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Fixed point theorems for weak contractions in the sense of Berinde on partial metric spaces

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ABSTRACT

In this paper, we introduce the notion of (δ, L) weak contraction and (φ, L) weak contraction in the sense of Berinde in partial metric space. Then we give some fixed point results in partial metric space using these new concepts.

1. Introduction

There are a lot of fixed point theorems for different type contractions in the literature. The fundamental linear contractive conditions are given by Banach [7], Kannan [28], Chatterjea [21], Reich [38], Zamfirescu [45], Hardy and Rogers [24] and the fundamental nonlinear contractive conditions are given by Boyd and Wong [19] and Matkowski [30]. In addition to the above, some fixed point theorems are proved using the φ-weak contraction, which is called weak contraction in some papers, in the sense of Alber and Guerre-Delabriere [3,39] and using the (ψ, φ)-weak contraction [22,37]. On the other hand Berinde [8,10,11] defined weak contraction (or (δ, L)-weak contraction) mappings in a metric space as follow:

Definition 1. Let (X, d) be a metric space and $T : X \to X$ is a self operator. T is said to be a weak contraction (or (δ, L)-weak contraction) if there exist a constant $\delta \in (0, 1)$ and some $L \geqslant 0$ such that

$$d(Tx, Ty) \leqslant \delta d(x, y) + Ld(y, Tx) \tag{1.1}$$

for all $x, y \in X$.

Note that, by the symmetry property of the distance, the weak contraction condition implicitly includes the following dual one

$$d(Tx, Ty) \leqslant \delta d(x, y) + Ld(x, Ty) \tag{1.2}$$

for all $x, y \in X$. So, in order to check the weak contractiveness of a mapping T, it is necessary to check both (1.1) and (1.2).

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In [8] and [11], Berinde shows that any Banach, Kannan, Chatterjea and Zamfirescu mappings are weak contraction. Using the concept of weak contraction mappings, Berinde [8] proved the following fixed point theorem:

Theorem 1. *Let (X,d) be a complete metric space and $T : X \to X$ a (δ, L)-weak contraction, then T has a fixed point.*

Also, Berinde shows that any (δ, L)-weak contraction mapping is a Picard operator.

After then, some fixed point and common fixed point theorems were given in [1,6,12–15,17,33–35,42,43] as a generalization of Theorem 1. Also using (δ, L)-weak contraction, some fixed point theorems were given in different type spaces such as ordered metric space, cone metric space and orbitally complete metric space (see [2,16,18,20,23,36,41]).

Again, Berinde [9] introduced the nonlinear type weak contraction using a comparison function and proved the following fixed point theorem. A map $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$, where $\mathbb{R}^+ = [0,\infty)$, is called comparison function if it satisfies:

(i) φ is monotone increasing,
(ii) $\lim_{n\to\infty} \varphi^n(t) = 0$ for all $t \in \mathbb{R}^+$.

If φ satisfies (i) and

(iii) $\sum_{n=0}^{\infty} \varphi^n(t)$ converges for all $t \in \mathbb{R}^+$,

then φ is said to be (c)-comparison function.

We can find some properties and some examples of comparison and (c)-comparison functions in [11].

Definition 2. Let (X,d) be a metric space and $T : X \to X$ is a self operator. T is said to be a weak φ-contraction (or (φ, L)-weak contraction) if there exist a comparison function φ and some $L \geqslant 0$ such that

$$d(Tx, Ty) \leqslant \varphi\big(d(x,y)\big) + Ld(y, Tx) \tag{1.3}$$

for all $x, y \in X$.

Similar to the case of weak contraction, in order to check the weak φ-contractiveness of a mapping T, it is necessary to check both (1.3) and

$$d(Tx, Ty) \leqslant \varphi\big(d(x,y)\big) + Ld(x, Ty) \tag{1.4}$$

for all $x, y \in X$.

Clearly any weak contraction is a weak φ-contraction, but the converse may not be true. Also the class of weak φ-contractions includes Matkowski type nonlinear contractions.

Theorem 2. *Let (X,d) be a complete metric space and $T : X \to X$ a (φ, L)-weak contraction with φ is (c)-comparison function, then T has a fixed point.*

The aim of this paper is to introduce the concept of weak and weak φ-contractions in the sense of Berinde on partial metric space and to prove some fixed point theorems in this interesting space. For this, we recall the definition and some properties of partial metric spaces. See [25,27,31,32,40,44] for details.

The notion of partial metric space, where the distance of a point in the self may not be zero, was introduced by Matthews [31] as follows:

A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:

(p_1) $x = y \Longleftrightarrow p(x,x) = p(x,y) = p(y,y)$ (T_0-separation axiom),
(p_2) $p(x,x) \leqslant p(x,y)$ (small self-distance axiom),
(p_3) $p(x,y) = p(y,x)$ (symmetry),
(p_4) $p(x,y) \leqslant p(x,z) + p(z,y) - p(z,z)$ (modified triangular inequality).

A partial metric space (for short PMS) is a pair (X,p) such that X is a nonempty set and p is a partial metric on X. It is clear that, if $p(x,y) = 0$, then, from (p_1) and (p_2), $x = y$. But if $x = y$, $p(x,y)$ may not be 0. A basic example of a PMS is the pair $(\mathbb{R}^+, p)$, where $p(x,y) = \max\{x,y\}$ for all $x, y \in \mathbb{R}^+$. For another example, let I denote the set of all intervals $[a,b]$ for any real numbers $a \leqslant b$. Let $p : I \times I \to \mathbb{R}^+$ be the function such that $p([a,b],[c,d]) = \max\{b,d\} - \min\{a,c\}$. Then (I,p) is a PMS. Other examples of PMS can be found in [26,31].

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family open p-balls

$$\big\{B_p(x,\varepsilon)\colon\ x \in X,\ \varepsilon > 0\big\},$$

where

$$B_p(x,\varepsilon)=\{y\in X:\ p(x,y)<p(x,x)+\varepsilon\}$$

for all $x\in X$ and $\varepsilon>0$.

If p is a partial metric on X, then the functions $p^s, p^w : X\times X\to\mathbb{R}^+$ given by

$$p^s(x,y)=2p(x,y)-p(x,x)-p(y,y) \tag{1.5}$$

and

$$p^w(x,y)=p(x,y)-\min\{p(x,x),p(y,y)\} \tag{1.6}$$

are ordinary equivalent metrics on X.

A sequence $\{x_n\}$ in a PMS (X,p) converges, with respect to τ_{p^s}, to a point $x\in X$ if and only if $\lim_{m,n\to\infty}p(x_n,x_m)=\lim_{n\to\infty}p(x,x_n)=p(x,x)$. A sequence $\{x_n\}$ in a PMS (X,p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty}p(x_n,x_m)$. If $\lim_{m,n\to\infty}p(x_n,x_m)=0$, then $\{x_n\}$ is said to be a 0-Cauchy sequence in (X,p). A PMS (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x\in X$ such that $p(x,x)=\lim_{n,m\to\infty}p(x_n,x_m)$. (X,p) is called 0-complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x\in X$ such that $p(x,x)=0$. Therefore, (X,p) is 0-complete if and only if every 0-Cauchy sequence converges with respect to τ_{p^s} [40].

It is clear that every 0-Cauchy sequence in (X,p) is a Cauchy sequence in (X,p). Therefore, if (X,p) is complete then it is 0-complete. We can find a basic example in [40] which shows that 0-complete partial metric space may not be complete.

A sequence $\{x_n\}$ is a Cauchy sequence in a partial metric space (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s). Also, (X,p) is complete if and only if (X,p^s) is compete [31].

After the definition of partial metric space, Matthews [31] proved the partial metric version of Banach fixed point theorem. Then, Valero [44], Oltra and Valero [32], Altun et al. [4,5] and Karapinar and Erhan [29] gave some generalizations of the result of Matthews.

2. The main result

In this section, we prove that any weak contraction or weak φ-contraction T has a fixed point in a PMS and show that some operators are satisfying the weak contraction conditions.

Definition 3. Let (X,p) be a partial metric space. A map $T:X\to X$ is called (δ,L)-weak contraction if there exist a $\delta\in[0,1)$ and some $L\geqslant 0$ such that

$$p(Tx,Ty)\leqslant\delta p(x,y)+Lp^w(y,Tx) \tag{2.1}$$

for all $x,y\in X$.

Because of the symmetry property of the distance, the (δ,L)-weak contraction condition implicitly includes the following dual one

$$p(Tx,Ty)\leqslant\delta p(x,y)+Lp^w(x,Ty) \tag{2.2}$$

for all $x,y\in X$. Consequently, in order to check the (δ,L)-weak contractiveness of T, it is necessary to check both (2.1) and (2.2).

Remark 1. In Definition 3, if p is an ordinary metric, then the inequalities (2.1) and (2.2) turn to (1.3) and (1.4), respectively.

Any Banach contraction is a (δ,L)-weak contraction. In the following we show that any Kannan and Chatterjea contractions are (δ,L)-weak contraction.

Proposition 1. *Let (X,p) be a partial metric space and $T:X\to X$ be a Kannan mapping, i.e. there exists $a\in[0,\frac{1}{2})$ such that*

$$p(Tx,Ty)\leqslant a\big[p(x,Tx)+p(y,Ty)\big] \tag{2.3}$$

for all $x,y\in X$. Then T is a (δ,L)-weak contraction.

Proof. By using (2.3)and triangle inequality, we get

$$p(Tx,Ty)\leqslant a\big[p(x,Tx)+p(y,Ty)\big]\leqslant a\big[p(x,y)+p(y,Tx)-p(y,y)+p(y,Tx)+p(Tx,Ty)-p(Tx,Tx)\big]$$

and then

$$\begin{aligned}(1-a)p(Tx,Ty) &\leqslant ap(x,y)+2ap(y,Tx)-a\big[p(y,y)+p(Tx,Tx)\big]\\ &\leqslant ap(x,y)+2ap(y,Tx)-2a\min\big\{p(y,y),p(Tx,Tx)\big\}\\ &= ap(x,y)+2a\big[p(y,Tx)-\min\big\{p(y,y),p(Tx,Tx)\big\}\big]\\ &= ap(x,y)+2ap^w(y,Tx).\end{aligned}$$

So, we have

$$p(Tx,Ty)\leqslant \frac{a}{1-a}p(x,y)+\frac{2a}{1-a}p^w(y,Tx)$$

and hence, in view of $0<a<\frac{1}{2}$, (2.1) holds with $\delta=\frac{a}{1-a}$ and $L=\frac{2a}{1-a}$. Since (2.3) is symmetric with respect to x and y, (2.2) also holds. □

Proposition 2. *Let (X,p) be a partial metric space and $T:X\to X$ be a Chatterjea mapping, i.e. there exists $b\in[0,\frac{1}{2})$ such that*

$$p(Tx,Ty)\leqslant b\big[p(x,Ty)+p(y,Tx)\big] \tag{2.4}$$

for all $x,y\in X$. Then T is a (δ,L)-weak contraction.

Proof. Using

$$p(x,Ty)\leqslant p(x,y)+p(y,Tx)+p(Tx,Ty)-p(y,y)-p(Tx,Tx)$$

by (2.4), we get

$$p(Tx,Ty)\leqslant b\big[p(x,Ty)+p(y,Tx)\big]\leqslant b\big[p(x,y)+p(y,Tx)+p(Tx,Ty)-p(y,y)-p(Tx,Tx)+p(y,Tx)\big]$$

and then

$$(1-b)p(Tx,Ty)\leqslant bp(x,y)+2bp(y,Tx)-2b\min\big\{p(y,y),p(Tx,Tx)\big\}=bp(x,y)+2bp^w(y,Tx).$$

So, we have

$$p(Tx,Ty)\leqslant \frac{b}{1-b}p(x,y)+\frac{2b}{1-b}p^w(y,Tx)$$

which is (2.1), with $\delta=\frac{b}{1-b}<1$ and $L=\frac{2b}{1-b}\geqslant 0$. The symmetry of (2.4) also implies (2.2). □

Definition 4. Let (X,p) be a partial metric space. A map $T:X\to X$ is called (φ,L)-weak contraction if there exist a comparison function φ and some $L\geqslant 0$ such that

$$p(Tx,Ty)\leqslant \varphi\big(p(x,y)\big)+Lp^w(y,Tx) \tag{2.5}$$

for all $x,y\in X$.

As above, because of the symmetry of the distance, the (φ,L)-weak contraction condition implicitly includes the following dual one

$$p(Tx,Ty)\leqslant \varphi\big(p(x,y)\big)+Lp^w(x,Ty) \tag{2.6}$$

for all $x,y\in X$. Consequently, in order to check the (φ,L)-weak contractiveness of T, it is necessary to check both (2.5) and (2.6).

Now we give our main result.

Theorem 3. *Let (X,p) be a 0-complete partial metric space and $T:X\to X$ be (φ,L) weak contraction with a (c)-comparison function, then T has a fixed point.*

Proof. Let $x_0\in X$ and $x_n=Tx_{n-1}$ for all $n\in\mathbb{N}$. Since T is (φ,L)-weak contraction, we have

$$p(Tx_{n-1},Tx_n)\leqslant \varphi\big(p(x_{n-1},x_n)\big)+Lp^w(x_n,Tx_{n-1})=\varphi\big(p(x_{n-1},x_n)\big)$$

and so

$$p(x_n,x_{n+1})\leqslant \varphi\big(p(x_{n-1},x_n)\big).$$

We obtain by induction

$$p(x_n,x_{n+1})\leqslant \varphi^n\big(p(x_0,x_1)\big)$$

for all $n \in \mathbb{N}$. By triangle rule, for $m > n$, we have

$$\begin{aligned} p(x_n, x_m) &\leqslant \sum_{k=n}^{m-1} p(x_k, x_{k+1}) - \sum_{k=n}^{m-2} p(x_{k+1}, x_{k+1}) \\ &\leqslant \sum_{k=n}^{m-1} p(x_k, x_{k+1}) \\ &\leqslant \sum_{k=n}^{\infty} p(x_k, x_{k+1}) \\ &\leqslant \sum_{k=n}^{\infty} \varphi^k\big(p(x_0, x_1)\big). \end{aligned}$$

Since φ is a (c)-comparison function, then $\sum_{k=0}^{\infty} \varphi^k(p(x_0, x_1))$ is convergent and so $\{x_n\}$ is a 0-Cauchy sequence in X. Since X is 0-complete and then $\{x_n\}$ converges, with respect to τ_p, to a point $z \in X$ such that

$$\lim_{n\to\infty} p(x_n, z) = p(z, z) = 0.$$

Now we claim that $p(z, Tz) = 0$. Suppose the contrary, then $p(z, Tz) > 0$. Since φ is (c)-comparison function, then $\varphi(t) < t$ for $t > 0$. Also since $\lim_{n\to\infty} p(x_n, z) = p(z, z) = 0$, then there exists $n_0 \in \mathbb{N}$ such that for all $n \geqslant n_0$, $p(x_n, z) < \frac{p(z,Tz)}{2}$. Therefore, we have

$$\begin{aligned} p(z, Tz) &\leqslant p(z, x_{n+1}) + p(x_{n+1}, Tz) \\ &= p(z, x_{n+1}) + p(Tx_n, Tz) \\ &\leqslant p(z, x_{n+1}) + \varphi\big(p(x_n, z)\big) + Lp^w(z, x_{n+1}) \\ &\leqslant p(z, x_{n+1}) + \varphi\left(\frac{p(z, Tz)}{2}\right) + Lp^w(z, x_{n+1}) \\ &< p(z, x_{n+1}) + \frac{p(z, Tz)}{2} + Lp^w(z, x_{n+1}) \end{aligned}$$

and letting $n \to \infty$ we obtain

$$0 < p(z, Tz) < \frac{p(z, Tz)}{2},$$

which is a contradiction. Therefore $p(Tz, z) = 0$ and $z = Tz$. $\square$

Since any (δ, L)-weak contraction is a (φ, L)-weak contraction, we obtain the following corollary:

Corollary 1. *Let (X, p) be a 0-complete partial metric space and $T : X \to X$ be (δ, L)-weak contraction, then T has a fixed point in X.*

Example 1. Let $X = [0, 1]$ and

$$p(x, y) = \begin{cases} \max\{x, y\}, & x \neq y, \\ 0, & x = y. \end{cases}$$

It is clear that p is a partial metric and (X, p) is 0-complete. Also $p^w(x, y) = p(x, y)$. Define $T : X \to X$ by

$$Tx = \begin{cases} x^2, & x \in [0, \frac{1}{2}), \\ 0, & x \in [\frac{1}{2}, 1), \\ 1, & x = 1. \end{cases}$$

Now we show that (2.1) and (2.2) are satisfied for $\delta = \frac{1}{2}$ and $L = 1$. In order to show both (2.1) and (2.2), it is sufficient to show that

$$p(Tx, Ty) \leqslant \delta p(x, y) + L \min\big\{p^w(y, Tx), p^w(x, Ty)\big\}$$

for all $x, y \in X$. Consider the following six cases:

Case 1. Let $x = y$, then $p(Tx, Ty) = 0$ and so the result is clear. Therefore we will assume $x \neq y$ in the following cases.

Case 2. Let $x, y \in [\frac{1}{2}, 1)$, then $p(Tx, Ty) = 0$ and so the result is clear.

Case 3. Let $x, y \in [0, \frac{1}{2})$, then

$$p(Tx, Ty) = \max\{x^2, y^2\} \leqslant \frac{1}{2}\max\{x, y\} = \frac{1}{2}p(x, y) \leqslant \frac{1}{2}p(x, y) + \min\{p^w(y, Tx), p^w(x, Ty)\}.$$

Case 4. Let $x \in [\frac{1}{2}, 1)$ and $y = 1$, then

$$\begin{aligned} p(Tx, Ty) = 1 &< \frac{1}{2} + 1 \\ &= \frac{1}{2}\max\{x, 1\} + \min\{\max\{1, 0\}, \max\{x, 1\}\} \\ &= \frac{1}{2}p(x, y) + \min\{p(y, Tx), p(x, Ty)\} \\ &= \frac{1}{2}p(x, y) + \min\{p^w(y, Tx), p^w(x, Ty)\}. \end{aligned}$$

Case 5. Let $x \in [0, \frac{1}{2})$ and $y = 1$, then

$$\begin{aligned} p(Tx, Ty) = 1 &< \frac{1}{2} + 1 \\ &= \frac{1}{2}\max\{x, 1\} + \min\{\max\{1, x^2\}, \max\{x, 1\}\} \\ &= \frac{1}{2}p(x, y) + \min\{p(y, Tx), p(x, Ty)\} \\ &= \frac{1}{2}p(x, y) + \min\{p^w(y, Tx), p^w(x, Ty)\}. \end{aligned}$$

Case 6. Let $x \in [0, \frac{1}{2})$ and $y \in [\frac{1}{2}, 1)$, then

$$p(Tx, Ty) = \max\{x^2, 0\} = x^2 \leqslant \frac{1}{2}x \leqslant \frac{1}{2}\max\{x, y\} = \frac{1}{2}p(x, y) \leqslant \frac{1}{2}p(x, y) + \min\{p^w(y, Tx), p^w(x, Ty)\}.$$

Therefore all conditions of Corollary 1 are satisfied, so T has a fixed point. Note that since

$$p(T0, T1) = 1 = p(0, 1),$$

then the Banach contraction is not satisfied and so the result of Matthews is not applicable to this example.

In the above, we show that if T is a (φ, L)-weak contraction then it has a fixed point. But in order to guarantee the uniqueness of the fixed point of T, we have to consider an additional (φ, L)-weak contractive type condition, as in the following theorem.

Theorem 4. *Let (X, p) be a 0-complete partial metric space and $T : X \to X$ be (φ, L)-weak contraction. Suppose T also satisfies the following condition: there exist a comparison function φ_1 and some $L_1 \geqslant 0$ such that*

$$p(Tx, Ty) \leqslant \varphi_1\big(p(x, y)\big) + L_1 p^w(x, Tx) \tag{2.7}$$

holds, for all $x, y \in X$. Then T has a unique fixed point in X.

Proof. Suppose that, there are two fixed point z and w of T. If $p(z, w) = 0$, it is clear that $z = w$. Assume that $p(z, w) > 0$. By (2.7) with $x = z$ and $y = w$, we have

$$\begin{aligned} 0 < p(z, w) &= p(Tz, Tw) \\ &\leqslant \varphi_1\big(p(z, w)\big) + L_1 p^w(z, Tz) \\ &= \varphi_1\big(p(z, w)\big) \\ &< p(z, w) \end{aligned}$$

which is a contradiction. Therefore T has a unique fixed point. □

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