

# Optimization of pilot tones using differential evolution algorithm in MIMO-OFDM systems

Muhammet Nuri SEYMAN<sup>1</sup>, Necmi TAŞPINAR<sup>2,\*</sup>

<sup>1</sup>*Department of Electronic Communication, Vocational High School, Kırıkkale University,  
71100 Kırıkkale-TURKEY*

<sup>2</sup>*Department of Electrical and Electronics Engineering, Erciyes University,  
38039 Kayseri-TURKEY*

*e-mails: mnseyman@kku.edu.tr, taspinar@erciyes.edu.tr*

Received: 21.08.2010

## Abstract

*In this paper, we propose a differential evolution (DE) algorithm for optimizing the placement and power of the pilot tones that are utilized by a least square (LS) algorithm for channel estimation in multiple-input and multiple-output orthogonal frequency-division multiplexing (MIMO-OFDM) systems. Computer simulations demonstrated that the performance of the LS algorithm was increased by optimizing the pilot tones with the DE algorithm instead of locating them orthogonally. We used the upper bound of the mean square error (MSE) as a fitness function of the DE algorithm for optimization tasks. With the use of an upper bound, it is not necessary to compute the matrix inversion that is needed in computing the MSE.*

**Key Words:** *Differential evolution (DE), channel estimation, MIMO-OFDM, optimization*

## 1. Introduction

Wireless communication systems require high supporting data rate transmission and quality of service. Orthogonal frequency division multiplexing (OFDM) is regarded as a promising solution for supporting these requirements. OFDM is a multicarrier modulation scheme that divides the available bandwidth into a number of orthogonal subcarriers that are modulated independently. High bit rate data transmission can, therefore, be provided by using bandwidth efficiently [1]. Additionally, multiple antenna architecture on the transmitter and receiver side, which is called the multiple-input multiple-output (MIMO) technique, is a suitable choice to improve the capacity of OFDM without additional power or bandwidth consumption [2]. Due to the many advantages of MIMO-OFDM, it has been standardized for various digital communication systems such as terrestrial digital audio broadcasting (DAB-T), terrestrial digital video broadcasting (DVB-T), wireless local area networks (WLANs), and 4G wireless cellular systems.

---

\*Corresponding author: Department of Electronic Communication, Vocational High School, Kırıkkale University, 71100 Kırıkkale-TURKEY

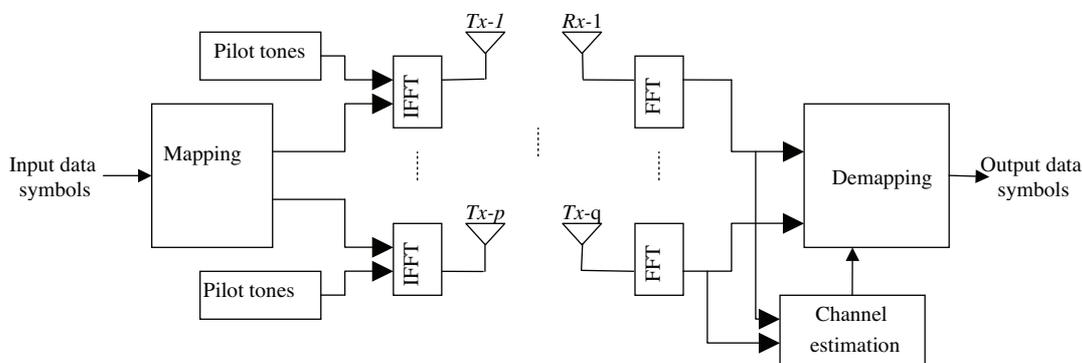
However, in these systems, knowledge of the channel state information (CSI) at the receiver is necessary for interference cancellation, coherent detection, and demodulation. To this end, channel estimation methods based on pilot tones can be applied to obtain CSI in OFDM systems [3-8]. In this method, pilot tones are inserted into each subcarrier at one OFDM symbol or inserted to the subset of subcarriers with a specific period [3]. However, the design of the location of pilot tones and power will directly affect the performance of the channel estimation algorithms. The optimization of the pilot tones for OFDM systems has been investigated in many papers recently [4-8]. Optimum pilot design and the placement of the pilot symbols by minimizing the Cramer-Rao bound were considered for channel estimation in [4]. In [5], the pilot locations for OFDM systems were optimized using bounds on the objective function based on the optimizing symbol error rate (SER) and the channel capacity. By minimizing the mean square error (MSE) of least square (LS) channel estimation, optimal uniform and nonuniform placement of pilot tones for MIMO-OFDM systems were presented in [6] and [7], respectively. In [8], optimal training sequences were derived using the lower bound on the variance of the channel estimate. Application of evolutionary computing techniques such as differential evolution in digital communication systems can also be found in the literature. In [9], Mannoni et al. optimized irregular Gallager codes for OFDM transmission using differential evolution. In [10], a differential evolution algorithm was proposed to reduce the training signal sequence. In [11], differential evolution was applied to design infinite impulse response (IIR) filters.

The aim of this paper is to propose an optimization of the power of pilot tones and their location affecting the performance of the comb-type LS channel estimation technique in MIMO-OFDM systems by using a differential evolution optimizer.

This paper is organized as follows: the background of MIMO-OFDM systems and the MSE of LS estimation technique are introduced in Section 2. In Section 3, the differential evolution algorithm is presented. In Section 4, the objective function of the differential evolution (DE) algorithm is derived. Simulation results and conclusions are given in Section 5 and Section 6, respectively.

## 2. System model

The block diagram of a MIMO-OFDM system that has  $N_t$  transmit antennas,  $N_r$  receive antennas, and  $N$  subcarriers is presented in Figure 1. At each transmit antenna, a data stream, including pilot tones, is turned into OFDM symbols by taking the inverse fast Fourier transform (IFFT). A cyclic prefix (CP) is added and then symbols are transmitted.



**Figure 1.** Simplified block diagram of MIMO-OFDM system.

At the  $q$ th receiver antenna, after removing the CP and applying the fast Fourier transform (FFT), the received signal vectors are expressed as follows.

$$Y^q(n) = \sum_{p=0}^{N_t-1} X_{diag}^p(n) F h^{p,q} + W^q(n) \quad q = 1, 2, \dots, N_r \quad p = 1, 2, \dots, N_q \quad (1)$$

Let  $X^p(n) = D^p(n) + B^p(n)$ , where  $D^p(n)$  is some arbitrary  $N \times 1$  data vector,  $B^p(n)$  is some arbitrary  $N \times 1$  pilot tone vector at time index  $n$ ,  $W^q(n)$  is additive white Gaussian noise,  $h^{p,q}$  is an  $L \times 1$  vector representing the channel impulse response of length  $L$  from the  $p$ th transmit antenna to the  $q$ th receive antenna,  $F$  is an  $N \times N$  unitary DFT matrix, and  $(\cdot)_{diag}$  is a diagonal matrix with column vector  $(\cdot)$ .

$$F = \begin{bmatrix} F_{0,0} & F_{0,1} & \dots & F_{0,N-1} \\ F_{1,0} & F_{1,1} & \dots & F_{1,N-1} \\ \dots & \dots & \dots & \dots \\ F_{N-1,0} & F_{N-1,1} & \dots & F_{N-1,N-1} \end{bmatrix}, \quad (2)$$

where the  $u$ th column and  $v$ th line of  $F_{u,v}$  are equal to  $(1/\sqrt{N})e^{-j2\pi(uv/N)}$ . Eq. (1) can then be rewritten as follows:

$$Y^q(n) = \sum_{p=0}^{N_t-1} D_{diag}^p(n) F h^{p,q} + \sum_{p=0}^{N_t-1} B_{diag}^p(n) F h^{p,q} + W^q(n). \quad (3)$$

If we write Eq. (3) in simplified form, we have:

$$Y^q = G h^q + A h^q + W^q, \quad (4)$$

where  $h^q = [h^{q,1^T}, \dots, h^{q,N_t^T}]^T$  is a channel impulse response vector of  $N_t L$  length,  $G = [D_{diag}^1 F, \dots, D_{diag}^{N_t} F]$  is a  $K \times N_t L$  matrix,  $A = [B_{diag}^1 F, \dots, B_{diag}^{N_t} F]$  is a  $K \times N_t L$  matrix, and  $(\cdot)^T$  is the transpose operation. For a MIMO system,  $N_t \times N_r$  individual channel impulse responses must be estimated. For this reason, one of the channel impulse responses,  $\hat{h}_q$ , can be estimated using a LS algorithm as:

$$\hat{h}_q = A^t Y^q = h_q + (A^H A)^{-1} A^H W^q, \quad (5)$$

where  $(\cdot)^t$  is the matrix pseudoinverse and equals  $A^t = (A^H A)^{-1} A^H$ , and  $(\cdot)^H$  is the Hermitian matrix. It is assumed that pilot sequences are designed as  $M \times N_t L$  matrix  $A$ , which has a full column rank of  $N_t L$  that requires  $M \geq N_t L$  where  $M$  is the set of pilot tones. A minimum number of pilot tones,  $M = L N_t$ , is needed.

For optimal pilot tone design, the MSE of LS is derived as:

$$MSE = \frac{1}{L N_t} \varepsilon \left\{ \left\| \hat{h}^q - h^q \right\|^2 \right\} = \frac{1}{L N_t} tr \left\{ A^t \varepsilon \left\{ W^q W^{qH} \right\} A^{tH} \right\}, \quad (6)$$

where  $(\cdot)^t$  is the matrix pseudoinverse,  $tr(\cdot)$  is the trace operator, and  $\varepsilon(\cdot)$  is the expectation. For zero-mean white noise,  $\varepsilon \left\{ W^q W^{qH} \right\} = \sigma^2 I_M$  where  $I_M$  is an  $M \times M$  identity matrix, and  $\sigma$  is the noise variance. In that case, the MSE can be defined as:

$$MSE = \frac{\sigma^2}{L N_t} tr \left\{ (A^H A)^{-1} \right\}. \quad (7)$$

The minimum MSE can be achieved if  $A^H A = P I_{L N_t}$ , where  $P$  is a fixed power dedicated for the pilot. The minimum MSE is then obtained as [6-7]:

$$MSE = \frac{\sigma^2}{P}. \quad (8)$$

### 3. Differential evolution algorithm

The DE algorithm is a population-based stochastic parallel direct search technique for the global optimization problem [12]. DE uses differential information of individuals in the current population of solutions to create new candidate solutions. Use of a few control parameters and fast convergence are the main advantages of DE in finding optimal points of problems.

The algorithm steps are given in the Table [11]. DE starts with the random initializing of the initial population. The mutation, crossover, and selection operator are then applied to improve the population [12-13].

Table. DE algorithm steps.

Step 1.	<i>Initialization</i>
Step 2.	<i>Evaluation</i>
Step 3.	<i>REPEAT</i>
	<i>Mutation</i>
	<i>Recombination</i>
	<i>Evaluation</i>
	<i>Selection</i>
	<i>UNTIL (stop criteria are met)</i>

#### 3.1. Mutation

In mutation, new vectors are derived by the combination of randomly chosen vectors from the current populations at each generation, which is described as follows.

For each target vector  $x_i (i = 1, 2, \dots, m)$ , a mutant vector in generation  $G$  is defined by:

$$V_{i,G+1} = x_{r_1,G} + \eta(x_{r_2,G} - x_{r_3,G}), \quad (9)$$

where  $i, r_1, r_2, r_3 \in \{1, 2, \dots, m\}$  are mutually different random integer indices and  $\eta$  is a scaling factor that determines the difference of  $x_{r_1}$  and  $x_{r_2}$ .

#### 3.2. Crossover

Crossover generates new solutions by shuffling the competing vectors, and it increases the diversity of the population. The operation is given by:

$$U_{i,G+1} = \begin{cases} V_{i,G+1} & \text{if } rand_j(0,1) \leq C_r \vee j = k \\ x_{i,G} & \text{else} \end{cases}, \quad (10)$$

where  $C_r$  is the crossover constant and  $rand_j(0,1)$  is the  $j$ th evaluation of random number generation.  $k \in \{1, 2, \dots, D\}$  is the random parameter index.

### 3.3. Selection

Selection decides whether the trial individual  $U_{i,G+1}$  will be a member of the population of the next generation ( $G + 1$ ) or not.

$$x_{i,G+1} = \begin{cases} U_{i,G+1}, & \text{if } f(U_{i,G+1}) < f(x_{i,G}) \\ x_{i,G}, & \text{else} \end{cases} \quad (11)$$

where  $f(\cdot)$  is the fitness function.

## 4. Fitness function of differential evaluation

When we use the MSE function in Eq. (7) as an objective function for the DE algorithm, the computational load of the optimizer will increase because of the matrix inversion of Eq. (8). In that case, we reduce the computational complexity of Eq. (8) using the Gershgorin circle theorem [14] because  $A$  is full rank and the eigenvalues of  $AA^H$  are positive and real. When this theorem is considered, the upper bound of MSE that we use as an objective function for DE is obtained by:

$$\text{tr} \{(AA^H)^{-1}\} = \sum_{i=1}^L \frac{1}{\lambda_i} \leq \begin{cases} \frac{L}{P - R_{\max}} & , P > R_{\max} \\ +\infty & , P \leq R_{\max} \end{cases} \quad (12)$$

where  $P(i = 1, \dots, L)$  is the diagonal elements of matrix  $(AA^H)$ ,  $\lambda_i(i = 1, \dots, L)$  is the eigenvalues, and  $R_{\max} = \max(R_i)$  is the maximum radius of the Gershgorin disk, whose definition is:

$$R_i = \sum_{\substack{j=1 \\ j \neq i}}^L |b_{ij}|. \quad (13)$$

In regards to Eq. (12), we use  $\frac{R_{\max}}{P}$  as the fitness function for DE.

In order to optimize the positions of pilot tones, the populations called pilot positions are first initialized at random values between 0 and 127. Possible combinations of pilot positions are tested using the fitness function  $\frac{R_{\max}}{P}$ . In order to get the best positions of the pilots, their locations are improved by the mutation, crossover, and selection operators mentioned in Section 3. The process is repeated until the termination criteria are reached, which was determined as a maximum of 2000 iterations in our simulations. At the end of the iterations, the best locations were chosen and 3.2 s was required as the convergence time when a Pentium D 3.0 computer was used. The power of the pilot tones was optimized by a continuous DE algorithm as mentioned above, but for this process, populations were initialized at random values between 0 and 1.

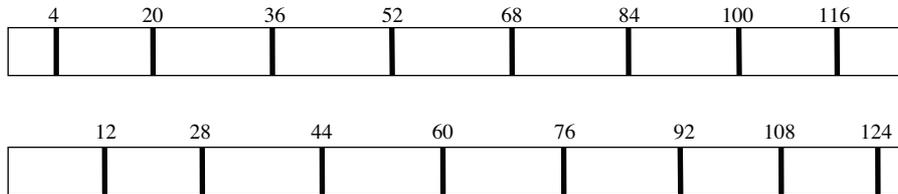
## 5. Simulation results

The simulations were performed for a  $2 \times 2$  MIMO-OFDM system with  $N = 128$  subcarriers, a cyclic prefix size of 32, 16 pilot tones, and a power of the pilot tones that was normalized to 1 dB. QPSK modulation was used. An 8-tap channel model, whose taps were independent, identically distributed, and correlated in time with a correlation function according to Jakes' model,  $r_{hh}(\tau) = \sigma_h^2 J_0(2\pi f_d \tau)$ , was used. Maximum relative delays of channels of 600 ns and powers of paths of -2 dB were chosen for the channel profile, while  $f_d = 5$

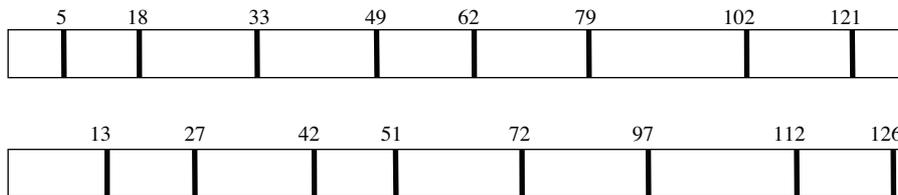
and  $f_d = 40$  Hz Doppler shifts were assumed over the channel. For optimizing the location of pilot tones and their power, we chose the differential evolution parameters of a population size of 20, scale factor of 0.8, and crossover probability of 0.8.

In order to evaluate the performance of the pilot tone design using DE, we simulated various pilot tones:

- a) Equipowered and equispaced orthogonal pilot tones, shown in Figure 2.
- b) Equipowered and optimal placement of pilot tones using DE, shown in Figure 3.
- c) Optimal power and placement of pilot tones using DE.

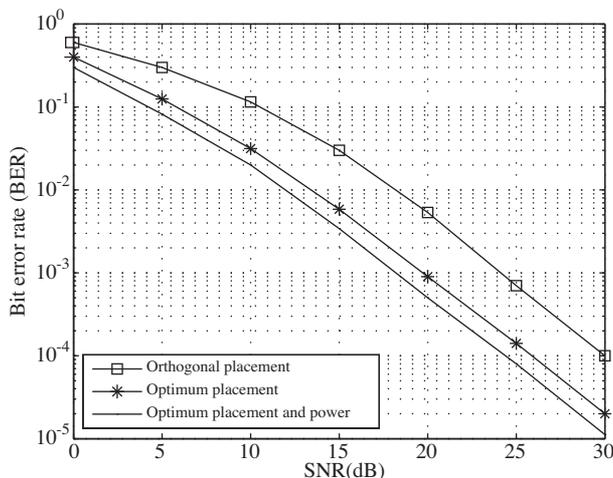


**Figure 2.** The placement of orthogonal pilot tones for 2 transmit antennas.

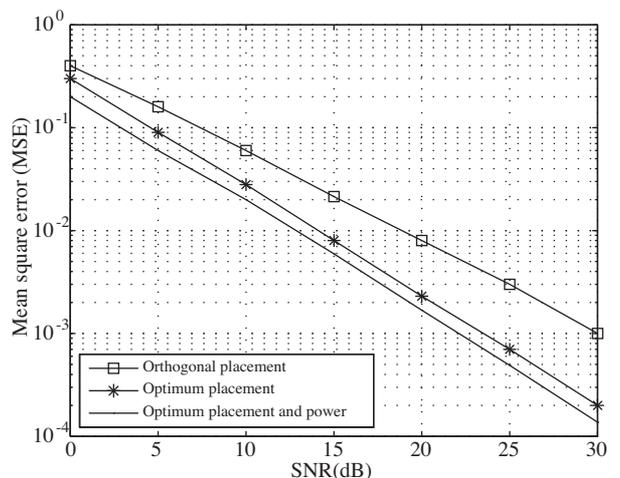


**Figure 3.** The placement of optimized pilot tones for 2 transmit antennas.

Figures 4 and 5 illustrate the bit error rate (BER) versus the signal-to-noise ratio (SNR) and the MSE versus the SNR of various pilot signals at an  $f_d = 5$  Hz Doppler shift, respectively. As can be seen, optimizing the pilot tones using a DE algorithm increases the performance of the LS channel estimation method when compared to orthogonal placement of the pilot tones.



**Figure 4.** BER versus SNR ( $f_d = 5$  Hz).



**Figure 5.** MSE versus SNR ( $f_d = 5$  Hz).

When Figure 4 is considered, at increased SNR values, both the BER and MSE decrease further. For instance, at a 10-dB SNR, the BER difference between the orthogonal placement and optimum placement of

the pilot tones is less than  $10^{-1}$ , whereas this difference is approximately  $10^{-1}$  at a 30-dB SNR. Furthermore, by not only optimizing the placement of pilot tones but also optimizing their power, the estimation performance is increased. Joint optimization (both placement and power) has more than a 1-dB gain in SNR than optimum placement of pilot tones at an MSE of  $10^{-1}$ . However, power optimization has a low impact on the performance. As a matter of fact, optimization of pilot tone placements has considerable effects on channel estimation performance.

To see the effect of increasing Doppler shifts, simulation results of BER versus SNR are shown in Figure 6, and MSE versus SNR is shown in Figure 7 for an  $f_d = 40$  Hz Doppler shift. It can be seen that increased Doppler shifts decrease the performance of the LS method. When Figures 4 and 6 are considered, it is seen that orthogonal pilot tones have a 5-dB loss in SNR at a BER of  $10^{-3}$  when simulated at  $f_d = 40$  Hz and  $f_d = 5$  Hz Doppler shifts. However, this difference is approximately 4 dB in the optimal placement of the pilot tones. Simulation results show that the optimization of pilot tones makes the system robust to the effect of Doppler shifts. Optimization of pilot tones in MIMO-OFDM systems can also be done to make the system more robust against estimation errors when the channel parameters are changed. Moreover, the DE algorithm avoids exhaustive searches for the optimization of pilot tone locations. The exhaustive search for pilot position for orthogonal pilots is  $C_{128}^{16} \approx 2.26041 \times 10^{28}$  in a system that has 128 subcarriers and 16 pilot tones, whereas the number of searches in the DE algorithm is just  $2000 \times 20 = 4 \times 10^4$  for a size of 2000 generations and 20 populations. Furthermore, we examined the rough computational complexity of orthogonal and optimal placement based on DE in terms of the number of multiplications and additions in order to represent the computational advantage of our proposal. The computational complexity was computed in terms of  $N_t$  (number of transmitter antennas),  $N_r$  (number of receiver antennas),  $n$  (population size),  $N_{iteration}$  (number of iterations), and  $M$  (number of pilot tones). For orthogonal placement as presented in [6], in order to get the orthogonality of the pilot tones,  $N_t N_r M^4$  multiplications are required. When MSE in Eq. (8) is used as the objective function, matrix inversion must also be computed; as a consequence,  $M^3$  multiplications and additions are needed [15]. However, in the DE algorithm, we avoid computing this matrix inversion by use of the fitness function mentioned in Section 4. For the proposed DE algorithm, at first  $n(N_t N_r)$  multiplications are required for the fitness of each position in population  $n$ . In order to improve the population,  $\mu$  additional

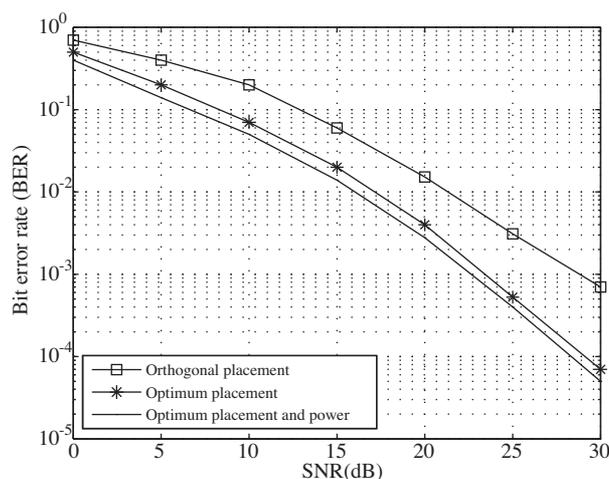


Figure 6. BER versus SNR ( $f_d = 40$  Hz).

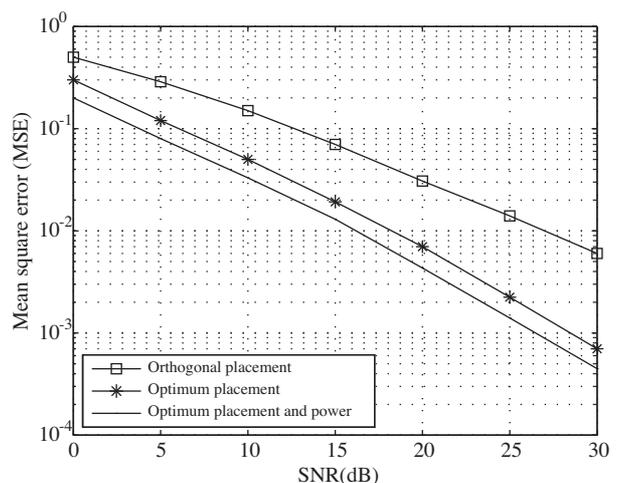


Figure 7. MSE versus SNR ( $f_d = 40$  Hz).

multiplications per iteration are needed. As a result,  $n(N_t N_r + \mu)N_{iteration}$  multiplications are required in the DE algorithm. As can be seen from the above computational analysis, optimizing pilot tones using DE is less complex than placing them orthogonally.

## 6. Conclusion

In this paper, a DE algorithm was proposed in order to optimize not only the placements of pilot tones but also their power for LS channel estimation algorithm in MIMO-OFDM systems. Compared with the orthogonal placement of pilot tones, the optimization of the location and power of pilot tones based on the DE algorithm offers better performance and robustness to different Doppler shifts. By using the DE algorithm, the exhaustive search for pilot position is considerably lessened compared to the orthogonal pilot tones. Moreover, by using the upper bound of MSE instead of using MSE directly for the fitness function of the DE algorithm, computing the matrix inversion can be avoided.

## Acknowledgement

We would like to thank the referees for their excellent constructive comments to improve the paper.

## References

- [1] R.V. Nee, R. Prasad, OFDM for Wireless Multimedia Communications, Boston, Artech House, 2000.
- [2] A.J. Paulraj, D.A. Gore, R.U. Nabar, H. Bölcskei, "An overview of MIMO communications - a key to gigabit wireless", Proceedings of the IEEE, Vol. 92, pp. 198-218, 2004.
- [3] S. Coleri, M. Ergen, A. Puri, A. Bahai, "Channel estimation techniques based on pilot arrangement in OFDM systems", IEEE Transactions on Broadcasting, Vol. 48, pp. 223-229, 2002.
- [4] M. Dong, L. Tong, "Optimal design and placement of pilot symbols for channel estimation", IEEE Transactions on Signal Processing, Vol. 50, pp. 3055-3068, 2002.
- [5] A.Y. Panah., R.G. Vaughan, R.W. Heath Jr., "Optimizing pilot locations using feedback in OFDM systems", IEEE Transactions on Vehicular Technology, Vol. 58, pp. 2803-2814, 2009.
- [6] I. Barhumi, G. Leus, M. Moonen, "Optimal training design for MIMO OFDM systems in mobile wireless channels", IEEE Transactions on Signal Processing, Vol. 51, pp. 1615-1623, 2003.
- [7] D. Hu, L. Yang, L. He, Y. Shi, "Optimal pilot sequence design for multiple-input multiple-output OFDM systems", Proceedings of the IEEE Global Telecommunications Conference (GLOBECOM'05), Saint Louis, USA, pp. 2260-2264, 2005.
- [8] J.H. Manton, "Optimal training sequences and pilot tones for OFDM systems", IEEE Communications Letters, Vol. 5, pp. 151-153, 2001.
- [9] V. Mannoni, D. Declercq, G. Gelle, "Optimized irregular Gallager codes for OFDM transmission", Proceedings of the IEEE 13th International Symposium on Personal, Indoor and Mobile Radio Communications, Lisbon, Portugal, Vol. 1, pp. 222-226, 2002.

- [10] Z. Wu, H. Huang, X. Zhang, B. Yang, H. Dong, "Adaptive equalization using differential evolution", Proceedings of the IEEE World Congress on Computational Intelligence, pp. 1962-1967, Hong Kong, 2008.
- [11] N. Karaboga, "Digital IIR filter design using differential evolution algorithm", EURASIP Journal on Applied Signal Processing, Vol. 8, pp. 1269-1276, 2005.
- [12] R. Storn, K. Price, "Differential evolution - a simple and efficient heuristic for global optimization over continuous spaces", Journal of Global Optimization, Vol. 11, pp. 341-359, 1997.
- [13] S. Rahnamayan, H.R. Tizhoosh, M.M.A. Salama, "Opposition-based differential evolution", IEEE Transactions on Evolutionary Computation, Vol. 12, pp. 64-79, 2008.
- [14] R.A. Horn, C.R. Johnson, Matrix Analysis, Cambridge, Cambridge University Press, 1985.
- [15] G.H. Golub, C.F.V. Loan, Matrix Computations, Baltimore, Johns Hopkins University Press, 1996.