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SEMI-PARALLEL TENSOR PRODUCT SURFACES IN SEMI-EUCLIDEAN SPACE $\mathbb{E}_2^4$

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Abstract. In this article, the tensor product surfaces are studied that arise from taking the tensor product of a unit circle centered at the origin in Euclidean plane $\mathbb{E}^2$ and a non-null, unit planar curve in Lorentzian plane $\mathbb{E}_1^2$. Also we have shown that the tensor product surfaces in 4-dimensional semi-Euclidean space with index 2, $\mathbb{E}_2^4$, satisfying the semi-parallelity condition $\overline{R}(X,Y).h = 0$ if and only if the tensor product surface is a totally geodesic surface in $\mathbb{E}_2^4$.

1. Introduction

B. Y. Chen initiated the study of the *tensor product immersion* of two immersions of a given Riemannian manifold [6]. This concept originated from the investigation of the quadratic representation of submanifold. Inspired by Chen's definition, F. Decruyenaere, F. Dillen, L. Verstraelen and L. Vrancken studied in [8] the tensor product of two immersions of, in general, different manifolds. Under some conditions, this realizes an immersion of the product manifold.

Let M and N be two differentiable manifolds and assume that

$$f : M \to \mathbb{E}^m,$$

and

$$g : N \to \mathbb{E}^n$$

are two immersions. Then the direct sum and tensor product maps are defined respectively by

$$f \oplus h : M \times N \to \mathbb{E}^{m+n}$$
$$(p,q) \to f(p) \oplus h(q) = (f^1(p), \dots, f^m(p), h^1(q), \dots, h^n(q))$$

and

$$f \otimes h : M \times N \to \mathbb{E}^{mn}$$
$$(p,q) \to f(p) \otimes h(q) = (f^1(p)h^1(q), \dots, f^1(p)h^n(q), \dots, f^m(p)h^n(q))$$

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Necessary and sufficient conditions for $f \otimes h$ to be an immersion were obtained in [7]. It is also proved there that the pairing $(\oplus, \otimes)$ determines a structure of a semiring on the set of classes of differentiable manifolds transversally immersed in Euclidean spaces, modulo orthogonal transformations. Some semirings were studied in [8]. In the special case, a tensor product surface is obtained by taking the tensor product of two curves. In many papers, minimality and totally reality properties of a tensor product surfaces were studied for example [2], [10], [11], [12]. The relations between a tensor product surface and a Lie group was shown in [15], [16]. In [2], Bulca and Arslan studied tensor product surfaces in 4- dimensional Euclidean space $\mathbb{E}^4$and they show that tensor product surfaces satisfying the semi-parallelity condition $\overline{R}(X,Y).h = 0$ are totally umbilical surface.

In this article, we investigate a tensor product surface M which is obtained from two curves. One of them is a unit circle centered at the origin in Euclidean plane $\mathbb{E}^2$ and a non-null, unit planar curve in Lorentzian plane $\mathbb{E}^2_1$. Firstly, we investigated some geometric properties of the tensor product surface in pseudo-Euclidean 4-space $\mathbb{E}^4_2$ then we obtain the sufficient and necessary conditions for the surface satisfying the semi parallelity condition $\overline{R}(X,Y).h = 0$.

We remark that the notions related with pseudo- Riemannian geometry are taken from [14].

2. Preliminaries

In the present section we give some definitons about Riemannian submanifolds from [5] and [4]. Let $\iota : M \rightarrow \mathbb{E}^n$ be an immersion from an $m-$dimensional connected Riemannian manifold M into an $n-$ dimensional Euclidean space $\mathbb{E}^n$. We denote by g the metric tensor of $\mathbb{E}^n$ as well as induced metric on M. Let $\bar{\nabla}$ be the Levi- Civita connection of $\mathbb{E}^n$ and ∇ the induced connection on M. Then the Gaussian and Weingarten formulas are given by

$$\begin{aligned} \bar{\nabla}_X Y &= \nabla_X Y + h(X,Y) \\ \bar{\nabla}_X N &= -A_N X + \nabla^{\perp}_X N \end{aligned} \tag{2.1}$$

where X, Y are vector fields tangent to M and N is normal to M. Moreover, h is the second fundamental form, $\nabla^{\perp}$ is linear connection induced in the normal bundle $T^{\perp}M$, called normal connection and A_N is the shape operator in the direction of N that is related with h by

$$< h(X,Y), N >=< A_N X, Y > . \tag{2.2}$$

If the set $\{X_1, .., X_m\}$ is a local basis for $\chi(M)$ and $\{N_1, ..., N_{n-m}\}$ is an orthonormal local basis for $\chi^{\perp}(M)$, then h can be written as

$$h = \sum_{\alpha=1}^{n-m} \sum_{i,j=1}^{m} h^{\alpha}_{ij} N_{\alpha}, \tag{2.3}$$

where

$$h_{ij}^{\alpha} =< h(X_i, X_j), N_{\alpha} > .$$

The covariant differentiation $\bar{\nabla}h$ of the second fundamental form h on the direct sum of the tangent bundle and the normal bundle $TM \oplus T^{\perp}M$ of M is defined by

$$(\bar{\nabla}_X h)(Y, Z) = \nabla^{\perp}_X h(Y, Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z), \tag{2.4}$$

for any vector fields X, Y and Z tangent to M. Then we have the Codazzi equation as

$$(\bar{\nabla}_X h)(Y, Z) = (\bar{\nabla}_Y h)(X, Z). \tag{2.5}$$

We denote by R the curvature tensor associated with ∇;

$$R(X, Y)Z = -\nabla_X \nabla_Y Z + \nabla_Y \nabla_X Z + \nabla_{[X,Y]} Z, \tag{2.6}$$

and denote by $R^{\perp}$ the curvature tensor associated with $\nabla^{\perp}$

$$R^{\perp}(X, Y)\eta = \nabla^{\perp}_Y \nabla^{\perp}_X \eta - \nabla^{\perp}_X \nabla^{\perp}_Y \eta - \nabla^{\perp}_{[X,Y]} \eta. \tag{2.7}$$

The equations Gauss and Ricci are given by

$$< R(X, Y)Z, W >=< h(X, W), h(Y, Z) > - < h(X, Z), h(Y, W) >, \tag{2.8}$$

$$< \bar{R}(X, Y)\eta, \xi > - < R^{\perp}(X, Y)\eta, \xi >=< [A_{\eta}, A_{\xi}]X, Y >, \tag{2.9}$$

for any vector fields X, Y, Z, W tangent to M and ξ, η normal vector fields to M.

The Gaussian curvature of M is defined by

$$K =< h(X_1, X_1), h(X_2, X_2) > - \|h(X_1, X_2)\|^2 \tag{2.10}$$

where the set $\{X_1, X_2\}$ is a linearly independent subset of $\chi(M)$.

The normal curvature K_N of M is defined by

$$K_N = \left\{ \sum_{1=\alpha<\beta}^{n-2} < R^{\perp}(X_1, X_2)N_{\alpha}, N_{\beta} >^2 \right\}^{1/2} \tag{2.11}$$

where $\{N_{\alpha}, N_{\beta}\}$ is an orthonormal basis of $\chi^{\perp}(M)$. From (2.11) we conclude that $K_N = 0$ if and only if $\nabla^{\perp}$ is a flat normal connection of M .

Further, the mean curvature vector $\vec{H}$ of M is defined by

$$\vec{H} = \frac{1}{m} \sum_{\alpha=1}^{n-m} tr(A_{N_{\alpha}}) N_{\alpha} \tag{2.12}$$

Let us consider the product tensor $\bar{R}.h$ of the curvature tensor $\overline{R}$ with the second fundamental form h is defined by

$$(\bar{R}(X, Y).h)(Z, T) = \bar{\nabla}_X(\bar{\nabla}_Y h(Z, T)) - \bar{\nabla}_Y(\bar{\nabla}_X h(Z, T)) - \bar{\nabla}_{[X,Y]} h(Z, T)) \tag{2.13}$$

for all X, Y, Z, T tangent to M.

The surface M is said to be semi - parallel (or semi-symmetric) if $\bar{R}.h = 0$, i.e. $\bar{R}(X,Y).h = 0$ [9], [17]. It is easily seen that

$$(\bar{R}(X,Y).h)(Z,T) = R^{\perp}(X,Y)h(Z,T) - h(R(X,Y)Z,T) - h(Z,R(X,Y)T) \tag{2.14}$$

Lemma 2.1. [9] *Let $M \subset \mathbb{E}^n$ be a smooth surface given with the patch X(u,v). Then the following equalities are hold;*

$$\left.\begin{aligned}
(\bar{R}(X_1,X_2).h)(X_1,X_1) = {} & \left(\sum_{\alpha=1}^{n-2} h_{11}^{\alpha}(h_{22}^{\alpha} - h_{11}^{\alpha} + 2K)\right) h(X_1,X_2) \\
& + \sum_{\alpha=1}^{n-2} h_{11}^{\alpha} h_{12}^{\alpha} (h(X_1,X_1) - h(X_2,X_2)) \\
(\bar{R}(X_1,X_2).h)(X_1,X_2) = {} & \left(\sum_{\alpha=1}^{n-2} h_{12}^{\alpha}(h_{22}^{\alpha} - h_{11}^{\alpha})\right) h(X_1,X_2) \\
& + \left(\sum_{\alpha=1}^{n-2} h_{12}^{\alpha} h_{12}^{\alpha} - K\right) (h(X_1,X_1) - h(X_2,X_2)) \\
(\bar{R}(X_1,X_2).h)(X_2,X_2) = {} & \left(\sum_{\alpha=1}^{n-2} h_{22}^{\alpha}(h_{22}^{\alpha} - h_{11}^{\alpha} - 2K)\right) h(X_1,X_2) \\
& + \sum_{\alpha=1}^{n-2} h_{22}^{\alpha} h_{12}^{\alpha} (h(X_1,X_1) - h(X_2,X_2))
\end{aligned}\right\} \tag{2.15}$$

Semi parallel surfaces classified by J. Deprez [9].

Theorem 2.1. [9]*Let M be a surface in n- dimensional Euclidean space $\mathbb{E}^n$. Then M is semi-parallel if and only if locally;*

i) M is aquivalent to 2-sphere, or

ii)M has trivial normal connection, or

iii) M is an isotropic surface in $\mathbb{E}^5 \subset \mathbb{E}^n$ satisfying $\|H\|^2 = 3K$.

3. Tensor product surfaces of a Euclidean plane curve and a Lorentzian plane curve

Minimal and pseudo-minimal tensor product surfaces of a Lorentzian plane curve and a Euclidean plane curve was studied by I. Mihai and et al. in [13]. They also gave some examples of non-minimal pseudo-umbilical tensor product surfaces. It is well konown that the tensor product of two immersions is not commutative.Thus the tensor product surfaces of a Euclidean plane curve and a Lorentzian plane curve is a new surface in 4-dimensional semi-Euclidean space with index 2.

In the following section, we will consider the tensor product immersions which is obtained from a Euclidean plane curve and a Lorentzian plane curve. Let $c_1 : \mathbb{R} \to \mathbb{E}^2$ be a Euclidean plane curve and $c_2 : \mathbb{R} \to \mathbb{E}_1^2$ be a non-null Lorentzian plane curve. Put $c_1(t) = (\alpha_1(t), \alpha_2(t))$ and $c_2(s) = (\beta_1(s), \beta_2(s))$.

Then their tensor product surface is given by

$$x = c_1 \otimes c_2 : \mathbb{R}^2 \to \mathbb{E}_2^4$$

$$x(t,s)=(\alpha_1(t)\beta_1(s),\alpha_1(t)\beta_2(s),\alpha_2(t)\beta_1(s),\alpha_2(t)\beta_2(s)).$$

The metric tensor on $\mathbb{E}_1^2$ and $\mathbb{E}_2^4$ is given by

$$g=-dx_1^2+dx_2^2$$

and

$$\mathbf{g}=-dx_1^2+dx_2^2-dx_3^2+dx_4^2,$$

respectively.

If we take c_1 as a Euclidean unit circle $c_1(t)=(\cos t,\sin t)$ at centered origin and $c_2(s)=(\alpha(s),\beta(s))$ is a spacelike or timelike curve with unit speed then the surface patch becomes

$$M:x(t,s)=(\alpha(s)\cos t,\beta(s)\cos t,\alpha(s)\sin t,\beta(s)\sin t) \tag{3.1}$$

An orthonormal frame tangent to M is given by

$$\begin{aligned} e_1 &= \frac{1}{\|c_2\|}\frac{\partial x}{\partial t} \\ &= \frac{1}{\|c_2\|}(-\alpha(s)\sin t,-\beta(s)\sin t,\alpha(s)\cos t,\beta(s)\cos t), \\ e_2 &= \frac{\partial x}{\partial s} \\ &= (\alpha^{'}(s)\cos t,\beta^{'}(s)\cos t,\alpha^{'}(s)\sin t,\beta^{'}(s)\sin t). \end{aligned} \tag{3.2}$$

The normal space of M is spanned by

$$\begin{aligned} n_1 &= (\beta^{'}(s)\cos t,\alpha^{'}(s)\cos t,\beta^{'}(s)\sin t,\alpha^{'}(s)\sin t), \\ n_2 &= \frac{1}{\|c_2\|}(-\beta(s)\sin t,-\alpha(s)\sin t,\beta(s)\cos t,\alpha(s)\cos t) \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} \mathbf{g}(e_1,e_1) &= -\mathbf{g}(n_2,n_2)=\frac{g(c_2(s),c_2(s))}{\|c_2\|^2}=\varepsilon_1, \\ \mathbf{g}(e_2,e_2) &= -\mathbf{g}(n_1,n_1)=g(c_2^{'}(s),c_2^{'}(s))=\varepsilon_2 \end{aligned} \tag{3.4}$$

and $\varepsilon_1=\mp1$, $\varepsilon_2=\mp1$.

By covariant differentiation with respect to e_1 and e_2 a straightforward calculation gives

$$\begin{aligned}\bar{\nabla}_{e_1} e_1 &= a\varepsilon_2 e_2 - b\varepsilon_2 n_1\\ \bar{\nabla}_{e_1} e_2 &= -a\varepsilon_1 e_1 - b\varepsilon_1 n_2\\ \bar{\nabla}_{e_1} n_1 &= -b\varepsilon_1 e_1 - a\varepsilon_1 n_2\\ \bar{\nabla}_{e_1} n_2 &= -b\varepsilon_2 e_2 + a\varepsilon_2 n_1\end{aligned} \tag{3.5}$$

$$\begin{aligned}\bar{\nabla}_{e_2} e_1 &= -b\varepsilon_1 n_2\\ \bar{\nabla}_{e_2} e_2 &= -c\varepsilon_2 n_1\\ \bar{\nabla}_{e_2} n_1 &= -c\varepsilon_2 e_2\\ \bar{\nabla}_{e_2} n_2 &= -b\varepsilon_1 e_1\end{aligned} \tag{3.6}$$

where a, b and c are Christoffel symbols and as in follows

$$a = a(s) = \frac{\alpha\alpha^{'} - \beta\beta^{'}}{\|c_2\|^2}, \tag{3.7}$$

$$b = b(s) = \frac{\alpha\beta^{'} - \alpha^{'}\beta}{\|c_2\|^2}, \tag{3.8}$$

$$c = c(s) = \alpha^{'}\beta^{''} - \alpha^{''}\beta^{'}. \tag{3.9}$$

In addition, from (2.3) second fundamental form of this structure is written as,

$$h = \sum_{i,j,\alpha=1}^{2} \varepsilon_\alpha h_{ij}^{\alpha} n_\alpha, \tag{3.10}$$

where

$$\begin{aligned}&h_{11}^1 = b && h_{11}^2 = 0\\ &h_{12}^1 = h_{21}^1 = 0 && h_{12}^2 = h_{21}^2 = b\\ &h_{22}^1 = c && h_{22}^2 = 0\end{aligned} \tag{3.11}$$

By considering equations (3.8) and 3.9, we conclude that

Corollary 3.1. *If $b = 0$ then c is also zero.*

Also by using Corollary 3.1 and (3.11), we have

Corollary 3.2. *M is a totally geodesic surface in $\mathbb{E}_2^4$ if and only if $b = 0$ which means that c_2 is a straightline passing through the origin.*

If $b = 0$, from (3.8), we get $c_2(s) = \beta(s)(\lambda, 1)$. Since M is a non-degenerate surface, the position vector of c_2 cannot be a null then $\lambda \neq \pm 1$. In this case, we can write the parametric equation of tensor product surface M as follows

$$M : x(t, s) = (\lambda\beta(s)\cos t, \beta(s)\cos t, \lambda\beta(s)\sin t, \beta(s)\sin t),\ \lambda \neq \pm 1,\ \lambda \in \mathbb{R}.$$

Indeed, this surface fully lies in a cone surface passing through the origin (but not light cone) in 4-dimensional semi-Euclidean space with index 2, $\mathbb{E}_2^4$, with equation $-x_1^2 + \lambda^2 x_2^2 - x_3^2 + \lambda^2 x_4^2 = 0$ where $\lambda \neq \pm 1$ and $\lambda \in \mathbb{R}$.

The induced covariant differentiation on M as in follows,

$$\left.\begin{array}{rcl}\nabla_{e_1}e_1 & = & a\varepsilon_2 e_2,\\ \nabla_{e_1}e_2 & = & -a\varepsilon_1 e_1,\\ \bar{\nabla}_{e_2}e_1 & = & 0,\\ \bar{\nabla}_{e_2}e_2 & = & 0.\end{array}\right\} \tag{3.12}$$

$$\left.\begin{array}{rcl}\nabla^{\perp}_{e_1}n_1 & = & a\varepsilon_2 n_2,\\ \nabla^{\perp}_{e_1}n_2 & = & a\varepsilon_2 n_1,\end{array}\right\} \tag{3.13}$$

$$\left.\begin{array}{rcl}\nabla^{\perp}_{e_2}n_1 & = & 0,\\ \nabla^{\perp}_{e_2}n_2 & = & 0\end{array}\right\} \tag{3.14}$$

where the equalities (3.13) and (3.14) define the normal connection on M.

Lemma 3.1. *Let $x = c_1 \otimes c_2$ be a tensor product immersion of a Euclidean unit circle c_1 at centered origin and unit speed non-null Lorentzian curve c_2 in $\mathbb{E}^2_1$. Then the shape operators of M in direction of n_1 and n_2 are given by respectively,*

$$A_{n_1} = \begin{bmatrix} b\varepsilon_1 & 0 \\ 0 & c\varepsilon_2 \end{bmatrix}, \qquad A_{n_2} = \begin{bmatrix} 0 & b\varepsilon_1 \\ b\varepsilon_2 & 0 \end{bmatrix}. \tag{3.15}$$

By a simple calculation, we see that Gauss and Ricci equations of M are identical and they are given by as follow

$$a' - a^2\varepsilon_1 = b^2\varepsilon_1 - bc\varepsilon_2, \tag{3.16}$$

and Codazzi equation of M is

$$b' = 2ab\varepsilon_1 - ac\varepsilon_2. \tag{3.17}$$

Thus we give the following theorem.

Theorem 3.1. *If M is a tensor product surface of a Euclidean unit circle at centered origin and a non-null unit speed Lorentzian curve in $\mathbb{E}^2_1$ then the Christoffel symbols of M satisfy the following Riccati equation*

$$(a+b)' = \varepsilon_1 (a+b)^2 - c\varepsilon_2 (a+b). \tag{3.18}$$

Theorem 3.2. *Let M be a tensor product surface given with the surface patch (3.1). Then there exist following relation between Gaussian curvature K and normal curvature K_N*

$$K_N = |K| = \left|b^2\varepsilon_1 - bc\varepsilon_2\right|$$

Theorem 3.3. *Let M be a tensor product surface given with the surface patch (3.1) . Then the followings are equivalent,*

i) $\nabla^{\perp}$ is a flat connection,
ii) $K_N = K = 0$,
iii) $b = 0$ or $\varepsilon_1 b = \varepsilon_2 c$.

Now, we suppose that M is a semi parallel surface, i.e., $\bar{R}.h = 0$. From (2.15) we get

$$\left.\begin{array}{rcl} b^2\varepsilon_1(c - b + 2b\varepsilon_1 - 2c\varepsilon_2) & = & 0, \\ b\varepsilon_2(b - b\varepsilon_1 + c\varepsilon_2)(c - b) & = & 0, \\ b\varepsilon_1(2b^2\varepsilon_1 + bc - c^2 - 2bc\varepsilon_2) & = & 0, \end{array}\right\}. \tag{3.19}$$

Theorem 3.4. *Let M be a tensor product surface given with the surface patch (3.1). Then M is a semi parallel surface if and only if*
i) For $\varepsilon_1 = \varepsilon_2$, either $b = 0$ or $b = c$,
ii)For $\varepsilon_1 \neq \varepsilon_2$, $b = 0$.

Corollary 3.3. *Let M be a tensor product surface given with the surface patch (3.1) with $\varepsilon_1 \neq \varepsilon_2$ then M is a semi parallel surface if and only if M is a a totally geodesic surface in $\mathbb{E}^4_2$.*

References

[1] K. Arslan, B. Bulca, B. Kılıc, Y. H. Kim , C. Murathan and G. Ozturk, Tensor Product Surfaces with Pointwise 1-Type Gauss Map, Bull. Korean Math.Soc. **48** (2011), 601-609.
[2] K. Arslan and C. Murathan, Tensor product surfaces of pseudo-Euclidean planar curves, Geometry and topology of submanifolds, VII (Leuven, 1994/Brussels, 1994) World Sci. Publ., River Edge, NJ (1995), 71-74.
[3] B. Bulca and K. Arslan, Semiparallel tensor product surfaces in $\mathbb{E}^4$, Int. Electron. J. Geom., 7,1,(2014), 36-43.
[4] M. do Carmo, Riemannian geometry, Birkhauser, 1993.
[5] B. Y. Chen, Geometry of Submanifolds, M. Dekker, New York 1973.
[6] B. Y. Chen, Differential Geometry of semiring of immersions, I: *General Theory* Bull. Inst. Math. Acad. Sinica **21** (1993), 1-34.
[7] F. Decruyenaere, F. Dillen, I. Mihai and L. Verstraelen, Tensor products of spherical and equivariant immersions Bull. Belg. Math. Soc.- Simon Stevin **1** (1994), 643-648.
[8] F. Decruyenaere, F. Dillen, L. Verstraelen and L. Vrancken, The semiring of immersions of manifolds, Beitrage Algebra Geom. **34** (1993), 209-215.
[9] J. Deprez, Semi- parallel Surfaces in Euclidean Space, J. Geom., 25 (1985), 192-200.
[10] K. İlarslan and E. Nesovic, Tensor product surfaces of a Lorentzian space curve and a Lorentzian plane curve, Bull. Inst. Math. Acad. Sinica **33** (2005), 151-171.
[11] K. İlarslan and E. Nesovic, Tensor product surfaces of a Euclidean space curve and a Lorentzian plane curve, Differential Geometry - Dynamical Systems **9** (2007),47-57.
[12] I. Mihai, and B. Rouxel, Tensor Product Surfaces of Euclidean Plane Curves, Results in Mathematics, 27 (1995), no.3-4, 308-315.
[13] I. Mihai, I. Van de Woestyne, L. Verstraelen and J. Walrave, Tensor product surfaces of a Lorentzian plane curve and a Euclidean plane curve. Rend. Sem. Mat. Messina Ser. II 3(18) (1994/95), 147–158.
[14] B. O'Neill, Semi - Riemannian Geometry, with applications to relavity, Academic Press. New York, (1983)

[15] S. Özkaldı Karakuş and Y. Yayli, Bicomplex number and tensor product surfaces in $\mathbb{R}_2^4$, Ukrainian Math. J. 64 (2012), no. 3, 344–355.

[16] S. Özkaldi and Y. Yayli, Tensor product surfaces in $\mathbb{R}^4$ and Lie groups, Bull. Malays. Math. Sci. Soc. (2) 33 (2010), no. 1, 69–77.

[17] Z. I., Szabo, Structure theorems on Riemannian spaces satisfying $R(X,Y)R = 0$, I. The local version, J. Differential Geometry, 17 (1982), 531-582.

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