers				
	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$j = 1$	(2.73; 2.84; 2.9)	(2.91; 3.18; 3.39)	(3.55; 3.75; 2.98)	(3.1; 3.16; 3.18)	(3.47; 3.7; 3.87)
$j = 2$	(3.34; 3.48; 3.54)	(3.65; 3.98; 4.25)	(4.32; 4.57; 3.65)	(3.79; 3.86; 3.98)	(4.35; 4.63; 4.71)
$j = 3$	(4.13; 4.29; 4.37)	(3.55; 3.87; 4.13)	(4.26; 4.5; 4.5)	(4.68; 4.77; 3.87)	(4.22; 4.5; 4.64)

TABLE 34: Inventory capacity in retailer r.

Retailers				
$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
5000	4600	4800	4900	4500

TABLE 35: Backorder cost of product j in retailer r at period t.

Products	Retailers				
	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$j = 1$	(17.76; 18.47; 18.82)	(18.94; 20.66; 22.04)	(23.05; 24.38; 19.38)	(20.15; 20.53; 20.66)	(22.54; 24.04; 25.15)
$j = 2$	(20.06; 20.85; 21.25)	(21.91; 23.9; 25.49)	(25.9; 27.4; 21.88)	(22.75; 23.18; 23.9)	(26.07; 27.81; 28.26)
$j = 3$	(25.16; 26.16; 26.66)	(21.63; 23.6; 25.17)	(25.97; 27.47; 27.45)	(28.54; 29.08; 23.6)	(25.74; 27.46; 28.33)

TABLE 36: Total demand of customer c for product j from retailer r at period t.

	High		Medium		Low	
	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
			$j = 1$			
$r = 1$	(540; 650; 730)	29	(378; 455; 511)	26.5	(243; 293; 329)	24.1
$r = 2$	(570; 640; 710)	31.2	(399; 448; 497)	28.2	(257; 288; 320)	25.4
$r = 3$	(550; 640; 720)	31.9	(385; 448; 504)	24.6	(248; 288; 324)	23.3
$r = 4$	(570; 680; 710)	30.6	(399; 476; 497)	26.4	(257; 306; 320)	24.1
$r = 5$	(550; 610; 700)	30.6	(385; 427; 490)	24.4	(248; 275; 315)	23.9
			$j = 2$			
$r = 1$	(540; 650; 720)	29.1	(378; 455; 504)	29.4	(249; 299; 332)	25.6
$r = 2$	(510; 570; 730)	29.8	(357; 399; 511)	25.6	(235; 263; 336)	25.2
$r = 3$	(550; 570; 700)	30.1	(385; 399; 490)	27.2	(253; 263; 322)	25.4
$r = 4$	(540; 640; 720)	29.1	(378; 448; 504)	28.4	(249; 295; 332)	23.5
$r = 5$	(490; 630; 720)	31.6	(343; 441; 504)	29.5	(226; 290; 332)	24.1
			$j = 3$			
$r = 1$	(530; 600; 660)	29.7	(371; 420; 462)	28.4	(255; 288; 317)	24.1
$r = 2$	(470; 550; 680)	29.5	(329; 385; 476)	26.6	(226; 264; 327)	25.5
$r = 3$	(510; 570; 690)	30.1	(357; 399; 483)	24.6	(245; 274; 332)	26.1
$r = 4$	(530; 590; 680)	30.1	(371; 413; 476)	27.7	(255; 284; 327)	25.9
$r = 5$	(450; 560; 650)	29.9	(315; 392; 455)	24.5	(216; 269; 312)	26

TABLE 37: Price of product j at period t.

$j = 1$	$j = 2$	$j = 3$
(600; 675; 750)	(760; 800; 840)	(800; 825; 850)

$\overline{\mathrm{TCPW}}_{pwkt}$: Transportation capacity of path k from plant p to warehouse w at period t

FTCP_{pwkt}: Fixed cost of using path k from plant p to warehouse w at period t

$\widetilde{\mathrm{VTCP}}_{jpwkt}$: Unit transportation cost of product j from plant p to warehouse w by using path k at period t

RTC_j: Required capacity to transport a unit product j

$\overline{\mathrm{TCWR}}_{wrkt}$: Transportation capacity of path k from warehouse w to retailer r at period t

FTCW_{wrkt}: Fixed cost of using path k from warehouse w to retailer r at period t

$\widetilde{\mathrm{VTCW}}_{jwrkt}$: Unit transportation cost of product j from warehouse w to retailer r by using path k at period t
$\widetilde{\mathrm{WHC}}_{jwt}$: Unit holding cost of product j in warehouse w at period t
WIC_w: Inventory capacity in warehouse w
$\widetilde{\mathrm{HCR}}_{jrt}$: Unit holding cost of product j in retailer r at period t
RIC_r: Inventory capacity in retailer r
$\widetilde{\mathrm{BCR}}_{jrt}$: Backorder cost of product j in retailer r at period t
$\overline{\overline{\mathrm{CDP}}}_{jrt}$: Total demand of customer c for product j from retailer r at period t
$\widetilde{\mathrm{POP}}_{jt}$: Price of product j at period t.

Decision Variables

TRQ_{ispkt}: Quantity of raw material i transported from supplier s to plant p by using path k at period t
SRP_{ipt}: Inventory level of raw material i stored in plant p at the end of the period t
RPQ_{jpt}: Quantity of product j manufactured at regular time in plant p at period t
OPQ_{jpt}: Quantity of product j manufactured at overtime in plant p at period t
TPQP_{jpwkt}: Quantity of product j transported from plant p to warehouse w by using path k at period t
SLP_{jpt}: Inventory level for product j in plant p at the end of the period t
SLW_{jwt}: Inventory level for product j in warehouse w at the end of the period t
TPQW_{jwrkt}: Quantity of product j transported from warehouse w to retailer r by using path k at period t
SLR_{jrt}: Inventory level for product j in retailer r at the end of the period t
BLR_{jrtc}: Backorder level for product j in retailer r for customer c at the end of the period t
SPQ_{jrtc}: Quantity of product j sold from retailer r to customer c at period t
UKS_{spkt}: {1, if path k is used between supplier s and plant p at period t; 0, otherwise}
UKP_{pwkt}: {1, if path k is used between plant p and warehouse w at period t; 0, otherwise}
UKW_{wrkt}: {1, if path k is used between warehouse w and retailer r at period t; 0, otherwise}.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This paper has been supported by the Scientific and Technological Research Council of Turkey.

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