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A hybrid spherical fuzzy logarithmic decomposition of criteria importance and alternative ranking technique based on Adaptive Standardized Intervals model with application

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ABSTRACT

This study presents a hybrid fuzzy Multi-Attribute Group Decision Making (MAGDM) model with application to commercial insurance selection. The proposed hybrid model uses Spherical Fuzzy (SF) sets using Yager t-norm and t-conorm operations. The Logarithmic Decomposition of Criteria Importance (LODECI) method is used for criterion weighting due to its efficacy in stabilizing scenarios that may challenge other weighting techniques. For alternative ranking, the study employs the Alternative Ranking Technique based on Adaptive Standardized Intervals (ARTASI), offering enhanced flexibility in handling uncertainties inherent in expert evaluations. The combination of these methods and the utilization of SF sets gives rise to the proposed SF-LODECI-ARTASI hybrid model. The paper systematically delineates the procedural steps involving SF sets, Yager t-norm and t-conorm operations, SF-LODECI, and SF-ARTASI methods. Subsequently, the developed hybrid model is applied to a case study of commercial insurance selection, supported by a numerical example. The research application results emphasize the consistency of these findings with other alternative methods. Additionally, sensitivity scenarios are constructed to scrutinize the robustness of the proposed hybrid model. The study concludes by elucidating implications and contributions to the existing literature.

1. Introduction

Commercial insurance is designed to protect businesses against the negative consequences of potential risks they may encounter. It aims to minimize damage and secure firms in adverse situations caused by various risks associated with commercial activities. Commercial insurance involves the creation of insurance packages based on the purchase of multiple policies. These policies typically cover areas such as property insurance, liability insurance, workers' insurance, transportation insurance, and business loss insurance.

In recent times, the selection of commercial insurance has been treated as a decision problem within the framework of Multi-Attribute Group Decision Making (MAGDM) [1]. MAGDM is an approach in multi-attribute decision-making (MADM) involving experts or decision-makers in the decision model to address and resolve the decision problem [2,3]. In the context of selecting commercial insurance, MAGDM allows for the integration of different perspectives from business managers and stakeholders. It also facilitates the determination of different weights for identified criteria and enables decision-making based on these weights. Moreover, it aids in creating decision matrices and the operation of decision-making processes based on these matrices. Consequently, MAGDM enhances the acceptability level of the decision for all stakeholders involved in the business [4–7].

The criteria influencing the selection of insurance can be both quantitative and qualitative [8]. Fuzzy logic, particularly based on Spherical Fuzzy (SF) sets, comes into play when dealing with qualitative criteria [9]. SF sets provide a more flexible representation of uncertainty due to their well-defined mathematical properties compared to traditional crisp sets or fuzzy sets utilizing alternative membership functions. On the other hand, calculating operations involving SF sets can be computationally complex and may incur higher costs compared to other types of fuzzy sets. Additionally, they may lack interpretability. Fuzzy logic, with its ability to handle uncertainty, allows for a departure from rigid classifications based on precise evaluations [10]. The degree of fuzziness typically ranges between 0 and 1, representing a numerical quantification of uncertainty [11]. Fuzzy logic employs fuzzy sets and operations within specific fuzzy logic rules to accomplish its processes [11].

SF sets, which define uncertainty within a spherical region rather than pinpointing specific points, offer flexible solutions to cope with

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uncertainties in MAGDM problems [12,13]. SF sets contribute to a more comprehensive consideration of criteria weights by providing greater flexibility in the weighting process [14]. SF sets also better represent the uncertainties and conflicts in expert opinions, fostering a more collaborative environment. SF sets have been employed in various research domains such as performance-emission optimization [12], the selection of the most effective coronavirus disease of 2019 (COVID-19) virus protector [15], municipal solid waste management problems [16], hospital site selection [17], offshore wind power station site selection [18], sustainable supplier selection [9], advanced manufacturing system selection [19], evaluation of cloud computing services [20], assessment of waste management [21], renewable energy [22], evaluation of hydrogen storage systems [23], distribution center location selection [24], industrial robot selection [25], process mining technology selection [26], diabetic retinopathy detection [27], assessment of soil fertility [28], energy storage systems performance evaluation [29], logistic autonomous vehicles assessment [30], clean energy barriers evaluation [31], human–machine interface design evaluation [32], evaluating electrical vehicle charging station location ([33], medical waste disposal planning [34], blockchain platform evaluation [35], evaluation of circular economy business models [36], sustainable energy planning [37], emergency location selection [38], autonomous vehicles project [39], sustainable resilient supplier selection [40], risk analysis [41] and similar research topics.

SF sets are utilized in various methods for criteria weighting and alternative ranking. In criteria weighting methods, SF sets are used to create fuzzy sets based on the evaluation of criteria, determine weights, and calculate the weighted aggregations of criteria. In alternative ranking methods, SF sets play a role in establishing fuzzy sets during the evaluation of alternatives by decision-makers, obtaining the weighted aggregations of alternatives, and ranking alternatives specific to the decision problem. The academic literature reveals diverse applications of SF sets in various criteria weighting and alternative ranking methods, including technique for order preference by similarity to ideal solution (TOPSIS) [14], analytic hierarchy process (AHP) [19], preference ranking organization method for enrichment evaluation (PROMETHEE) [17], weighted aggregated sum product assessment (WASPAS) [42], additive ratio assessment (ARAS) [43], viekriterijumsko kompromisno rangiranje (VIKOR) [22], criteria importance through intercriteria correlation (CRITIC) [44], decision-making trial and evaluation laboratory (DEMATEL) [45], elimination and choice translating reality (ELECTRE) [46], and measurement of alternatives and ranking according to compromise solution (MARCOS) [47].

The main motivation of this research is to develop a new hybrid model based on SF sets for the commercial insurance selection problem. While various methods based on SF sets have been applied to the commercial insurance selection problem in the literature, this research aims to propose new methods using SF sets to overcome the limitations of existing approaches and enhance the decision process further. In this context, SF-LODECI-ARTASI, a hybrid model, is suggested for the commercial insurance selection problem within the MAGDM framework. The Logarithmic Decomposition of Criteria Importance (LODECI) method for criteria weighting [48], based on SF sets, is proposed to address the conflicts arising from Entropy and MEREC methods and to prevent stability issues encountered in these methods. The Alternative Ranking Technique based on the Adaptive Standardized Intervals (ARTASI) method [49], based on SF sets, is introduced to extend the traditional range of 0 to 1 used in alternative ranking methods, allowing for more precise evaluations. Therefore, in this research, the SF-LODECI-ARTASI hybrid model is proposed to provide decision support for the commercial insurance selection problem within the MAGDM approach.

1.1. Aims and contributions of the study

This research aims to propose a MAGDM approach for the commercial insurance selection process. While previous studies in the literature have addressed the commercial insurance selection using MAGDM, our objective is to innovate by incorporating recently developed methods and highlighting the advantages of these approaches. The primary focus is on introducing a novel hybrid model for the commercial insurance selection process and formulating an algorithm specific to this hybrid model. Spherical fuzzy sets are employed in this hybrid method, along with Yager t-norm and t-conorm operations. For criterion weighting, the LODECI method is preferred due to its effectiveness in stabilizing situations that may be unstable in other criterion weighting methods. In the alternative ranking, the ARTASI method is utilized to expand the uncertainty levels in expert evaluations, providing more flexible solutions. The amalgamation of these two methods and spherical fuzzy sets gives rise to the SF-LODECI-ARTASI method.

This research contributes to the existing body of knowledge in the following ways:

(i) *Development of a Novel Hybrid Model*: The study introduces a new hybrid model, the SF-LODECI-ARTASI method, for the commercial insurance selection process. This model incorporates spherical fuzzy sets, Yager t-norm and t-conorm operations, LODECI, and ARTASI methods, offering a fresh perspective in literature.

(ii) *Stabilized Criterion Weighting*: The LODECI method is chosen as the criterion weighting approach in the hybrid model due to its effectiveness in stabilizing situations that may be unstable in other criterion weighting methods. This enhances the reliability of the decision model.

(iii) *Flexible Alternative Ranking*: The ARTASI method in alternative ranking allows for the expansion of uncertainty levels in expert evaluations, enabling the generation of more flexible and adaptable solutions in the commercial insurance selection process.

(iv) *Application and Comparison*: The developed hybrid model is applied to a practical example of commercial insurance selection using a dataset from research conducted by [1]. The selection of this dataset aims to facilitate the comparability of the results with findings from the new hybrid model.

(v) *Consistency with Previous Research*: The study observes consistency between the obtained results and those from other research in the field, validating the robustness of the proposed hybrid model.

(vi) *Robustness Analysis*: Various sensitivity scenarios are created to test the robustness of the proposed hybrid model. The outcomes of these scenarios provide support for the resilience and stability of the suggested hybrid model.

(vii) *Implications and Insights*: The research concludes by presenting implications and insights derived from the study, contributing not only a novel approach based on spherical fuzzy sets but also offering an alternative model for addressing the commercial insurance selection problem.

1.2. Organization of the study

This paper consists of five sections. Section 2 presents the methodological framework, where SF preliminaries, Yager t-norm and t-conorm operations, and the steps of the SF-LODECI-ARTASI hybrid model are elucidated. Section 3 comprises a numerical example, illustrating an application related to commercial insurance selection. In Section 4, results and implications are expounded, covering sensitivity analyses, comparative analyses, research findings, and managerial insights. Section 5 is the conclusion, where research limitations and recommendations for future studies are presented.

2. Methodological framework

2.1. Preliminaries of SF sets

Definition 1. In the designated domain of discourse, denoted as $\mathcal{D}$, the expression $\tilde{A}$, as defined by the formulation in Eq. (1), signifies the existence of a set of fuzzy sets operating within the encompassing context of set $\mathcal{D}$ [22].

$$\tilde{A} = \left\{ \left\langle d, \mu_{\tilde{A}}(d), \nu_{\tilde{A}}(d), \pi_{\tilde{A}}(d) \mid d \in \mathcal{D} \right\rangle \right\} \tag{1}$$

The functions $\{\mu_{\tilde{A}}(d) : \tilde{A} \to [0,1], \nu_{\tilde{A}}(d) : \tilde{A} \to [0,1]\, and\, \pi_{\tilde{A}}(d) : \tilde{A} \to [0,1]\}$ can be construed as representing the degrees of membership, non-membership, and the indeterminacy of hesitancy, respectively.

These functions are rigorously defined under the constraint that the inequality $0 \le \mu_{\tilde{A}}(d)^2 + \nu_{\tilde{A}}(d)^2 + \pi_{\tilde{A}}(d)^2 \le 1$ is upheld for all elements d belonging to the set $\mathcal{D}$. These functions may be succinctly represented as $\tilde{A} = \left(\mu_{\tilde{A}}(d), \nu_{\tilde{A}}(d), \pi_{\tilde{A}}(d)\right)$.

Definition 2. In a hypothetical scenario wherein $\tilde{A} = \left(\mu_{\tilde{A}}(d), \nu_{\tilde{A}}(d), \pi_{\tilde{A}}(d)\right)$ represents an SF set operating within the universal set $\mathcal{D}$, the derivation of the score function, denoted as $\mathbb{S}(\tilde{A})$, is conducted utilizing the formulation provided in Eq. (2) [50]:

$$\mathbb{S}(\tilde{A}) = \left(\frac{1}{3}\right)\left(2 + \left(\mu_{\tilde{A}}(d)\right) - \left(\nu_{\tilde{A}}(d)\right) - \left(\pi_{\tilde{A}}(d)\right)\right); \mathbb{S}(\tilde{A}) \in [0,1]. \tag{2}$$

Definition 3. Within the specified context, two distinct SF sets ($\tilde{A}_1$ and $\tilde{A}_2$) are defined over the universal set $\mathcal{D}$, featuring their respective elements as follows: $\tilde{A}_1 = \left(\mu_{\tilde{A}_1}(d), \nu_{\tilde{A}_1}(d), \pi_{\tilde{A}_1}(d)\right)$ and $\tilde{A}_2 = \left(\mu_{\tilde{A}_2}(d), \nu_{\tilde{A}_2}(d), \pi_{\tilde{A}_2}(d)\right)$. In instances where $\mathbb{S}(\tilde{A})$ signifies the score functions of $\tilde{A}_{\mathfrak{a}}(\mathfrak{a} = 1, 2)$, the establishment of ordinal relations is achieved as follows: If $\mathbb{S}(\tilde{A}_1) > \mathbb{S}(\tilde{A}_2)$, then $\tilde{A}_1$ is deemed greater than $\tilde{A}_2$.

2.2. Yager t-norm and t-conorm operations in SF sets

This section provides an exposition on Yager Operations and delineates their applications to SF sets.

Definition 4. The Yager t-norm and t-conorm, expressed in Eqs. (3) and (4) respectively, as presented by [51]:

$$\Gamma(\mathfrak{x}, \mathfrak{y}) = 1 - min\left(1, \left((1-\mathfrak{x})^{\psi} + (1-\mathfrak{y})^{\psi}\right)^{1/\psi}\right) \tag{3}$$

$$\Gamma^*(\mathfrak{x}, \mathfrak{y}) = min\left(1, \left((\mathfrak{x})^{\psi} + (\mathfrak{y})^{\lambda}\right)^{1/\psi}\right) \tag{4}$$

where $\psi > 0; \mathfrak{x}, \mathfrak{y} \in [0,1]$.

Consider two SF sets, denoted as $\tilde{A}_1$ and $\tilde{A}_2$, defined over the universal set $\mathcal{D}$, with their respective components given by $\tilde{A}_1 = \left(\mu_{\tilde{A}_1}(d), \nu_{\tilde{A}_1}(d), \pi_{\tilde{A}_1}(d)\right)$ and $\tilde{A}_2 = \left(\mu_{\tilde{A}_2}(d), \nu_{\tilde{A}_2}(d), \pi_{\tilde{A}_2}(d)\right)$. The Yager product $(\tilde{A}_1 \cap_{\Gamma,\Gamma^*} \tilde{A}_2)$ and Yager sum $(\tilde{A}_1 \cup_{\Gamma,\Gamma^*} \tilde{A}_2)$ incorporating the t-norm (Γ) and t-conorm (Γ^*) are formulated as Eqs. (5) and (6), respectively.

$$\tilde{A}_1 \cap_{\Gamma,\Gamma^*} \tilde{A}_2 = \Big(\Gamma\left(\mu_{\tilde{A}_1}(d), \mu_{\tilde{A}_2}(d)\right), \Gamma^*\left(\nu_{\tilde{A}_1}(d), \nu_{\tilde{A}_2}(d)\right), \Gamma^*\left(\pi_{\tilde{A}_1}(d), \pi_{\tilde{A}_2}(d)\right)\Big), \tag{5}$$

$$\tilde{A}_1 \cup_{\Gamma,\Gamma^*} \tilde{A}_2 = \Big(\Gamma^*\left(\mu_{\tilde{A}_1}(d), \mu_{\tilde{A}_2}(d)\right), \Gamma\left(\nu_{\tilde{A}_1}(d), \nu_{\tilde{A}_2}(d)\right), \Gamma\left(\pi_{\tilde{A}_1}(d), \pi_{\tilde{A}_2}(d)\right)\Big), \tag{6}$$

Definition 5. Consider two SF sets, denoted as $\tilde{A}_1$ and $\tilde{A}_2$, defined over the universal set $\mathcal{D}$, with their respective components given by $\tilde{A}_1 = \left(\mu_{\tilde{A}_1}(d), \nu_{\tilde{A}_1}(d), \pi_{\tilde{A}_1}(d)\right)$ and $\tilde{A}_2 = \left(\mu_{\tilde{A}_2}(d), \nu_{\tilde{A}_2}(d), \pi_{\tilde{A}_2}(d)\right)$. The protocols governing the interplay among two SF sets, employing the Yager t-norm and Yager t-conorm, are expounded as follows, with the stipulation that $\psi > 0$ *and* $\mathfrak{w} > 0$ [52]:

$$\text{(i)}\ \tilde{A}_1 \otimes \tilde{A}_2 = \begin{pmatrix} \sqrt{1 - min\left(1, \left(\left(1 - \left(\mu_{\tilde{A}_1}(d)\right)^2\right)^{\psi} + \left(1 - \left(\mu_{\tilde{A}_2}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)}, \\ \sqrt{1 - min\left(1, \left(\left(1 - \left(\nu_{\tilde{A}_1}(d)\right)^2\right)^{\psi} + \left(1 - \left(\nu_{\tilde{A}_2}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)}, \\ \sqrt{min\left(1, \left(\left(\left(\pi_{\tilde{A}_1}(d)\right)^2\right)^{\psi} + \left(\left(\pi_{\tilde{A}_2}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)} \end{pmatrix},$$

$$\text{(ii)}\ \tilde{A}_1 \oplus \tilde{A}_2 = \begin{pmatrix} \sqrt{min\left(1, \left(\left(\left(\mu_{\tilde{A}_1}(d)\right)^2\right)^{\psi} + \left(\left(\mu_{\tilde{A}_2}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)}, \\ \sqrt{1 - min\left(1, \left(\left(1 - \left(\nu_{\tilde{A}_1}(d)\right)^2\right)^{\psi} + \left(1 - \left(\nu_{\tilde{A}_2}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)}, \\ \sqrt{1 - min\left(1, \left(\left(1 - \left(\pi_{\tilde{A}_1}(d)\right)^2\right)^{\psi} + \left(1 - \left(\pi_{\tilde{A}_2}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)} \end{pmatrix},$$

$$\text{(iii)}\ \mathfrak{w}\tilde{A}_1 = \begin{pmatrix} \sqrt{min\left(1, \left(\mathfrak{w}\left(\mu_{\tilde{A}_1}(d)\right)^{2\psi}\right)^{1/\psi}\right)}, \\ \sqrt{1 - min\left(1, \left(\mathfrak{w}\left(1 - \left(\nu_{\tilde{A}_1}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)}, \\ \sqrt{1 - min\left(1, \left(\mathfrak{w}\left(1 - \left(\pi_{\tilde{A}_1}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)} \end{pmatrix}$$

$$\text{(iv)}\ \tilde{A}_1^{\mathfrak{w}} = \begin{pmatrix} \sqrt{1 - min\left(1, \left(\mathfrak{w}\left(1 - \left(\mu_{\tilde{A}_1}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)}, \\ \sqrt{\sqrt{1 - min\left(1, \left(\mathfrak{w}\left(1 - \left(\nu_{\tilde{A}_1}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)}}, \\ \sqrt{min\left(1, \left(\mathfrak{w}\left(\pi_{\tilde{A}_1}(d)\right)^{2\psi}\right)^{1/\psi}\right)} \end{pmatrix}$$

Theorem 1. *Consider two SF sets, denoted as $\tilde{A}_1$ and $\tilde{A}_2$, defined over the universal set $\mathcal{D}$, with their respective components given by $\tilde{A}_1 = \left(\mu_{\tilde{A}_1}(d), \nu_{\tilde{A}_1}(d), \pi_{\tilde{A}_1}(d)\right)$ and $\tilde{A}_2 = \left(\mu_{\tilde{A}_2}(d), \nu_{\tilde{A}_2}(d), \pi_{\tilde{A}_2}(d)\right)$. In the case of the Yager t-norm (Γ) and t-conorm (Γ^*), the processes entail the collaborative interaction between two SF sets in the subsequent manner [52]:*

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = \tilde{A}_2 \oplus \tilde{A}_1$,
(ii) $\tilde{A}_1 \otimes \tilde{A}_2 = \tilde{A}_2 \otimes \tilde{A}_1$,
(iii) $\mathfrak{w}\left(\tilde{A}_1 \oplus \tilde{A}_2\right) = \mathfrak{w}\tilde{A}_1 \oplus \mathfrak{w}\tilde{A}_2 for \mathfrak{w} > 0$,
(iv) $\left(\tilde{A}_1 \otimes \tilde{A}_2\right)^{\mathfrak{w}} = \tilde{A}_1^{\mathfrak{w}} \otimes \tilde{A}_2^{\mathfrak{w}}$ for $\mathfrak{w} > 0$,
(v) $\mathfrak{w}_1\tilde{A}_1 \oplus \mathfrak{w}_2\tilde{A}_1 = \left(\mathfrak{w}_1 + \mathfrak{w}_2\right)\tilde{A}_1$ for $\mathfrak{w}_1, \mathfrak{w}_2 > 0$,
(vi) $\tilde{A}_1^{\mathfrak{w}_1} \otimes \tilde{A}_1^{\mathfrak{w}_2} = \tilde{A}_1^{(\mathfrak{w}_1 + \mathfrak{w}_2)}$ for $\mathfrak{w}_1, \mathfrak{w}_2 > 0$.

Proof. The proof of Theorem 1 has been established in the study conducted by [52].

2.3. SF-Yager Weighted Arithmetic mean (SFYWA) aggregation operator

In this section, SFYWA is described.

Definition 6. Let us consider the SF set $\tilde{A}_{\mathcal{X}}$, denoted as $\tilde{A}_{\mathcal{X}} = \left(\mu_{\tilde{A}_{\mathcal{X}}}(d), \nu_{\tilde{A}_{\mathcal{X}}}(d), \pi_{\tilde{A}_{\mathcal{X}}}(d)\right)$, where it constitutes an SF set. Additionally, the associated weight vector is denoted as $\mathfrak{w}_{\mathcal{X}} = (\mathfrak{w}_1, \mathfrak{w}_2, \dots, \mathfrak{w}_{\mathfrak{p}})$, with $\mathfrak{w}_{\mathcal{X}}\ \epsilon\ [0,1]$ and $\sum_{\mathcal{X}=1}^{\mathfrak{p}} \mathfrak{w}_{\mathcal{X}} = 1$. Within this framework, the SFYWA aggregation operator is expressed by Eq. (7):

$$SFYWA\left(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_{\mathfrak{p}}\right) = \oplus_{\mathcal{X}=1}^{\mathfrak{p}} \mathfrak{w}_{\mathcal{X}}\tilde{A}_{\mathcal{X}} \tag{7}$$

Theorem 2. *For Definition 6, the SFYWA aggregation operator theorem is stated as Eq. (8):*

$$SFYWA\left(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_{\mathfrak{p}}\right) = \oplus_{\mathcal{X}=1}^{\mathfrak{p}} \mathfrak{w}_{\mathcal{X}}\tilde{A}_{\mathcal{X}}$$

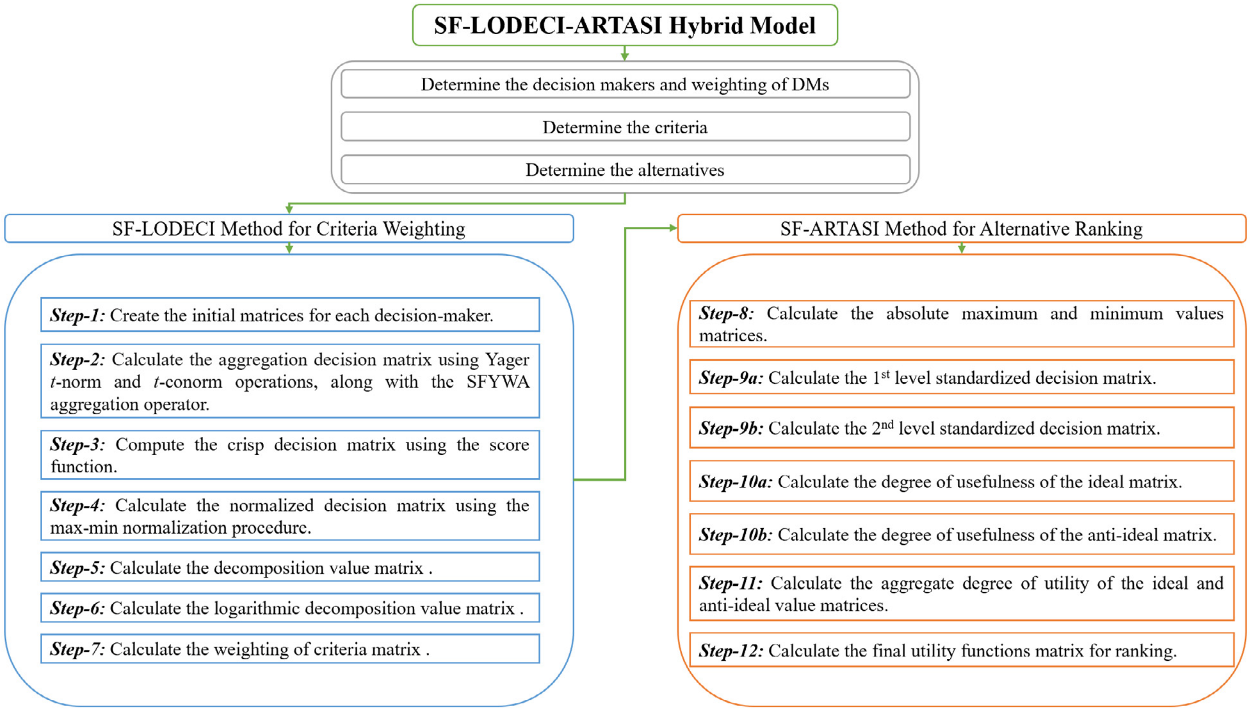

Fig. 1. Methodological flowchart.

$$= \left(\begin{array}{c} \sqrt{min\left(1, \left(\sum_{\mathcal{X}=1}^{\mathfrak{p}} \mathfrak{w}_{\mathcal{X}} \left(\mu_{\tilde{A}_{\mathcal{X}}}(d)\right)^{2\psi}\right)^{1/\psi}\right)}, \\ \sqrt{1 - min\left(1, \left(\sum_{\mathcal{X}=1}^{\mathfrak{p}} \mathfrak{w}_{\mathcal{X}} \left(1 - \left(v_{\tilde{A}_{\mathcal{X}}}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)}, \\ \sqrt{1 - min\left(1, \left(\sum_{\mathcal{X}=1}^{\mathfrak{p}} \mathfrak{w}_{\mathcal{X}} \left(1 - \left(\pi_{\tilde{A}_{\mathcal{X}}}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right)} \end{array} \right) \tag{8}$$

Proof. The proof of Theorem 2 has been established in the study conducted by [52].

2.4. The novel SF-LODECI-ARTASI hybrid method based on SFYWA aggregation operator

In this study, the methodological flow of the proposed SF-LODECI-ARTASI hybrid model is illustrated in Fig. 1. This encompasses two main sections. The first section entails the execution of criterion weighting through SF-LODECI. In the second section, alternative ranking processes are conducted using SF-ARTASI. Subsections provide detailed explanations of SF sets, Yager t-norm and t-conorm operations, and the procedural steps of the SF-LODECI-ARTASI hybrid model.

The SF-LODECI-ARTASI hybrid method has been developed for the decision-making problem. Let $(\mathcal{P}_i) = \{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_m\}\ (i = 1, 2, \ldots, m)$ be alternatives and $(\mathcal{C}_j) = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_n\}\ (j = 1, 2, \ldots, n)$ be criteria and $(\mathcal{DM}_k) = \{\mathcal{DM}_1, \mathcal{DM}_2, \ldots, \mathcal{DM}_{\mathfrak{q}}\}\ (k = 1, 2, \ldots, \mathfrak{q})$ for the decision-making problem. The steps of the SF-LODECI-ARTASI hybrid method are as follows:

Criteria weighting using SF-LODECI:

Step 1: Each alternative $(\mathcal{P}_i)$ undergoes evaluation by each decision maker $(\mathcal{DM}_k)$ concerning each criterion $(\mathcal{C}_j)$, employing the linguistic variables (LVs) illustrated in Table 1. Following this evaluation, the LVs are converted into corresponding SF sets, as detailed in Table 1, leading to the creation of the initial decision matrices $\left(\tilde{\mathcal{B}}^{(\mathcal{DM}_k)} = \left[\tilde{\mathcal{B}}_{ij}^{(\mathcal{DM}_k)}\right]_{mxn}\right)$ where

$$\tilde{A}_{\tilde{\mathcal{B}}_{ij}^{(\mathcal{DM}_k)}} = \left(\mu_{\tilde{\mathcal{B}}_{ij}^{(\mathcal{DM}_k)}}(d), v_{\tilde{\mathcal{B}}_{ij}^{(\mathcal{DM}_k)}}(d), \pi_{\tilde{\mathcal{B}}_{ij}^{(\mathcal{DM}_k)}}(d)\right)$$
$(i = 1, \ldots, m; j = 1, \ldots, n; k = 1, \ldots, \mathfrak{q})$.

Table 1
Linguistic variables for SF sets [23].

Linguistic variable	*SF Sets*		
	$\mu(d)$	$v(d)$	$\pi(d)$
Superior (S)	0.9	0.1	0.1
Extremely Good (EG)	0.8	0.2	0.2
Very Good (VG)	0.7	0.3	0.3
Good (G)	0.6	0.4	0.4
Fair (F)	0.5	0.5	0.5
Poor (P)	0.4	0.6	0.4
Very Poor (VP)	0.3	0.7	0.3
Extremely Poor (EP)	0.2	0.8	0.2
Inferior (I)	0.1	0.9	0.1

Step 2: Using the SFYWA aggregation operator presented in Eq. (9) calculate the aggregated decision matrix $\left(\tilde{\mathcal{B}} = \left[\tilde{\mathcal{B}}_{ij}\right]_{mxn}\right)$. In Eq. (9), the expert's weight vector is denoted as $\mathfrak{w}_k = (\mathfrak{w}_1, \mathfrak{w}_2, \ldots, \mathfrak{w}_{\mathfrak{q}})$, with $\mathfrak{w}_k \epsilon [0, 1]$ and $\sum_{k=1}^{\mathfrak{q}} \mathfrak{w}_k = 1$.

$$SFYWA\left(\tilde{\mathcal{B}}^{(\mathcal{DM}_1)}, \tilde{\mathcal{B}}^{(\mathcal{DM}_2)}, \ldots, \tilde{\mathcal{B}}^{(\mathcal{DM}_{\mathfrak{q}})}\right) = \oplus_{k=1}^{\mathfrak{q}} \mathfrak{w}_k \tilde{\mathcal{B}}^{(\mathcal{DM}_k)}$$
$$= \left(\begin{array}{c} \sqrt{min\left(1, \left(\sum_{k=1}^{\mathfrak{q}} \mathfrak{w}_k \left(\mu_{\tilde{\mathcal{B}}^{(\mathcal{DM}_k)}}(d)\right)^{2\psi}\right)^{1/\psi}\right)}, \\ \sqrt{1 - min\left[1, \left(\sum_{k=1}^{\mathfrak{q}} \mathfrak{w}_k \left(1 - \left(v_{\tilde{A}_{\tilde{\mathcal{B}}^{(\mathcal{DM}_k)}}}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right]}, \\ \sqrt{1 - min\left[1, \left(\sum_{k=1}^{\mathfrak{q}} \mathfrak{w}_k \left(1 - \left(\pi_{\tilde{A}_{\tilde{\mathcal{B}}^{(\mathcal{DM}_k)}}}(d)\right)^2\right)^{\psi}\right)^{1/\psi}\right]} \end{array} \right) for\ \psi > 0 \tag{9}$$

Step 3: Using the score function $\left(\mathbb{S}\left(\tilde{\mathcal{B}}_{ij}\right)\right)$ stated in Eq. (10), calculate the crisp values. Then, obtain the crisp decision matrix $\left(\mathcal{E} = \left[\mathcal{E}_{ij}\right]_{mxn}\right)$.

$$\mathbb{S}\left(\tilde{\mathcal{B}}_{ij}\right) = \left(\frac{1}{3}\right)\left(2 + \left(\mu_{\tilde{\mathcal{B}}_{ij}}(d)\right) - \left(v_{\tilde{\mathcal{B}}_{ij}}(d)\right) - \left(\pi_{\tilde{\mathcal{B}}_{ij}}(d)\right)\right)$$
$$\text{for } (i = 1, \ldots, m; j = 1, \ldots, n) \tag{10}$$

Step 4: Using Eq. (11), calculate the normalized decision matrix $\left(\mathcal{F}=\left[\mathcal{F}_{ij}\right]_{mxn}\right)$.

$$\mathcal{F}_{ij}=\begin{pmatrix}\frac{\mathcal{E}_{ij}}{\mathcal{E}_j^{max}} if\ j\in benefit\ criteria\\ \frac{\mathcal{E}_j^{min}}{\mathcal{E}_{ij}} if\ j\in cost\ criteria\end{pmatrix}\ \text{for}\ (i=1,\dots,m;j=1,\dots,n) \tag{11}$$

Step 5: Using Eq. (12), calculate the decomposition value matrix $\left(\mathcal{G}=\left[\mathcal{G}_{ij}\right]_{mxn}\right)$.

$$\mathcal{G}_{ij}=\max\left\{\left|\mathcal{F}_{ij}-\mathcal{F}_{rj}\right|\right\}\ \text{for}\ r\neq i\ and\ \times(r=1,2,\dots m;i=1,\dots,m;j=1,\dots,n) \tag{12}$$

Step 6: Using Eq. (13), calculate the logarithmic decomposition value matrix $\left(\mathcal{H}=\left[\mathcal{H}_j\right]_n\right)$.

$$\mathcal{H}_j=\ln\left(1+\frac{\sum_{i=1}^{m}\mathcal{G}_{ij}}{m}\right)\ \text{for}\ (i=1,\dots,m;j=1,\dots,n) \tag{13}$$

Step 7: Using Eq. (14), calculate the final weighting of the criteria matrix $\left(\mathcal{W}=\left[\mathcal{W}_j\right]_n\right)$.

$$\mathcal{W}_j=\frac{\mathcal{H}_j}{\sum_{j=1}^{n}\mathcal{H}_j}\ \text{for}\ (j=1,\dots,n) \tag{14}$$

Alternative ranking using SF-ARTASI:

Step 8: The crisp decision matrix $\left(\mathcal{E}=\left[\mathcal{E}_{ij}\right]_{mxn}\right)$ calculated in Step 3, is an initial decision matrix for the SF-ARTASI method. Using a crisp decision matrix, calculate the absolute maximum values matrix $\left(\mathcal{E}^{max}=\left[\mathcal{E}_j^{max}\right]_n\right)$ and the absolute minimum values matrix $\left(\mathcal{E}^{min}=\left[\mathcal{E}_j^{min}\right]_n\right)$ with Eqs. (15) and (16), respectively.

$$\mathcal{E}_j^{max}=\left(\max_{1\le i\le m}\left(\mathcal{E}_{ij}\right)\right)+\left(\max_{1\le i\le m}\left(\mathcal{E}_{ij}\right)\right)^{1/m}\ \text{for}\ (i=1,\dots,m;j=1,\dots,n) \tag{15}$$

$$\mathcal{E}_j^{min}=\left(\min_{1\le i\le m}\left(\mathcal{E}_{ij}\right)\right)-\left(\min_{1\le i\le m}\left(\mathcal{E}_{ij}\right)\right)^{1/m}\ \text{for}\ (i=1,\dots,m;j=1,\dots,n) \tag{16}$$

Step 9: In this step, the standardized decision matrix is calculated applied to two sub-steps.

Step 9a: First level standardized decision matrix $\left(\mathcal{I}=\left[\mathcal{I}_{ij}\right]_{mxn}\right)$ is calculated using Eq. (17).

$$\mathcal{I}_{ij}=\left(\left(\frac{\xi^{(u)}-\xi^{(l)}}{\mathcal{E}_j^{max}-\mathcal{E}_j^{min}}\right)\mathcal{E}_{ij}\right)+\left(\left|\frac{\left(\mathcal{E}_j^{max}\right)\left(\xi^{(l)}\right)-\left(\mathcal{E}_j^{min}\right)\left(\xi^{(u)}\right)}{\mathcal{E}_j^{max}-\mathcal{E}_j^{min}}\right|\right)\ \text{for}\ (i=1,\dots,m;j=1,\dots,n) \tag{17}$$

where $\mathcal{E}_j^{max}$ and $\mathcal{E}_j^{min}$ represent the absolute maximum values and the absolute minimum values, respectively, $\xi^{(u)}$ and $\xi^{(l)}$ represent the lower and upper limits of the standardized interval, while $\mathcal{E}_{ij}$ represent initial decision matrix value. In addition, the standardized interval $\left[\xi^{(l)},\xi^{(u)}\right]$ values equal $[1,100]$ [49].

Step 9b: Second level standardized decision matrix $\left(\mathcal{L}=\left[\mathcal{L}_{ij}\right]_{mxn}\right)$ is calculated using Eq. (18).

$$\mathcal{L}_{ij}=\begin{pmatrix}\left(\mathcal{L}_{ij}\right)=\left(-\mathcal{I}_{ij}+\max_{1\le i\le m}\left(\mathcal{I}_{ij}\right)+\min_{1\le i\le m}\left(\mathcal{I}_{ij}\right)\right);if\ j\in cost\ criteria\\ \left(\mathcal{L}_{ij}\right)=\left(\mathcal{I}_{ij}\right);if\ j\in benefit\ criteria\end{pmatrix} \tag{18}$$

Step 10: In this step, the degree of usefulness of alternatives for the ideal and anti-ideal values is calculated by applying two sub-steps.

Step 10a: The degree of usefulness of the ideal matrix $\left(\mathcal{M}^{+}=\left[\mathcal{M}_{ij}^{+}\right]_{mxn}\right)$ is calculated using Eq. (19).

$$\mathcal{M}_{ij}^{+}=\left(\left(\frac{\mathcal{L}_{ij}}{\max_{1\le i\le m}\left(\mathcal{L}_{ij}\right)}\right)\left(\mathcal{W}_j\right)\left(\xi^{(u)}\right)\right)\ \text{for}\ (i=1,\dots,m;j=1,\dots,n) \tag{19}$$

where $\xi^{(u)}=100$ and $\mathcal{W}_j$ are criterion weights.

Step 10b: The degree of usefulness of the anti-ideal matrix $\left(\mathcal{M}^{-}=\left[\mathcal{M}_{ij}^{-}\right]_{mxn}\right)$ is calculated using Eq. (20).

$$\mathcal{M}_{ij}^{-}=\left(-\mathcal{N}_{ij}+\max_{1\le i\le m}\left(\mathcal{N}_{ij}\right)+\min_{1\le i\le m}\left(\mathcal{N}_{ij}\right)\right)\ \text{for}\ (i=1,\dots,m;j=1,\dots,n) \tag{20}$$

where $\mathcal{N}_{ij}$ is the degree of usefulness. $\mathcal{N}_{ij}$ is calculated using Eq. (21).

$$\mathcal{N}_{ij}=\left(\left(\frac{\min_{1\le i\le m}\left(\mathcal{L}_{ij}\right)}{\mathcal{L}_{ij}}\right)\left(\mathcal{W}_j\right)\left(\xi^{(u)}\right)\right)\ \text{for}\ (i=1,\dots,m;j=1,\dots,n) \tag{21}$$

where $\xi^{(u)}=100$ and $\mathcal{W}_j$ are criterion weights.

Step 11: The aggregate degree of utility of the alternatives for the ideal value matrix $\left(\mathcal{O}^{+}=\left[\mathcal{O}_i^{+}\right]_m\right)$ and the anti-ideal value matrix $\left(\mathcal{O}^{-}=\left[\mathcal{O}_i^{-}\right]_m\right)$ using Eq. (22) and Eq. (23), respectively.

$$\mathcal{O}_i^{+}=\sum_{j=1}^{n}\mathcal{M}_{ij}^{+}\ \text{for}\ (i=1,\dots,m;j=1,\dots,n) \tag{22}$$

$$\mathcal{O}_i^{-}=\sum_{j=1}^{n}\mathcal{M}_{ij}^{-}\ \text{for}\ (i=1,\dots,m;j=1,\dots,n) \tag{23}$$

Step 12: The final utility functions matrix $\left(\mathcal{R}=\left[\mathcal{R}_i\right]_m\right)$ is calculated using Eq. (24). Then, the highest value of the final utility functions matrix is the best alternative.

$$\mathcal{R}_i=\left(\mathcal{O}_i^{+}+\mathcal{O}_i^{-}\right)\left(\left((\Omega)\left(f\left(\mathcal{O}_i^{+}\right)\right)^{\varphi}\right)+\left((1-\Omega)\left(f\left(\mathcal{O}_i^{-}\right)\right)^{\varphi}\right)\right)^{1/\varphi}\ \text{for}\ (i=1,\dots,m;\varphi\in[1,+\infty);\Omega\in[0,1]) \tag{24}$$

where $f\left(\mathcal{O}_i^{+}\right)$ and $f\left(\mathcal{O}_i^{-}\right)$ represent additive functions and calculating using $\left(f\left(\mathcal{O}_i^{+}\right)=\frac{\mathcal{O}_i^{+}}{\mathcal{O}_i^{+}+\mathcal{O}_i^{-}}\right)$ and $\left(f\left(\mathcal{O}_i^{-}\right)=\frac{\mathcal{O}_i^{-}}{\mathcal{O}_i^{+}+\mathcal{O}_i^{-}}\right)$, respectively.

The Algorithm steps comprising the SF-LODECI-ARTASI hybrid method are delineated in Table 2.

3. Numerical example

To ensure the comparability of the implementation of the SF-LODECI-ARTASI hybrid model, a numerical example was adopted from the sample application created by [1] in the literature. This example application explores the accurate selection of commercial insurance to optimize asset allocation. In the numerical example, a decision model is formulated using three decision-makers ($\mathcal{DM}_1$, $\mathcal{DM}_2$, $\mathcal{DM}_3$), four criteria (Ages $\left(\mathcal{C}_1\right)$, Healthy conditions $\left(\mathcal{C}_2\right)$,Vocational types $\left(\mathcal{C}_3\right)$, Economic conditions $\left(\mathcal{C}_4\right)$), and five alternatives (Critical illness insurance $\left(\mathcal{P}_1\right)$, Medical insurance $\left(\mathcal{P}_2\right)$, Life insurance $\left(\mathcal{P}_3\right)$, Travel insurance $\left(\mathcal{P}_4\right)$, Accident insurance $\left(\mathcal{P}_5\right)$). All criteria are utility-based. *Ages criterion* considers factors such as life expectancy, health risks associated with certain age groups, and likelihood of filing claims. *Health conditions criterion* is evaluated to assess the potential risks associated with insuring a specific individual. *Vocational type of criterion* may involve higher risks or exposure to hazards, which in turn can impact the probability of filing insurance claims. *Economic conditions criterion* in insurance selection refers to the broader financial landscape

Table 2
Algorithm.

Algorithm	*The fundamental objective of this algorithm is to demonstrate the SF-LODECI-ARTASI hybrid model developed utilizing the SFYWA aggregation operator and incorporating Yager t-norm and t-conorm operations*
Input	As a set of alternatives $(\mathcal{P}_i) = \{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_m\}$, criteria $(\mathcal{C}_j) = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_n\}$ and decision makers $(\mathcal{DM}_k) = \{\mathcal{DM}_1, \mathcal{DM}_2, \ldots, \mathcal{DM}_q\}$ for calculating weighting of criteria matrix $(\mathcal{W} = [\mathcal{W}_j]_n)$ and ranking of alternatives matrix $(\mathcal{R} = [\mathcal{R}_i]_m)$.
Output	Ranking of the alternatives.
Step 1	Each decision-maker evaluates each alternative based on each criterion according to Table 1. Subsequently, initial matrices $\left(\tilde{\mathcal{B}}^{(\mathcal{DM}_k)} = \left[\tilde{\mathcal{B}}_{ij}^{(\mathcal{DM}_k)}\right]_{m \times n}\right)$ are created for each decision-maker.
Step 2	Using the SFYWA aggregation operator presented in Eq. (9) calculate the aggregated decision matrix $\left(\tilde{\mathcal{B}} = [\tilde{\mathcal{B}}_{ij}]_{m \times n}\right)$ considering the decision-maker weights vector $\left(\mathfrak{w}_k = \left(\mathfrak{w}_1, \mathfrak{w}_2, \ldots, \mathfrak{w}_q\right)\right)$ and $(\psi = 1)$.
Step 3	Calculate the crisp decision matrix $\left(\mathcal{E} = [\mathcal{E}_{ij}]_{m \times n}\right)$ using the score function $(\mathbb{S}(\tilde{\mathcal{B}}_{ij}))$ (Eq. (10)).
Step 4	Calculate the normalized decision matrix $\left(\mathcal{F} = [\mathcal{F}_{ij}]_{m \times n}\right)$ using the max–min normalization procedure (Eq. (11)).
Step 5	Calculate the decomposition value matrix $\left(\mathcal{G} = [\mathcal{G}_{ij}]_{m \times n}\right)$ using Eq. (12).
Step 6	Calculate logarithmic decomposition value matrix $(\mathcal{H} = [\mathcal{H}_j]_n)$ using Eq. (13).
Step 7	Calculate the final weighting of criteria matrix $(\mathcal{W} = [\mathcal{W}_j]_n)$ using Eq. (14).
Step 8	Calculate the absolute maximum values matrix $\left(\mathcal{E}^{max} = \left[\mathcal{E}_j^{max}\right]_n\right)$ and the absolute minimum values matrix $\left(\mathcal{E}^{min} = \left[\mathcal{E}_j^{min}\right]_n\right)$ using Eq. (15) and Eq. (16), respectively.
Step 9a	Calculate the first level standardized decision matrix $\left(\mathcal{I} = [\mathcal{I}_{ij}]_{m \times n}\right)$ using Eq. (17).
Step 9b	Calculate the second level standardized decision matrix $\left(\mathcal{L} = [\mathcal{L}_{ij}]_{m \times n}\right)$ using Eq. (18).
Step 10a	Calculate the degree of usefulness of the ideal matrix $\left(\mathcal{M}^+ = \left[\mathcal{M}_{ij}^+\right]_{m \times n}\right)$ using Eq (19).
Step 10b	Calculate the degree of usefulness of the anti-ideal matrix $\left(\mathcal{M}^- = \left[\mathcal{M}_{ij}^-\right]_{m \times n}\right)$ using Eq (21) and Eq (20), respectively.
Step 11	Calculate the aggregate degree of utility of the ideal value matrix $\left(\mathcal{O}^+ = \left[\mathcal{O}_i^+\right]_m\right)$ and the anti-ideal value matrix $\left(\mathcal{O}^- = \left[\mathcal{O}_i^-\right]_m\right)$ using Eq. (22) and Eq. (23), respectively.
Step 12	Calculate the final utility functions matrix $(\mathcal{R} = [\mathcal{R}_i]_m)$ using Eq. (24) $((\Omega = 0.2)$ and $(\varphi = 1))$. The highest value of final utility functions matrix is the best alternative.
End.	

Table 3
The evaluations of decision-makers regarding commercial insurances.

Alternatives	$\mathcal{DM}_1$				$\mathcal{DM}_2$				$\mathcal{DM}_3$			
	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{P}_1$	VG	G	P	F	F	VG	VP	VG	VG	VP	EP	P
$\mathcal{P}_2$	S	G	EG	G	S	S	VG	EG	G	VG	EG	VG
$\mathcal{P}_3$	P	VP	VP	F	VG	VP	VP	G	VG	VP	EP	VG
$\mathcal{P}_4$	G	VG	VP	VG	VG	F	F	F	G	VG	P	VG
$\mathcal{P}_5$	VG	P	VG	F	VG	P	VG	F	VG	F	VG	F

and situations that could impact an individual's capacity to secure and sustain insurance coverage. The weights of the decision-makers are as follows: "$(\mathfrak{w}_1 = 0.4)$, $(\mathfrak{w}_2 = 0.4)$, $(\mathfrak{w}_3 = 0.2)$". The decision weights were directly derived from the study by [1]. The SF-LODECI-ARTASI hybrid model, based on the values presented in the numerical example, is applied using the Algorithm. In the subsequent subsection, findings obtained by sequentially applying all steps of the algorithm are presented in tabular form.

3.1. Selection of commercial insurance using the novel SF-LODECI-ARTASI hybrid model based on SFYWA aggregation operator

All steps encompassed within the Algorithm are sequentially executed as follows:

Step 1: Using Table 1, each decision-maker evaluates each alternative based on individual criteria. The assessments conducted by $\mathcal{DM}_1$, $\mathcal{DM}_2$, and $\mathcal{DM}_3$ are presented in Table 3. The decision-maker evaluations are sourced from the study by [1]. Subsequently, using Table 1, the LVs are transformed into SF sets. Matrices corresponding to SF sets are sequentially presented in Table 4. These matrices are considered as the initial matrices associated with each decision-maker.

Example. : The linguistic expression for the first expert's evaluation of the first alternative according to the first criterion as "VG" corresponds to the SF set as follows: (0.70,0.30,0.30)

Step 2: Using the SFYWA aggregation operator presented in Eq. (9), the aggregated decision matrix is calculated $(\psi = 1)$. The aggregated decision matrix $\left(\tilde{\mathcal{B}} = [\tilde{\mathcal{B}}_{ij}]_{mxn}\right)$ is shown in Table 5.

Example. : The calculation of the first alternative according to the first criterion is as follows:

$$\mu_{\tilde{\mathcal{B}}_{1,1}} = \sqrt{min\left(1, \left(0.4 * (0.70)^2 + 0.4 * (0.50)^2 + 0.2 * (0.70)^2\right)\right)} = 0.628$$

$$\nu_{\tilde{\mathcal{B}}_{1,1}} = \sqrt{1 - min\left(1, \left(\left(\left(0.4 * \left(1 - (0.30)^2\right)\right) + \left(0.4 * \left(1 - (0.50)^2\right)\right) + \left(0.2 * \left(1 - (0.30)^2\right)\right)\right)\right)\right)} = 0.392$$

$$\pi_{\tilde{\mathcal{B}}_{1,1}} = \sqrt{1 - min\left(1, \left(\left(\left(0.4 * \left(1 - (0.30)^2\right)\right) + \left(0.4 * \left(1 - (0.50)^2\right)\right) + \left(0.2 * \left(1 - (0.30)^2\right)\right)\right)\right)\right)} = 0.392$$

Step 3: Using the score function presented in Eq. (10), the crisp decision matrix is calculated. The crisp decision matrix $\left(\mathcal{E} = [\mathcal{E}_{ij}]_{mxn}\right)$ is shown in Table 6.

Example. : The calculation of the first alternative according to the first criterion is as follows:

$$\mathbb{S}\left(\tilde{\mathcal{B}}_{1,1}\right) = \left(\frac{1}{3}\right) * (2 + (0.628) - (0.392) - (0.392)) = 0.6143$$

Table 4
SF fuzzy sets for $\mathcal{DM}_1$, $\mathcal{DM}_2$ and $\mathcal{DM}_3$.

DMs	$\mathcal{P}$	$\mathcal{C}_1$			$\mathcal{C}_2$			$\mathcal{C}_3$			$\mathcal{C}_4$		
		$\mu_{\tilde{A}}(d)$	$v_{\tilde{A}}(d)$	$\pi_{\tilde{A}}(d)$	$\mu_{\tilde{A}}(d)$	$v_{\tilde{A}}(d)$	$\pi_{\tilde{A}}(d)$	$\mu_{\tilde{A}}(d)$	$v_{\tilde{A}}(d)$	$\pi_{\tilde{A}}(d)$	$\mu_{\tilde{A}}(d)$	$v_{\tilde{A}}(d)$	$\pi_{\tilde{A}}(d)$
$\mathcal{DM}_1$	$\mathcal{P}_1$	0.70	0.30	0.30	0.60	0.40	0.40	0.40	0.60	0.40	0.50	0.50	0.50
	$\mathcal{P}_2$	0.90	0.10	0.10	0.60	0.40	0.40	0.80	0.20	0.20	0.60	0.40	0.40
	$\mathcal{P}_3$	0.40	0.60	0.40	0.30	0.70	0.30	0.30	0.70	0.30	0.50	0.50	0.50
	$\mathcal{P}_4$	0.60	0.40	0.40	0.70	0.30	0.30	0.30	0.70	0.30	0.70	0.30	0.30
	$\mathcal{P}_5$	0.70	0.30	0.30	0.40	0.60	0.40	0.70	0.30	0.30	0.50	0.50	0.50
$\mathcal{DM}_2$	$\mathcal{P}_1$	0.50	0.50	0.50	0.70	0.30	0.30	0.30	0.70	0.30	0.70	0.30	0.30
	$\mathcal{P}_2$	0.90	0.10	0.10	0.90	0.10	0.10	0.70	0.30	0.30	0.80	0.20	0.20
	$\mathcal{P}_3$	0.70	0.30	0.30	0.30	0.70	0.30	0.30	0.70	0.30	0.60	0.40	0.40
	$\mathcal{P}_4$	0.70	0.30	0.30	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	$\mathcal{P}_5$	0.70	0.30	0.30	0.40	0.60	0.40	0.70	0.30	0.30	0.50	0.50	0.50
$\mathcal{DM}_3$	$\mathcal{P}_1$	0.70	0.30	0.30	0.30	0.70	0.30	0.20	0.80	0.20	0.40	0.60	0.40
	$\mathcal{P}_2$	0.60	0.40	0.40	0.70	0.30	0.30	0.80	0.20	0.20	0.70	0.30	0.30
	$\mathcal{P}_3$	0.70	0.30	0.30	0.30	0.70	0.30	0.20	0.80	0.20	0.70	0.30	0.30
	$\mathcal{P}_4$	0.60	0.40	0.40	0.70	0.30	0.30	0.40	0.60	0.40	0.70	0.30	0.30
	$\mathcal{P}_5$	0.70	0.30	0.30	0.50	0.50	0.50	0.70	0.30	0.30	0.50	0.50	0.50

Table 5
The aggregated decision matrix.

$\mathcal{P}$	$\mathcal{C}_1$			$\mathcal{C}_2$			$\mathcal{C}_3$			$\mathcal{C}_4$		
	$\mu_{\tilde{A}}(d)$	$v_{\tilde{A}}(d)$	$\pi_{\tilde{A}}(d)$	$\mu_{\tilde{A}}(d)$	$v_{\tilde{A}}(d)$	$\pi_{\tilde{A}}(d)$	$\mu_{\tilde{A}}(d)$	$v_{\tilde{A}}(d)$	$\pi_{\tilde{A}}(d)$	$\mu_{\tilde{A}}(d)$	$v_{\tilde{A}}(d)$	$\pi_{\tilde{A}}(d)$
$\mathcal{P}_1$	0.628	0.392	0.392	0.598	0.445	0.344	0.329	0.684	0.329	0.573	0.456	0.410
$\mathcal{P}_2$	0.849	0.200	0.200	0.752	0.293	0.293	0.762	0.245	0.245	0.706	0.313	0.313
$\mathcal{P}_3$	0.598	0.445	0.344	0.300	0.700	0.300	0.283	0.721	0.283	0.585	0.427	0.427
$\mathcal{P}_4$	0.642	0.363	0.363	0.628	0.392	0.392	0.410	0.607	0.410	0.628	0.392	0.392
$\mathcal{P}_5$	0.700	0.300	0.300	0.422	0.581	0.422	0.700	0.300	0.300	0.500	0.500	0.500

Table 6
The crisp decision matrix.

$\mathcal{P}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{P}_1$	0.6143	0.6033	0.4386	0.5689
$\mathcal{P}_2$	0.8162	0.7219	0.7572	0.6932
$\mathcal{P}_3$	0.6033	0.4333	0.4263	0.5772
$\mathcal{P}_4$	0.6384	0.6143	0.4645	0.6143
$\mathcal{P}_5$	0.7000	0.4729	0.7000	0.5000

Table 7
The normalized decision matrix.

$\mathcal{P}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{P}_1$	0.7526	0.8356	0.5793	0.8207
$\mathcal{P}_2$	1.0000	1.0000	1.0000	1.0000
$\mathcal{P}_3$	0.7392	0.6002	0.5630	0.8327
$\mathcal{P}_4$	0.7822	0.8509	0.6134	0.8862
$\mathcal{P}_5$	0.8577	0.6550	0.9244	0.7213

Step 4: Using Eq. (11), the normalized decision matrix is calculated. The normalized decision matrix $\left(\mathcal{F} = \left[\mathcal{F}_{ij}\right]_{mxn}\right)$ is presented in Table 7.

Example. : The calculation of the first alternative according to the first criterion is as follows:

$$\mathcal{F}_{1,1} = \frac{0.6143}{0.8162} = 0.7526$$

Step 5: Using Eq. (12), the decomposition value matrix is calculated. The decomposition value matrix $\left(\mathcal{G} = \left[\mathcal{G}_{ij}\right]_{mxn}\right)$ is presented in Table 8.

Example. : The calculation of the first alternative according to the first criterion is as follows:

$$\mathcal{G}_{1,1} = |0.7526 - 1| = 0.247369$$

Step 6: Using Eq. (13), the logarithmic decomposition value matrix is calculated. The logarithmic decomposition value matrix $\left(\mathcal{H} = \left[\mathcal{H}_{j}\right]_{n}\right)$ is presented in Table 9.

Table 8
The decomposition value matrix.

$\mathcal{P}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{P}_1$	0.247369	0.235407	0.420739	0.179279
$\mathcal{P}_2$	0.260843	0.399764	0.437029	0.278705
$\mathcal{P}_3$	0.260843	0.399764	0.437029	0.167347
$\mathcal{P}_4$	0.217801	0.250639	0.386634	0.164858
$\mathcal{P}_5$	0.118501	0.344994	0.361455	0.278705

Table 9
The decomposition value matrix.

$\mathcal{H}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{H}_j$	0.2036	0.2823	0.3426	0.1937

Table 10
The final weighting of criteria matrix.

$\mathcal{W}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{W}_j$	0.1992	0.2761	0.3351	0.1895

Example. : The calculation of the first criterion is as follows:

$$\mathcal{H}_1 = \ln\left(1 + \frac{(0.247369 + 0.260843 + 0.260843 + 0.217801 + 0.142342)}{5}\right) = 0.2036$$

Step 7: Using Eq. (14), the final weighting of criteria matrix is calculated. The final weighting of criteria matrix $\left(\mathcal{W} = \left[\mathcal{W}_{j}\right]_{n}\right)$ is presented in Table 10.

Example. : The calculation of the first criterion is as follows:
$$\mathcal{W}_1 = \frac{0.2036}{(0.2036+0.2823+0.3426+0.1937)} = 0.1992$$

Step 8: Using Eqs. (15) and (16), the absolute maximum values matrix $\left(\mathcal{E}^{max} = \left[\mathcal{E}^{max}_{j}\right]_{n}\right)$ and the absolute minimum values matrix $\left(\mathcal{E}^{min} = \left[\mathcal{E}^{min}_{j}\right]_{n}\right)$ are calculated, respectively. The absolute maximum values and minimum values matrices are presented in Table 11.

Table 11
The absolute maximum values and minimum values matrices.

$\mathcal{E}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{E}_j^{max}$	1.7764	1.6589	1.7031	1.6225
$\mathcal{E}_j^{min}$	−0.3006	−0.4127	−0.4169	−0.3706

Table 12
The first level standardized decision matrix.

$\mathcal{P}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{P}_1$	44.60792	49.55296	40.95195	47.66531
$\mathcal{P}_2$	54.23155	55.22366	55.82936	53.83832
$\mathcal{P}_3$	44.08376	41.43086	40.37594	48.07616
$\mathcal{P}_4$	45.75824	50.0785	42.1579	49.91831
$\mathcal{P}_5$	48.6939	43.32057	53.15706	44.24185

Table 13
The second level standardized decision matrix.

$\mathcal{P}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{P}_1$	44.60792	49.55296	40.95195	47.66531
$\mathcal{P}_2$	54.23155	55.22366	55.82936	53.83832
$\mathcal{P}_3$	44.08376	41.43086	40.37594	48.07616
$\mathcal{P}_4$	45.75824	50.0785	42.1579	49.91831
$\mathcal{P}_5$	48.6939	43.32057	53.15706	44.24185

Example. : The calculation of the first criterion is as follows:

$$\mathcal{E}_1^{max} = (0.8162) + (0.8162)^{\frac{1}{5}} = 1.7764$$

$$\mathcal{E}_1^{min} = (0.6033) - (0.6033)^{\frac{1}{5}} = -0.3006$$

Step 9a: Using Eq. (17), the first level standardized decision matrix is calculated. The first level standardized decision matrix $\left(\mathcal{I} = \left[\mathcal{I}_{ij}\right]_{mxn}\right)$ is presented in Table 12.

Example. : The calculation of the first alternative according to the first criterion is as follows:

$$\mathcal{I}_{1,1} = \left(\left(\frac{100-1}{1.7764-(-0.3006)}\right) * 0.6143\right) + \left(\frac{1.7764 * 1 - (-0.3006) * 100}{1.7764-(-0.3006)}\right) = 44.60792$$

Step 9b: Using Eq. (18), the second level standardized decision matrix is calculated. The second level standardized decision matrix $\left(\mathcal{L} = \left[\mathcal{L}_{ij}\right]_{mxn}\right)$ is presented in Table 13. Due to the absence of cost-based criteria among the criteria, comparable results can be achieved with a first level standardized decision matrix.

Example. : The calculation of the first alternative according to the first criterion is as follows:

$$\mathcal{L}_{1,1} = \mathcal{I}_{1,1} = 44.60792$$

Step 10a: Using Eq. (19), the degree of usefulness of the ideal matrix is calculated. The degree of usefulness of the ideal matrix $\left(\mathcal{M}^{+} = \left[\mathcal{M}_{ij}^{+}\right]_{mxn}\right)$ is presented in Table 14.

Example. : The calculation of the first alternative according to the first criterion is as follows:

$$\mathcal{M}_{1,1}^{+} = \left(\frac{44.60792}{54.23155}\right) * 0.1992 * 100 = 16.38545$$

Step 10b: Using Eq. (20), the degree of usefulness of the anti-ideal matrix is calculated. The degree of usefulness of the anti-ideal matrix $\left(\mathcal{M}^{-} = \left[\mathcal{M}_{ij}^{-}\right]_{mxn}\right)$ is presented in Table 15.

Example. : The calculation of the first alternative according to the first criterion is as follows:

$$\mathcal{N}_{1,1} = \left(\frac{44.08376}{44.60792}\right) * 0.1992 * 100 = 19.6859$$

Table 14
The degree of usefulness of the ideal matrix.

$\mathcal{P}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{P}_1$	16.38545	24.77693	24.58327	16.78
$\mathcal{P}_2$	19.92042	27.61234	33.51411	18.95314
$\mathcal{P}_3$	16.19291	20.71581	24.23749	16.92464
$\mathcal{P}_4$	16.80799	25.03971	25.3072	17.57315
$\mathcal{P}_5$	17.88631	21.66068	31.90994	15.57482

Table 15
The degree of usefulness of the anti-ideal matrix.

$\mathcal{P}$	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
$\mathcal{P}_1$	16.42670	25.24168	24.70888	16.93609
$\mathcal{P}_2$	19.92042	27.61234	33.51411	18.95314
$\mathcal{P}_3$	16.19291	20.71581	24.23749	17.08642
$\mathcal{P}_4$	16.92188	25.48395	25.65409	17.73007
$\mathcal{P}_5$	18.07890	21.92030	32.29565	15.57482

Table 16
The aggregate degree of utility of the alternatives for the ideal value matrix.

$\mathcal{O}$	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$
$\mathcal{O}_i^{+}$	82.53	100.00	78.07	84.73	87.03
$\mathcal{O}_i^{-}$	83.31	100.00	78.23	85.79	87.87

$$\mathcal{N}_{2,1} = \left(\frac{44.08376}{54.23155}\right) * 0.1992 * 100 = 16.1929$$

$$\mathcal{N}_{3,1} = \left(\frac{44.08376}{44.08376}\right) * 0.1992 * 100 = 19.9204$$

$$\mathcal{N}_{4,1} = \left(\frac{44.08376}{45.75824}\right) * 0.1992 * 100 = 19.1910$$

$$\mathcal{N}_{5,1} = \left(\frac{44.08376}{48.6939}\right) * 0.1992 * 100 = 18.0340$$

$$\mathcal{M}_{11}^{-} = (-19.6859 + 19.9204 + 16.1929) = 16.4270$$

Step 11: Using Eqs. (22) and (23), the aggregate degree of utility of the alternatives for the ideal value matrix $\left(\mathcal{O}^{+} = \left[\mathcal{O}_i^{+}\right]_m\right)$ and the anti-ideal value matrix $\left(\mathcal{O}^{-} = \left[\mathcal{O}_i^{-}\right]_m\right)$ are calculated, respectively. The aggregate degree of utility of the alternatives for the ideal value matrix and the anti-ideal value matrix are presented in Table 16.

Example. : The calculation of the first alternative is as follows:

$$\mathcal{O}_1^{+} = (16.38545 + 24.77693 + 24.58327 + 16.78000) = 82.53$$

$$\mathcal{O}_1^{-} = (16.42699 + 25.24168 + 24.70888 + 16.93609) = 83.31$$

Step 12: Using Eq. (24), the final utility functions matrix is calculated $((\Omega = 0.2)$ and $(\varphi = 1))$. The final utility functions matrix $\left(\mathcal{R} = \left[\mathcal{R}_i\right]_m\right)$ is presented in Table 17. The highest value of final utility functions matrix is the best alternative.

Example. : The calculation of the first alternative is as follows:

$$f\left(\mathcal{O}_1^{+}\right) = \frac{82.53}{82.53 + 83.31} = 0.4976$$

$$f\left(\mathcal{O}_1^{-}\right) = \frac{83.31}{82.53 + 83.31} = 0.5023$$

$$\mathcal{R}_1 = (82.53 + 83.31) * \left(\left((0.2) * (0.4976)^1\right) + \left((1 - 0.2) * (0.5023)^1\right)^{1/1}\right) = 83.156$$

Table 17
The aggregate degree of utility of the alternatives for the ideal value matrix.

$\mathcal{R}$	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$
$\mathcal{R}_i$	83.156	100.000	78.200	85.578	87.702
Rank	4	1	5	3	2

4. Results and implications

In this research, the SF-LODECI-ARTASI hybrid model is developed and practically demonstrated in the context of the commercial insurance selection problem. The algorithmic steps of the developed model are sequentially presented to derive findings. The research findings focus on two fundamental points: first, the results related to criterion weights, and second, the findings associated with alternative rankings.

According to the findings related to criterion weights, the prioritization of criteria is as follows: "Vocational types $(\mathcal{C}_3)$ > Healthy conditions $(\mathcal{C}_2)$ > Ages $(\mathcal{C}_1)$ > Economic conditions $(\mathcal{C}_4)$". Accordingly, the "vocational types" criterion is identified as the most crucial criterion, while the criterion with the lowest importance level is determined to be "economic conditions".

In terms of the findings related to alternative rankings, the ranking of insurances is as follows: "Medical insurance $(\mathcal{P}_2)$ > Accident insurance $(\mathcal{P}_5)$ > Travel insurance $(\mathcal{P}_4)$ > Critical illness insurance $(\mathcal{P}_1)$ > Life insurance $(\mathcal{P}_3)$". According to this ranking, the best alternative insurance is "Medical Insurance", and "Life Insurance" occupies the last position in the ranking.

It is crucial to assess the robustness of the obtained results and the proposed SF-LODECI-ARTASI hybrid model. Therefore, sensitivity and comparative analyses are conducted to compare the results. Subsections below detail sensitivity analyses and comparative analyses. Additionally, implications for managers and research insights are presented in the subsequent subsections of this section.

4.1. Sensitivity analysis

To assess the robustness of the proposed SF-LODECI-ARTASI hybrid model, sensitivity analysis scenarios are devised. These scenarios aid in observing the relationships between the obtained findings upon the algorithm's execution under different conditions and the research findings. The sensitivity analysis scenarios (SAS) are structured as follows:

(i) SAS-1: Is there a variation observed in alternative rankings when decision weights are considered equally important?
(ii) SAS-2: If each criterion were successively excluded from the decision model, would changes be observed in alternative rankings?
(iii) SAS-3: If each alternative were successively excluded from the decision model, would changes be observed in alternative rankings?
(iv) SAS-4: When altering the W value in the Yager t-norm and t-conorm operations of the SFYWA aggregation operator, does a change occur in alternative rankings?
(v) SAS-5: When X values are successively assigned 1, 2, and 3, and corresponding Y values are varied between 0 and 1, does a change occur in alternative rankings?

The SASs are sequentially applied, and the obtained findings are presented below:

(i) According to SAS-1, when decision weights are considered equal, there is no change in alternative rankings.
(ii) In SAS-2, each criterion was individually excluded, and alternative rankings are shown in Table 18. According to the results, only when the third criterion is excluded from the decision model, a change is observed in alternative rankings. However, the best alternative remains the same. In scenarios where other criteria are excluded, alternative rankings remain unchanged.
(iii) In SAS-3, each alternative was individually excluded, and alternative rankings remain unchanged. Table 19 presents the alternative rankings in scenarios where alternatives are excluded.
(iv) In SAS-4, ψ values were successively assigned as 2, 3, 4, and 5. Only when $\psi = 5$, a change is observed in alternative rankings. However, the best alternative remains the same, as illustrated in Fig. 2.
(v) In SAS 5, φ values were successively assigned as 1, 2, and 3. For each assigned φ value, Ω values varied from 0.1 to 0.9. According to the results, there is no change in alternative rankings.

4.2. Comparative analysis

Comparative analyses are conducted in this study to juxtapose the findings of the proposed hybrid model with other SF-based methods present in the literature. In the study by [1], the findings of the SF-CPT-TODIM method were compared with those of other methods. The results obtained in this research are also incorporated, leading to the formulation of Table 20. Upon comparison of the obtained findings, it is evident that similar results have been achieved.

Comparative analysis findings (The finding of the SF-LODECI-ARTASI hybrid method has been added to the table organized by [1] (see Fig. 3).

In Table 20, the comparison analysis findings reveal identical rankings for the alternatives. This consistency underscores the applicability and robustness of the newly proposed SF-LODECI-ARTASI hybrid method. The research aims to investigate the hybrid utilization of the LODECI criterion weighting method and ARTASI alternative ranking methods, along with the application of SF sets. The research findings, sensitivity analysis results, and comparison analysis findings strongly support the usability of the proposed method. Hence, the applicability of this hybrid method for various decision support problems can be clearly articulated.

4.3. Research implications

This study contributes to the existing literature on commercial insurance selection through the development of a novel hybrid method, SF-LODECI-ARTASI, incorporating spherical fuzzy sets and leveraging Yager t-norm and t-conorm operations. The implications derived from this research encompass several key aspects:

(i) *Methodological Advancements*: By introducing the SF-LODECI-ARTASI hybrid model, this research advances the methodological landscape of MAGDM approaches in the context of commercial insurance selection. The integration of spherical fuzzy sets and innovative aggregation operators offers a fresh perspective and contributes to the ongoing evolution of decision-making methodologies.
(ii) *Comparative Analysis*: The application of the developed hybrid model to a commercial insurance selection scenario, utilizing a dataset from a previous study, facilitates a comparative analysis of results. This not only ensures the robustness and effectiveness of the proposed method but also enables benchmarking against existing models in the literature.
(iii) *Consistency Validation*: The observed consistency between the findings of this study and those of previous research underscores the reliability and applicability of the SF-LODECI-ARTASI hybrid model. This consistency enhances the credibility of the proposed method in addressing the complexities of commercial insurance decision-making.

Table 18
SAS-2 findings.

SAS-2	Seneria	Ranking of Alternatives	Best Alternative
SAS-2a	1st criterion is excluded	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SAS-2b	2nd criterion is excluded	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SAS-2c	3rd criterion is excluded	$\mathcal{P}_2 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_5 > \mathcal{P}_3$	Medical insurance
SAS-2d	4th criterion is excluded	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance

Table 19
SAS-3 findings.

SAS-3	Seneria	Ranking of Alternatives	Best Alternative
SAS-3a	1st alternative is excluded	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SAS-3b	2nd alternative is excluded	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SAS-3c	3rd alternative is excluded	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SAS-3d	4th alternative is excluded	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SAS-3e	5th alternative is excluded	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance

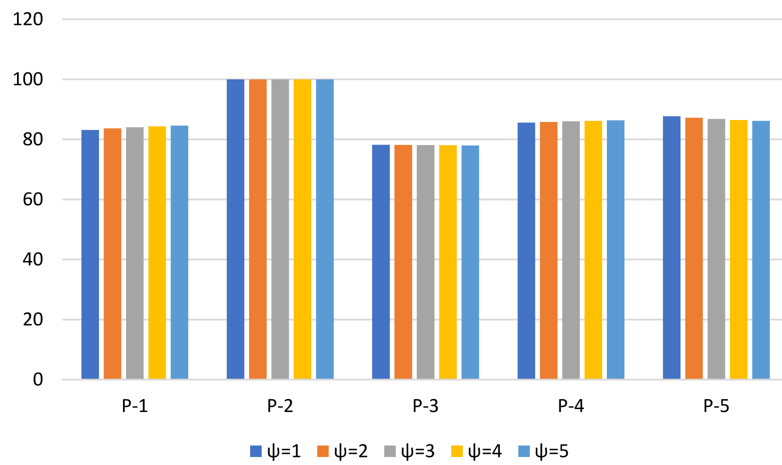

Fig. 2. SAS-4 findings.

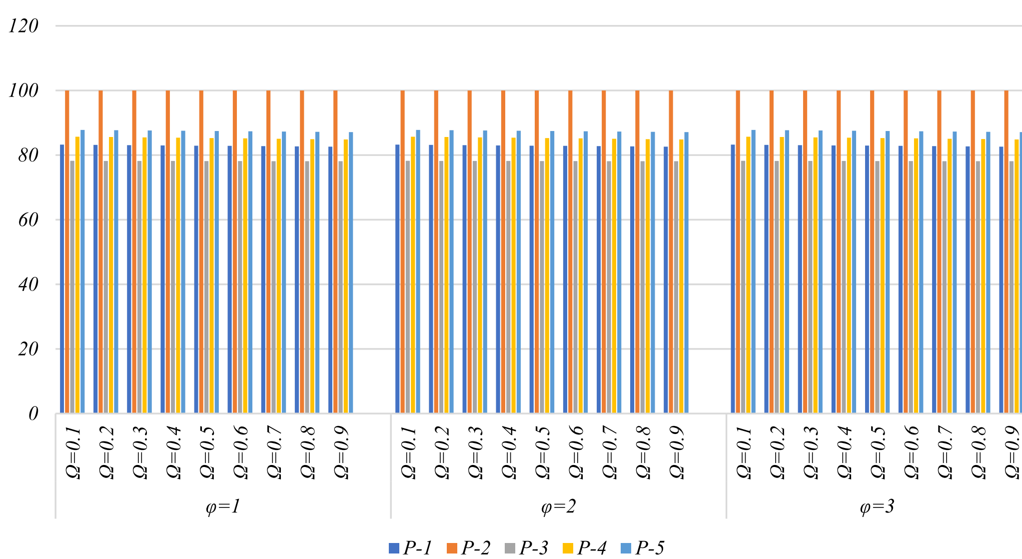

Fig. 3. SAS-5 findings.

(iv) *Robustness Testing*: The creation and examination of various sensitivity scenarios contribute to testing the robustness of the SF-LODECI-ARTASI hybrid model. This aspect is crucial for evaluating the model's performance under diverse conditions and ensures its suitability for real-world applications.

(v) *Practical Implications*: The study's findings offer practical insights for decision-makers involved in commercial insurance selection. The SF-LODECI-ARTASI hybrid model provides a flexible and adaptive framework, considering the inherent uncertainties and varying perspectives involved in decision-making processes.

(vi) *Extension of Decision Support Tools*: The incorporation of spherical fuzzy sets and advanced aggregation operators in the proposed hybrid model extends the repertoire of decision support tools available for commercial insurance decision-making. This extension enhances the decision-making toolkit for practitioners and researchers in the field.

Table 20
Comparative Analysis findings.

Methods	*Ranking*	*Best Alternative*
SWAM [14]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SWGM [14]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SFNWAA [53]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SFNWGA [53]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SFEWIA [54]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SFEWIG [54]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SF-TOPSIS [55]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SF-WASPAS [42]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SF-CODAS [56]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SF-VIKOR [57]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SF-CPT-TODIM [1]	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance
SF-LODECI-ARTASI	$\mathcal{P}_2 > \mathcal{P}_5 > \mathcal{P}_4 > \mathcal{P}_1 > \mathcal{P}_3$	Medical insurance

(vii) *Future Research Directions*: The research implications also highlight avenues for future investigations. The identified strengths and limitations of the SF-LODECI-ARTASI hybrid model pave the way for further refinements and extensions. Future research can explore additional applications and continue to enhance the sophistication of decision-making models in the domain of commercial insurance selection.

4.4. Managerial implications

This research endeavors to provide valuable managerial insights into the realm of commercial insurance selection using a MAGDM approach. The developed SF-LODECI-ARTASI hybrid model, incorporating spherical fuzzy sets and advanced aggregation techniques, offers practical implications and managerial guidance:

(i) *Innovative Decision-Making Framework*: The introduction of the SF-LODECI-ARTASI hybrid model signifies an innovative decision-making framework for managers involved in commercial insurance selection. This approach extends beyond conventional methodologies, integrating recent advancements to enhance the decision-making process.

(ii) *Enhanced Criteria Importance Assessment*: The utilization of the LODECI method for criteria weighting ensures a stable and effective assessment of the importance of various criteria. Managers can rely on this approach to make informed decisions considering the nuanced significance of each criterion.

(iii) *Flexible Alternative Ranking*: The ARTASI method empowers decision-makers to accommodate uncertainties in expert evaluations. This flexibility in alternative ranking enhances the adaptability of the model to varying levels of certainty and diverse perspectives.

(iv) *Application to Real-World Scenarios*: The practical application of the developed hybrid model to a commercial insurance selection case, using a dataset from a relevant study, demonstrates its applicability to real-world scenarios. Managers can draw parallels between the research findings and their own decision-making contexts for more informed choices.

(v) *Comparative Decision Analysis*: The comparative analysis of the SF-LODECI-ARTASI hybrid model against existing methodologies, as well as its consistency with previous research, provides managers with a benchmark for evaluating its effectiveness. This comparative insight aids in selecting the most suitable decision support tool for specific organizational needs.

(vi) *Robustness Testing for Decision Reliability*: The incorporation of various sensitivity scenarios in testing the robustness of the hybrid model ensures decision reliability under different conditions. Managers can gain confidence in the model's ability to withstand uncertainties and variations in decision inputs.

(vii) *Strategic Decision Support*: The managerial insights derived from this research contribute to strategic decision support in the domain of commercial insurance selection. The SF-LODECI-ARTASI hybrid model offers a nuanced and adaptive approach, aligning with the dynamic nature of the insurance landscape.

(viii) *Future Decision-Making Enhancement*: The study opens avenues for further enhancements in decision-making methodologies. Managers can anticipate the evolution of decision support tools, considering the integration of spherical fuzzy sets and advanced aggregation techniques for improved decision outcomes.

In summary, the SF-LODECI-ARTASI hybrid model, as proposed in this research, provides managers with a sophisticated and innovative tool for navigating the complexities of commercial insurance selection, fostering more informed and strategic decision-making processes.

5. Conclusion

In summary, this research endeavors to contribute to the field of commercial insurance selection by proposing a novel MAGDM approach, namely the SF-LODECI-ARTASI hybrid method. The literature review revealed existing studies employing MAGDM in the context of commercial insurance selection, utilizing various criteria weighting and alternative ranking methods. However, our research aims to innovate by incorporating recently developed methods and showcasing their advantages.

The development of a new hybrid model for the commercial insurance selection process, integrating spherical fuzzy sets, Yager t-norm and t-conorm operations, and advanced weighting and ranking techniques, results in the SF-LODECI-ARTASI method. The detailed steps of spherical fuzzy sets, Yager t-norm and t-conorm operations, SF-LODECI, and SF-ARTASI methods are elucidated in the study. Subsequently, the application of this hybrid model to a practical case of commercial insurance selection, utilizing a dataset from the research conducted by [1], ensures comparability of the obtained results with those of previous studies.

The research implementation demonstrates the consistency and reliability of the hybrid model, as evidenced by the congruence between the obtained findings and those from prior research. Furthermore, the robustness of the proposed model is rigorously tested through the creation of various sensitivity scenarios, reinforcing its resilience.

Despite the contributions and advancements made in this research on the development of the SF-LODECI-ARTASI hybrid method for commercial insurance selection, certain limitations should be acknowledged. These limitations pose opportunities for future research to enhance the robustness and applicability of the proposed model. The following are the notable research limitations:

(i) *Generalizability*: The research findings are based on the application of the proposed SF-LODECI-ARTASI hybrid model to a specific commercial insurance selection scenario using a dataset from a particular study [1]. The generalizability of the model to diverse contexts and datasets needs further exploration to validate its effectiveness across various insurance scenarios.

(ii) *Data Quality*: The reliability of the proposed hybrid model is contingent on the quality and representativeness of the dataset. In this study, the data were derived from a single source [1], and variations in data quality across different sources or real-world applications may impact the model's performance.
(iii) *Parameter Sensitivity*: The SF-LODECI-ARTASI hybrid method involves the determination of parameters, such as the selection of fuzzy set parameters and weighting coefficients. Sensitivity to these parameters might affect the model's outcomes, and a more comprehensive analysis of parameter sensitivity is warranted.
(iv) *Expert Involvement*: The effectiveness of the proposed model relies on expert input for criteria weighting and alternative ranking. The subjectivity inherent in expert judgments may introduce bias, and the model's performance could be influenced by the selection and number of experts involved.
(v) *Comparison with Benchmark Models*: While the study compares its results with those from previous research, a more extensive benchmarking against other existing decision-making models in the literature would strengthen the assessment of the SF-LODECI-ARTASI hybrid method's superiority.
(vi) *Dynamic Nature of Insurance Markets*: Commercial insurance markets are dynamic, influenced by evolving economic, regulatory, and technological factors. The static nature of the proposed model may limit its adaptability to changes in the insurance landscape over time.
(vii) *Algorithmic Complexity*: The application of the SF-LODECI-ARTASI hybrid method involves algorithmic complexity, especially in the utilization of spherical fuzzy sets. The computational efficiency and scalability of the model may pose challenges in scenarios with large datasets or real-time decision-making requirements.

Acknowledging these limitations provides a foundation for future researchers to address these concerns, refine the proposed model, and extend its applicability to a broader spectrum of commercial insurance decision-making scenarios.

The current research on the development of the SF-LODECI-ARTASI hybrid model for commercial insurance selection opens avenues for future investigations. To advance the field and address potential areas of improvement, the following future research suggestions are proposed:

(i) Investigate how the SF-LODECI-ARTASI hybrid model can be applied and how effective it is in various decision-making contexts outside the realm of commercial insurance. Evaluating its performance across different industry sectors and decision-making scenarios would provide valuable insights into its versatility and applicability beyond its initial domain.
(ii) Perform an extensive and thorough comparison between the SF-LODECI-ARTASI hybrid model and other contemporary decision-making models found in existing literature. This comprehensive analysis aims to elucidate the specific strengths and weaknesses of the SF-LODECI-ARTASI model in relation to its counterparts, thereby offering deeper insights into its performance and capabilities.
(iii) Improve the SF-LODECI-ARTASI hybrid model to enable it to dynamically respond to fluctuations and transformations within the commercial insurance market. This enhancement involves incorporating adaptive mechanisms that enable the model to evolve and maintain its effectiveness considering shifting economic conditions, regulatory requirements, and technological advancements.
(iv) Explore the potential for transforming the SF-LODECI-ARTASI hybrid model into a real-time decision support system. This investigation would involve assessing its feasibility and addressing any computational efficiency challenges to facilitate timely decision-making in dynamic and fast-paced environments.
(v) Create a user-friendly interface for the SF-LODECI-ARTASI hybrid model, designed to cater to a diverse audience of decision-makers with varying levels of technical proficiency. This interface development aims to enhance accessibility and usability, thereby facilitating the practical implementation and widespread adoption of the model in decision-making processes.
(vi) Perform an extensive sensitivity analysis to delve deeper into the effects of changing parameters on the performance of the SF-LODECI-ARTASI hybrid model. This comprehensive investigation aims to provide a more thorough understanding of how the model behaves across different scenarios, thereby enhancing its robustness and reliability in practical decision-making contexts.
(vii) Validate the SF-LODECI-ARTASI hybrid model across various cultural and regional contexts to assess its effectiveness and applicability in diverse decision-making environments. By understanding how cultural differences impact decision-making criteria and preferences, this validation process aims to enhance the model's cross-cultural relevance and utility.
(viii) Promote collaboration with industry stakeholders, policymakers, and practitioners to ensure the SF-LODECI-ARTASI hybrid model's practical relevance and acceptance. By involving stakeholders in the development process, tailored and impactful decision support tools can be created, aligning with the needs and requirements of real-world decision-making contexts.

By addressing these future research suggestions, scholars can contribute to the ongoing evolution of decision-making methodologies, ultimately enhancing the practical utility of the SF-LODECI-ARTASI hybrid model and similar approaches.

In conclusion, this study not only introduces a novel hybrid method grounded in spherical fuzzy sets but also presents an alternative model for addressing the challenges of commercial insurance selection. The findings contribute to the existing literature by offering a fresh perspective on decision-making methodologies, showcasing the adaptability and reliability of the SF-LODECI-ARTASI hybrid model.

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Ethical approval

This article does not contain any studies with human participants or animals performed by the author.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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