Mathematical and Computational Applications, Vol. 13, No. 2, pp. 101-112, 2008.

COMPARISON OF IBM-2 CALCULATIONS WITH X(5) CRITICAL POINT SYMMETRY FOR LOW-LYING STATES IN $^{144-154}$Nd

Sait İnan*, Nurettin Türkan**, İlyas İnci+, Davut Olgun++

*Celal Bayar University Demirci eğitim Fakültesi
saitinan@yahoo.com

**Bozok University Faculty of Arts and Science, 66200 Yozgat,Turkey
nurettin_turkan@yahoo.com

+Erciyes University Institute of Science Kayseri, Turkey
ilyasinci@yahoo.uc.com

++Kırıkkale University Institute of Science, Yahşihan, Kırıkkale, Turkey
dolgun@yahoo.com

Abstract- The X(5) would take place when moving continously from the pure U(5) symmetry to the SU(3) symmetry and it implies a definite relations among the level energies and among the E2 transition strengths. It was recently shown that a signature of phase transition is observed in the chain of Sm, Mo and Nd isotopes, where ^{152}Sm,^{104}Mo and ^{150}Nd display the predicted features of the X(5) symmetry and mark therefore the critical point. However, more detailed studies and experiments are needed to get ideas about this signature. Without entering into detail we have firstly compared the results obtained in our previous study [15] of $^{144-154}$Nd with that of the limits in X(5) symmetry and then given a clear descripton about the validity of the Hamiltonian parameters used in the study. At the end, we have concluded that some of Nd isotopes display X(5) symmetry features.

Key Words- critical symmetry, interacting boson model, even Nd.

1.INTRODUCTION

Dynamic symmetries have provided a useful tool to describe properties of several physical systems [1]. The most notable examples are the dynamic symmetries of the interacting boson model [1,2] in nuclear physics and those of the vibron model [1,3] in molecular physics. There three possible dynamic symmetries generally labeled by the first subalgebra U(5);harmonic vibrator [4], SU(3);symmetrically deformed rotor [5] and SO(6);triaxially soft rotor [6]. Nuclei may display behavior near these idealized limits and it is a recent approach to apply the ideas of a phase transition of the nuclear shape [1,7]. Definition of critical points of the shape change is stated as new benchmarks and the transition from a spherical harmonic vibrator to an axially deformed rotor has been described analytically [7] by introducing a dynamical symmetry, denoted as X(5). This dynamical symmetry arises when the potential in the Bohr Hamiltonian [5] is decoupled into two components-an infinite square well potential for the quadrupole deformation parameter β and a harmonic potential well for the triaxiality deformation parameter γ [8]. The signature of a phase transition between collective vibrator and axially deformed rotor has received considerable attention, in the frame of critical point properties in transitional nuclei [9]. The X(5) would take place when moving continously from the pure U(5) symmetry to the SU(3) symmetry and it implies a definite relations among the level energies and among the E2 transition strengths.

It was recently shown that a signature of phase transition is observed in the chain of Sm [7,10], Mo [11] and Nd [12-14] isotopes, where ^{152}Sm,^{104}Mo and ^{150}Nd display the predicted features of the X(5) symmetry and mark therefore the critical point. However, more detailed studies and experiments are needed to get ideas about this signature. Without entering into detail, which can be found in the Ref.[7], we can give purpose of the present study as follows; (i) To compare the results obtained in Ref.[15] with the limits of X(5) symmetry, (ii) To give a clear descripton about the validity of the Hamiltonian parameters used in this study, (iii)To get a brief conclusion about the relation between $^{144-154}$Nd and X(5) symmetry.

The outline of the remaining part of this paper is as follows; An approximate IBM-2 formulation is given without entering into detail and theoretical background is reviewed in section 2. The calculated $R_{4/2}$, $R_{0/2}$ and B(E2) values are compared with the results of some neighboring nuclei and with that of X(5) limits in section 3. The last section contains some concluding remarks.

2. THEORETICAL BACKGROUND

IBM Hamiltonian takes different forms [16] depending on the regions (SU(5), SU(3), SO(6)) of the traditional IBA triangle. The Hamiltonian that we consider is in the form of [17],

$$H=H_{sd} + \Sigma\theta_L[d^+ d^+ d^+]^{(L)} [\tilde{d}\,\tilde{d}\,\tilde{d}]^{(L)} \tag{1}$$

where H_{sd} is the standard Hamiltonian of the IBM [18,19],

$$H_{sd}= \in_d\eta_d + \kappa Q\cdot Q + \kappa' L\cdot L + \kappa'' P^+ \cdot P + q_3T_3\cdot T_3 + q_4T_4\cdot T_4 \tag{2}$$

In the IBA-2 model the neutrons' and protons' degrees of freedom are taken into account explicitly. Thus the Hamiltonian [2] can be written as ,

$$H = \varepsilon_\nu n_{d\nu} + \varepsilon_\pi n_{d\pi} + \kappa\, Q_\pi . Q_\nu + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu} \tag{3}$$

where $n_{d\rho}$ is the neutron (proton) d-boson number operator.

$$n_{d\rho} = d^+ \tilde{d} \;, \rho = \pi , \nu$$

$$\tilde{d}_{\rho m} = (-1)^m\, d_{\rho m} \tag{4}$$

where s^{+}_{ρ} , $d^{+}_{\rho m}$ and s_{ρ}, $d_{\rho m}$ represent the s and d-boson creation and annihilation operators. The rest of the operators in the equation(3) are defined as

$$Q_{\rho} = (s^{+}_{\rho}\tilde{d}_{\rho} + d^{+}_{\rho}s_{\rho})^{(2)} + \chi_{\rho}(d^{+}_{\rho}\tilde{d}_{\rho})^{(2)}$$

$$V_{\rho\rho} = \sum_{L=0,2,4} C_{L\rho}\left(\left(d^{+}_{\rho}d^{+}_{\rho}\right)^{(L)}\left(d^{+}_{\rho}\tilde{d}_{\rho}\right)^{(L)}\right)^{(0)} \quad ; \rho = \pi , \nu \tag{5}$$

and

$$M_{\pi\nu} ; \sum_{L=1,3} \xi_{L}\,(d^{+}_{\nu}d^{+}_{\pi})^{(L)}(d_{\nu}d_{\pi})^{(L)} + \xi_{2}(s_{\nu}\tilde{d}_{\pi} - s_{\pi}\tilde{d}_{\nu})^{(2)}.(s^{+}_{\nu}d^{+}_{\pi} - s^{+}_{\pi}d^{+}_{\nu})^{(2)} \tag{6}$$

In this case $M_{\pi\nu}$ affects only the position of the non-fully symmetric states relative to the symmetric ones. For this reason $M_{\pi\nu}$ is often referred to as the Majorana force.

The electric quadropole (E2) transitions are one of the important factors within the collective nuclear structure. In IBM-2 model, the general linear E2 operator is expressed as [2],

$$\begin{aligned} T(E2) &= e_{\nu}T_{\nu}(E2) + e_{\pi}T_{\pi}(E2) \\ &= e_{\nu}Q_{\nu} + e_{\pi}Q_{\pi} \end{aligned} \tag{7}$$

In these expression χ_{ρ} is an adimansional coefficient and e_{ρ} is the effective quadrupole charges. Below we show how B(E2;J→J') prescription is implemented in formulation.

$$B(E2;J\to J') = \frac{1}{2J+1}\,|(J'\|T(E2)\|J)|^{2} \tag{8}$$

3. RESULTS AND DISCUSSION

The energy ratios $R_{4/2}=E(4^{+}_{1})/E(2^{+}_{1})$ and $R_{0/2}=E(0^{+}_{2})/E(2^{+}_{1})$ are characteristics of different collective motions of the nucleus [8,18]. So,we firstly examined the energies of the yrast sequences in some even-even Nd nuclei. Table 1 shows the most appropriate Hamiltonian parameters of calculations for examining $^{144-154}$Nd nuclei.

Table 1. The most appropriate Hamiltonian parameters (taken from ref.[15]) of calculations for examining $^{144\text{-}154}$Nd nuclei.

$^{A}_{Z}X$	N_π	N_ν	N	Fit Number	ε	κ	χ_ν	χ_π	$C_{L\nu}$	$C_{L\pi}$
$^{144}_{60}Nd_{84}$	5	1	6	Fit 1	0.87	-0.095	-1.2	-1.2	0.90	0.00
				Fit 2	0.87	-0.105	1.1	1.2	2.00	0.00
$^{146}_{60}Nd_{86}$	5	2	7	Fit 1	0.81	-0.098	-1.1	-1.2	0.90	-0.60
				Fit 2	0.73	-0.085	1.2	1.1	0.00	0.00
$^{148}_{60}Nd_{88}$	5	3	8	Fit 1	0.66	-0.070	-1.1	-1.2	0.00	0.00
				Fit 2	0.70	-0.090	1.0	0.8	0.00	0.00
$^{150}_{60}Nd_{90}$	5	4	9	Fit 1	0.55	-0.091	-1.2	-1.2	0.90	-0.80
				Fit 2	0.45	-0.080	1.2	0.4	0.00	0.00
$^{152}_{60}Nd_{92}$	5	5	10	Fit 1	0.33	-0.068	-1.1	-1.2	0.00	0.00
				Fit 2	0.36	-0.090	0.2	0.1	2.00	0.00
$^{154}_{60}Nd_{94}$	5	6	11	Fit 1	0.34	-0.067	-1.1	-1.2	0.00	0.00
				Fit 2	0.40	-0.090	0.2	0.1	2.00	0.00

The experimental signatures for X(5) behavior are the following [8]. (a)The energies of the yrast states, $E(J^+_1)$, should show characteristic ratios lying between those of a vibrator and a rotor; (b) The strength of transitions between yrast states as reflected in the B(E2;J→J-2) values should increase with angular momentum J at a rate intermediate between the values for a vibrator and rotor; (c) The position of the first excited collective 0^+_2 state is 5.67 times the energy of 2^+_1 level; (d) the nonyrast states based on the 0^+_2 level have larger energy spacings than the yrast sequence; (e) The B(E2;J→J-2) values for intrasequence transitions should be lower for the nonyrast sequence relative to those of the yrast sequence; (f) intersequence B(E2) values should show a characteristic pattern. We will use all of the above points in our search for Nd nuclei of A≤140 displaying behavior similar to the X(5) predictions.

The calculated and experimental energy values are given in table 2 with $R_{4/2}$ and $R_{0/2}$ ratios. Fig.1 shows the $R_{4/2}$ and $R_{0/2}$ ratios as a function of neutron number changing from 84 to 94. An harmonic vibrator should have $R_{4/2}$ = 2.00, an axially symmetric rotor has $R_{4/2}$=3.33, while X(5) behavior should have $R_{4/2}$=2.91. The serached nuclei in the present study have $2.00 \le R_{4/2} \le 2.96$. As it is seen from the table 2 the calculated and experimental energy values are very close to X(5) predictions for ^{150}Nd, especially. Around N=90, the positions of the excited 0^+ states are also close to the X(5) prediction and we note that the spacings in the excited sequence follow the expected behavior.

In table 3, we present the calculated data with available experimental ones for $^{144\text{-}154}$Nd. In addition, fig.2 shows the energies of the yrast sequences (normalized to the energy of their respectively 2_1^+ levels) in those nuclei and compare them with the expected behavior for an harmonic vibrator, an axially deformed rotor, and the X(5) prediction.

Table 2. The calculated and experimental energy values of $^{144-154}$Nd nuclei with $R_{4/2}$ and $R_{0/2}$ ratios. $R_{4/2}$ values and ground-state band energies in $^{144-154}$Nd are compared to relevant analytical models, X(5), and IBM where these apply. X(5) values are taken from Refs. [21] and IBM values are taken from our previous study refered as [15]. Experiments are taken from Refs.[23-36].

J	X(5)	^{144}Nd		^{146}Nd		^{148}Nd		^{150}Nd		^{152}Nd		^{154}Nd	
		EXP	IBM	EXP	IBM	EXP	IBM	EXP	IBM	EXP	IBM	EXP	IBM
2_1^+	0.122	0.697	0.694	0.453	0.451	0.302	0.327	0.130	0.127	0,072	0,085	0.071	0.078
4_1^+	0.354	1.315	1.414	1.043	1.026	0.752	0.748	0.381	0.376	0.237	0.246	0.233	0.231
6_1^+	0.661	1.791	2.223	1.780	1.728	1,280	1,265	0.720	0.735	0.484	0.480	0.477	0.459
8_1^+	1.033	-	3.105	2.555	2.545	1.857	1.876	1.130	1.202	0.810	0.802	0.805	0.779
10_1^+	1.465	-	4.060	3.248	3.467	2.472	2.577	1.599	1.767	-	1.194	-	1.650
12_1^+	1.954	-	5.014	-	4.481	-	3.381	-	2.473	-	1.832	-	1.761
14_1^+	2.499	-	-	-	5.575	-	4.311	-	3.273	-	2.555	-	2.282
$R_{4/2}$	2.910	1.890	2.000	2.300	2.280	2.490	2.290	2.930	2.960	3.290	2.890	3.280	2.960
$R_{0/2}$	5.800	2,900	2.100	2.100	2.200	3.000	2.700	5.200	5.800	-	5.900	-	6.300

In fig.3 we present the B(E2;J→J-2) reduced transition strength which is normalized to their respective B(E2; $2_1^+ \to 0_1^+$) values and again compare them with the expected behavior for an harmonic vibrator, an axially deformed rotor, and the X(5) prediction.

It is clear from table 2 and fig.2 that the Nd nuclei with yrast energies that closely follow the X(5) prediction. However, as can be seen from fig.3, in most of the cases X(5) behavior can be excluded on the basis of the deduced yrast B(E2;J→J-2) values.

In fig.4 we compare the relative γ-band energies in $^{144-154}$Nd with X(5), with an axial rotor and with neighboring $^{146-156}$Sm nuclei. The results are quite interesting. They provide an extensive test of X(5) for the γ degree of freedom and the calculated values exhibit a good agreement with experimental ones. Moreover, as it is seen from the fig, X(5) agrees well with the data for the γ-band energies. However, ^{144}Nd, ^{146}Nd and ^{148}Nd deviate from X(5) slightly in the direction of the rotor. This striking disagreements need to be better understood.

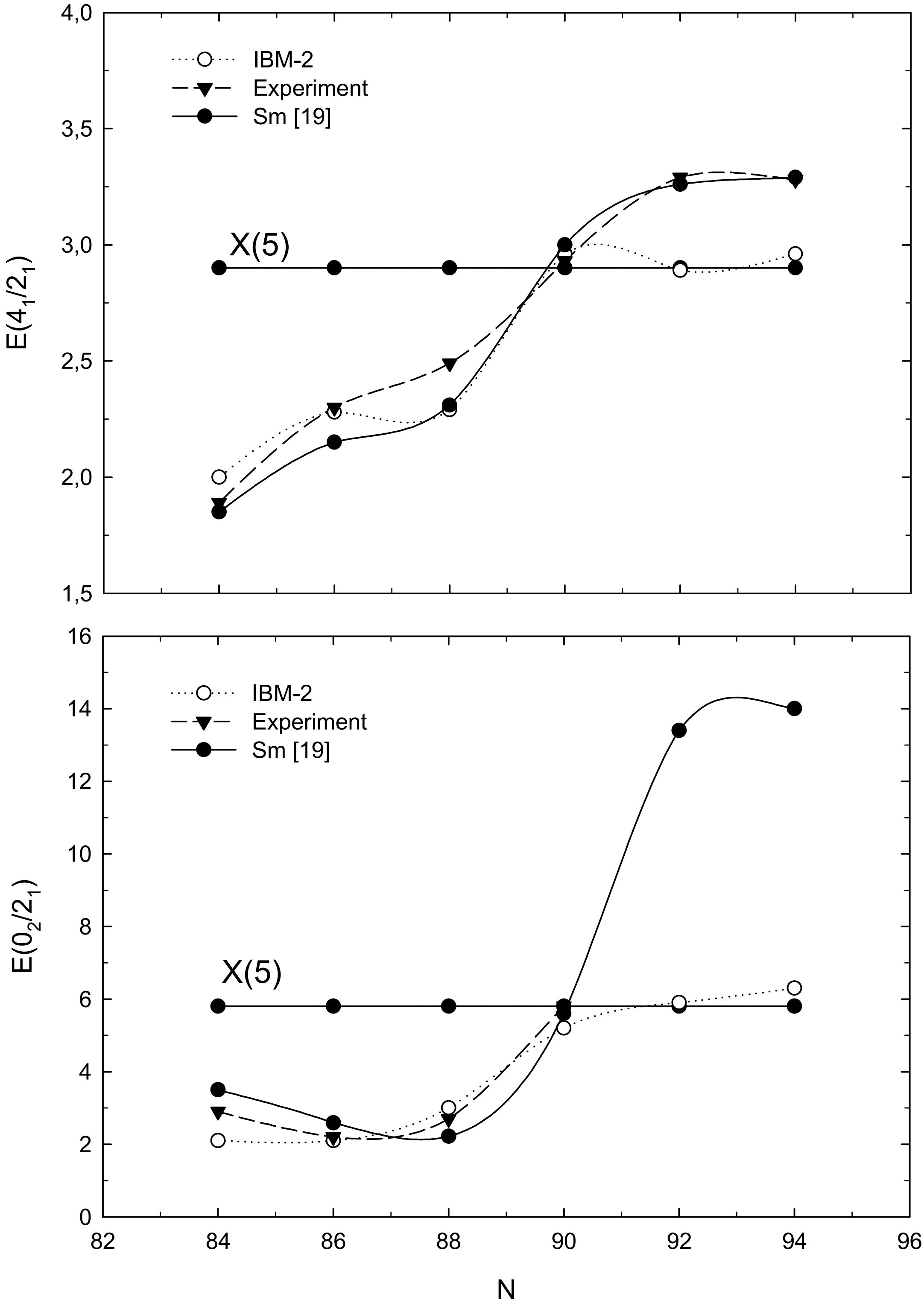

Fig. 1. The $R_{4/2}$ and $R_{0/2}$ ratios as a function of neutron number changing from 84 to 94.

Table 3. The calculated and experimental intrasequence B(E2;J→J-2) values for $^{144-154}$Nd. E2 transition rates are compared with the X(5) limit [1]. The B(E2) values are in Weisskopf units (1 $e^2b^2=2.62x10^{-3}$ W.u.). Experiments are taken from ref.[35-39].

$J_i \rightarrow J_f$	X(5)	^{144}Nd		^{146}Nd		^{148}Nd		^{150}Nd		^{152}Nd		^{154}Nd	
		EXP	IBM	EXP	IBM	EXP	IBM	EXP	IBM	EXP	IBM	EXP	IBM
$2^+_1\rightarrow0^+_1$	144	42	31	51	47	103	103	206	206	-	260	-	342
$4^+_1\rightarrow2^+_1$	230	32	50	76	76	164	164	313	305	-	381	-	503
$6^+_1\rightarrow4^+_1$	285	-	57	-	90	-	176	-	335	-	494	-	677
$8^+_1\rightarrow6^+_1$	328	-	55	-	89	-	180	-	332	-	493	-	684
$10^+_1\rightarrow8^+_1$	361	-	44	-	79	-	168	-	307	-	462	-	651
$2^+_2\rightarrow0^+_2$	115	-	3	-	11	-	13	-	46	-	77	-	114
$4^+_2\rightarrow2^+_2$	173	-	29	-	46	-	88	-	141	-	190	-	276
$0^+_2\rightarrow2^+_1$	90	-	41	-	57	-	88	-	53	-	31	-	36
$2^+_2\rightarrow0^+_1$	3	1.15	0.76	28	35	7,6	3.4	4.6	5.3	-	26	-	9.2
$2^+_2\rightarrow2^+_1$	12	36	42	34	44	46	60	14	42	-	191	-	267
$2^+_2\rightarrow4^+_1$	53	-	1	-	30	-	2.6	-	0.23	-	1.5	-	1
$4^+_2\rightarrow2^+_1$	1.4	-	0.4	-	0.7	-	0.0	-	0.53	-	1	-	0.2
$4^+_2\rightarrow4^+_1$	9	-	22	-	23	-	34	-	34	-	122	-	197
$4^+_2\rightarrow6^+_1$	40	-	0.3	-	1.2	-	2	-	0.0	-	0.04	-	6

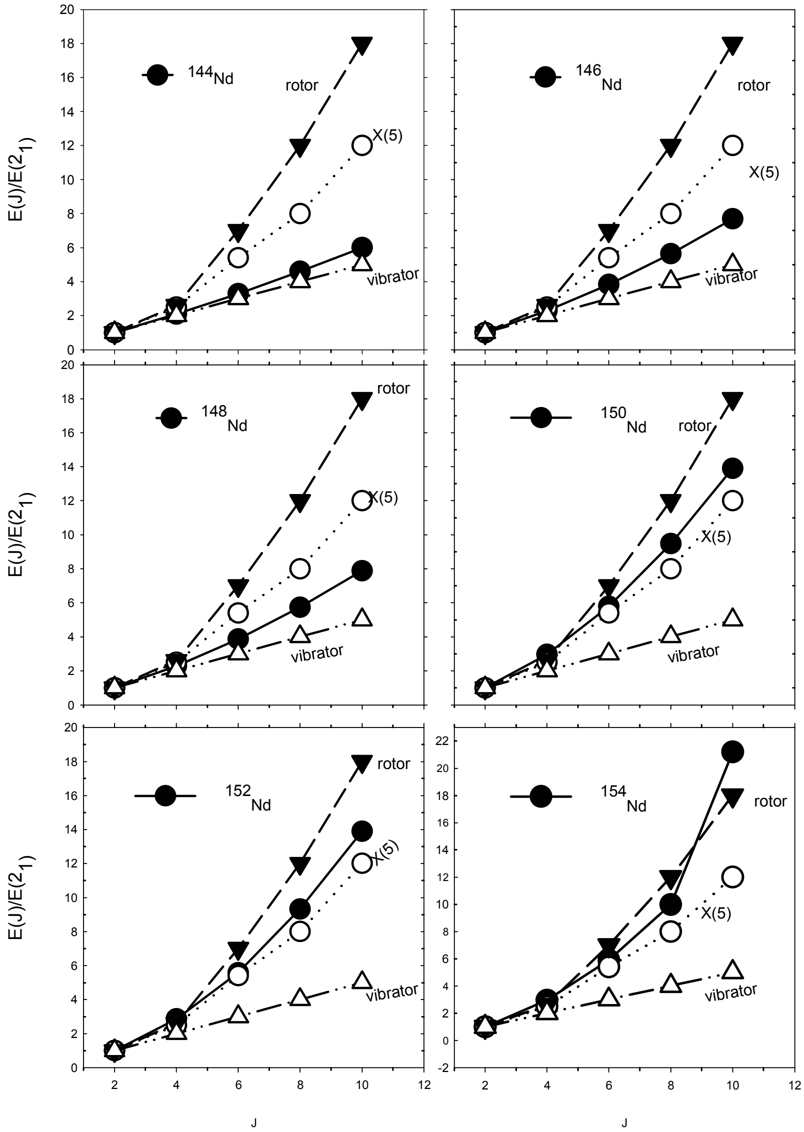

Fig. 2. The energies of the yrast sequences (normalized to the energy of their respectively 2_1^+ levels) in $^{144\text{-}154}$Nd nuclei.

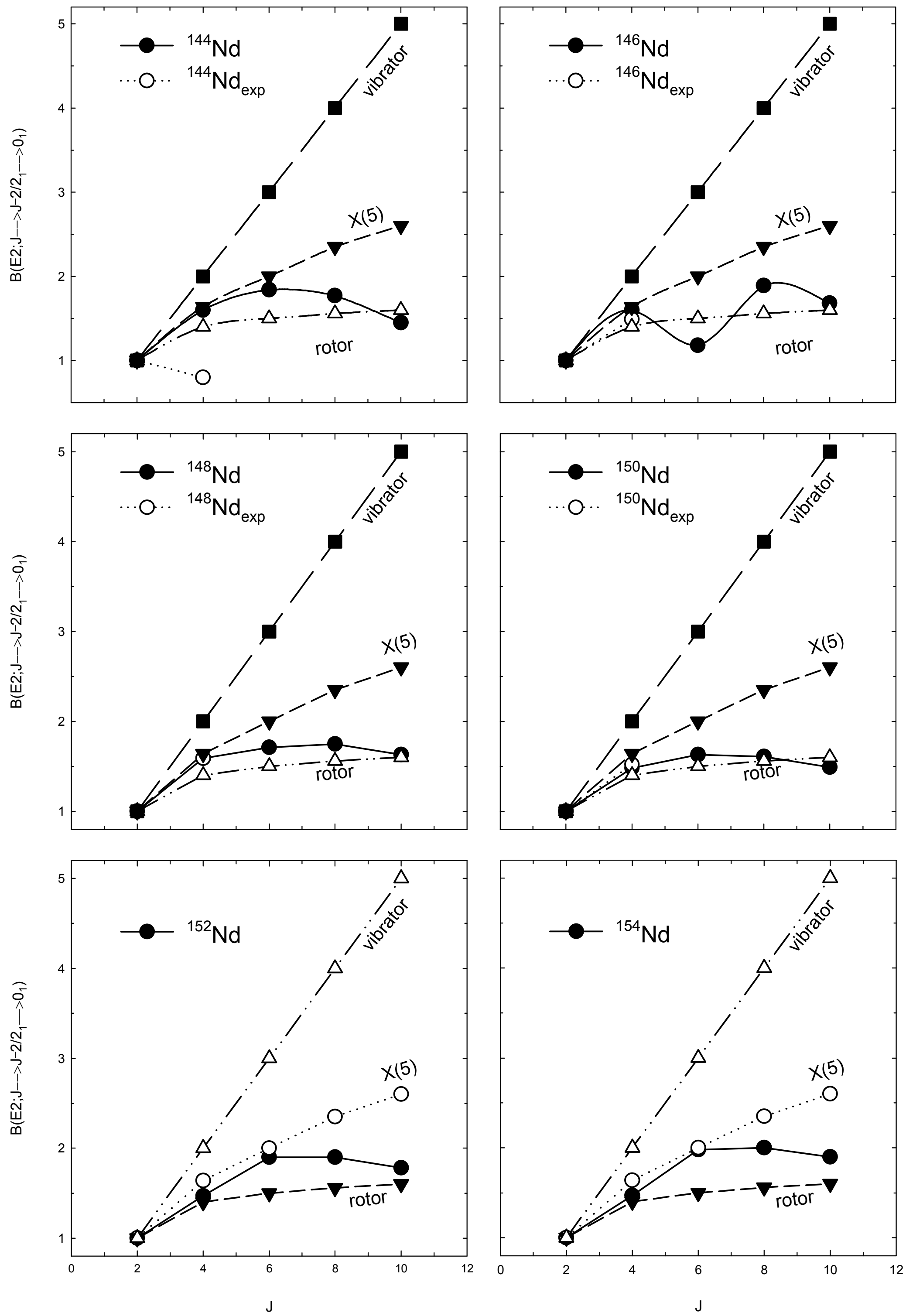

Fig. 3. The B(E2;J→J-2) reduced transition strength which is normalized to their respective B(E2; $2^+_1 \rightarrow 0^+_1$) values.

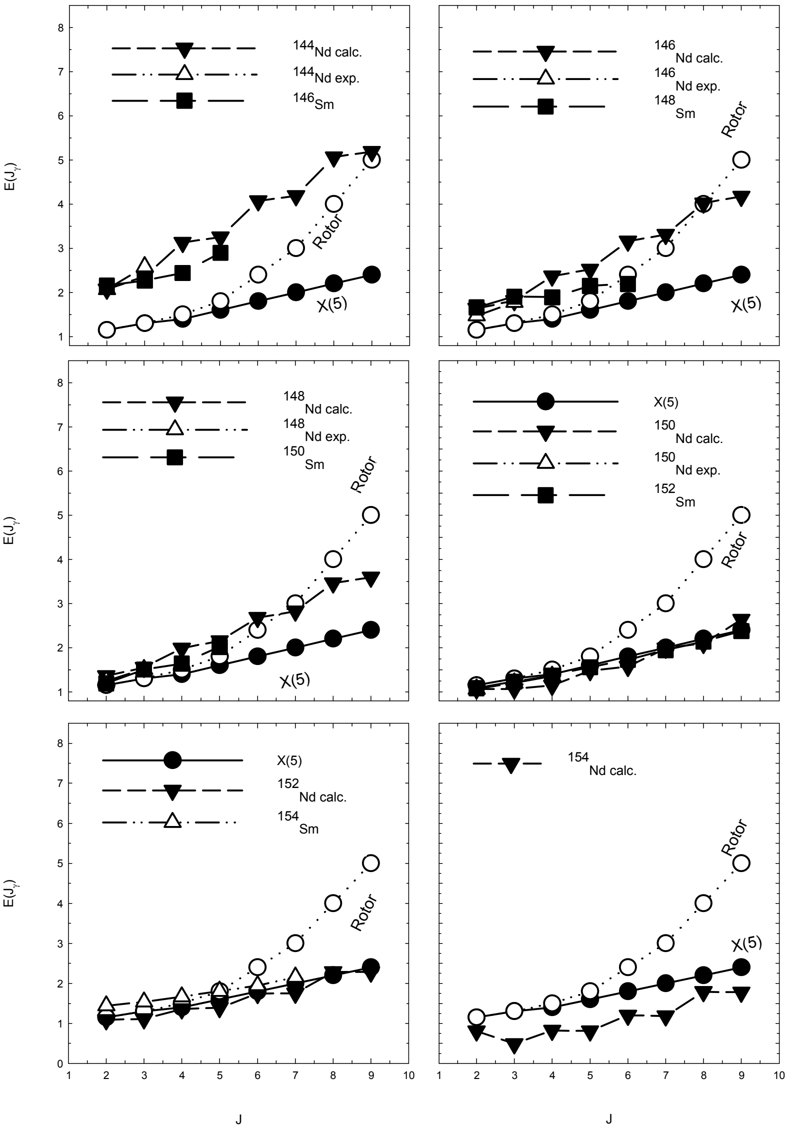

Fig. 4. Comparison of relative γ-band energies in $^{144\text{-}154}$Nd with X(5), with an axial rotor and with neighboring $^{146\text{-}156}$Sm nuclei.

3.CONCLUSION

We have searched the validity of our new parameters in IBM-2 formulation and theoretical background is reviewed for $^{144\text{-}154}$Nd . The calculated $R_{4/2}$, $R_{0/2}$ and B(E2) values are compared with the results of some neighboring nuclei and with that of X(5) limits. At the end, it was seen that some Nd nuclei, especially nuclei around N=90, with yrast energies follow the X(5) prediction closely. But X(5) behavior can be excluded on the basis of the deduced yrast B(E2;J→J-2) values in most of the cases. On the basis of the yrast state energies and yrast intraband transition strenghts, the best candidates were performed to be ^{148}Nd, ^{150}Nd and ^{152}Nd nuclei and the X(5) Picture reproduces the position of the first excited 0_2^+ in the nuclei with N=90.

We suggest that future experiments should focus on more detailed measurements of the excited states in ^{148}Nd and ^{152}Nd. Moreover, the detailed information on states above the collective 0^+_2 levels is needed. The present study will be important for understanding the collective excitations in transitional nuclei regarding the applicability of the IBM and the X(5) description.

4.REFERENCES

1. F. Iachello, Dynamic Symmetries at the Critical Point, *Phys. Rev. Lett.* **85**, 3580-3583, 2000.
2. F. Iachello and A. Arima, *The Interacting Boson Model,* Cambridge University Press.,Cambridge,1987.
3. F. Iachello, R. Levine, *Algebraic theory of Molecules,*Oxford Univ.Press, Oxford, 1995.
4. G. Scharff-Goldhaber, J. Weneser, System of Even-Even Nuclei, *Phys. Rev.* **98**, 212-214, 1955.
5. A. Bohr, quadrupole degree of freedom for the nuclear shape *Mat. Fys. Medd. K. Dan. Vidensk. Selsk.* **26**, 1-5, 1952.
6. L. Wilets, M. Jean, Surface Oscillations in Even-Even Nuclei, *Phys. Rev.* **102**, 788-796, 1956.
7. F. Iachello, Analytic description of critical point nuclei in a spherical-axially deformed shape phase transition, *Phys. Rev. Lett.* **87**, 037301, 2001.
8. R.M. Clark *et al.*, Searching for X(5) behavior in nuclei, *Phys. Rev.* C **68**, 037301, 2003.
9. V. Verner, P. von Brentano, R.F. Casten, J. Jolie, Singular character of critical points in nuclei, *Phys. Lett.* B **527**, 55-61, 2002.
10. R.F. Casten, N.V. Zamfir, Empirical Realization of a Critical Point Description in Atomic Nuclei, *Phys. Rev. Lett.* **87**,052503, 2001.
11. P.G. Bizzeti, A.M. Bizzeti-Sona, Evidence of X(5) symmetry for n_γ=0,1,2 bands in ^{104}Mo, *Phys. Rev.* C **66**, 031301, 2002.
12. R. Krücken *et al.*, $B(E2)$ Values in ^{150}Nd and the Critical Point Symmetry X(5), *Phys. Rev. Lett.* **88**, 232501, 2002.
13. R.M. Clark *et al.*, Reexamination of the *N*=90 transitional nuclei ^{150}Nd and ^{152}Sm *Phys. Rev.* C **67**, 041302, 2003.
14. R.F. Casten, N.V. Zamfir, R. Krücken, Comment on "Reexamination of the *N*=90 transitional nuclei ^{150}Nd and ^{152}Sm *Phys. Rev.* C **68**, 059801, 2003.
15. N. Turkan, İ. İnci, IBM-2 Calculations of Some Even-Even Neodymium Nuclei, *Phys. Scr.* **75**, 515-523, 2007.
16. A. Arima, T. Otsuka, F. Iachello, T. Talmi, Collective nuclear states as symmetric couplings of proton and neutron excitations, *Phys. Lett.* B **66**, 205-208, 1977.
17. K. Hayde, P. van Isacker, M. Waroquier, J. Moreau, Triaxial shapes in the interacting boson model, *Phys. Rev.* C **29**, 1420-1427, 1984.
18. C.A. Mallmann, System of Levels in Even-Even Nuclei, *Phys. Rev. Lett.* **2**, 507-509, 1959.
19. R.B. Firestone, *Table of Isotopes*, J.Wiley, USA, 1996.

20. F. Iachello, *Dronten Nuclear Structure Summer School* ,Plenum Press, New York, 1982.
21. K. Hayde, P. van Isacker, M. Waroquier, G. Wens, Y. Gigase, J. Stachel, g-boson excitations in the interacting boson model, *Nucl. Phys.* A **398**, 235-252, 1983.
22. N. Pietralla, O.M. Gorbachenko, Evolution of the β excitation in axially symmetric transitional nuclei, *Phys. Rev*. C **70**, 011304, 2004.
23. S. Raman, *Nucl. Data* B **2**, 47-48 , 1967.
24. M. Behar, Z.W. Grabowski, S. Raman, Angular correlation studies in ^{144}Nd, *Nucl.Phys*.A **219**, 516-524, 1974.
25. J. Dalmasso, H. Forest, G. Ardisson, ^{144}Nd levels fed in β decay of ^{144}Pr isomers, *Phys. Rev.* C **32**, 1006-1009, 1985.
26. A.V. Ramayya, Y. Yoshizawa, Collective nuclear states as symmetric couplings, *Phys. Rev.* B **137**, 13-16, 1965.
27. W.R. Daniels, F.O. Lawrence, D.C. Hoffman, Decay of 24 min ^{146}Pr, *Nucl.Phys.* A **118**, 467-477, 1968.
28. A.M. Demidov, L.I. Govor, Yu.K. Cherepantsev, T.J. Al-Janabi, H. Matti, *Proceedings of 4th. Intern. Symp. on Neutron-Capture Gamma-Ray Spectroscopy and Related Topics*, Grenoble, 1982.
29. T.E. Ward, N.A. Morcos, P.K. Kuroda**,** New Isotope ^{150}Pr, *Phys. Rev.* C **2**, 2410-2413, 1970.
30. S.W. Yates, N.R. Johnson, L.L. Riedinger, A.C. Kahler, Lifetimes of ground-band states in ^{150}Nd, *Phys. Rev.* C **17**, 634-638, 1978.
31. T. Karlewski, N. Hildebrand, M. Brugger, N. Kaffrell, N. Trautmann, G. Herrmann, Beta-decay of^{148,150,152}Pr into levels of^{148,150,152}Nd, *Z. Phys.* A **330**, 55-58, 1988.
32. J.B. Wilhelmy, S.G. Thompson, R.C. Jared, E. Cheifetz, Ground-State Bands in Neutron-Rich Even Te, Xe, Ba, Ce, Nd, and Sm Isotopes Produced in the Fission of ^{252}Cf, *Phys. Rev. Lett.* **25**, 1122-1125, 1970.
33. R. Chapman, W. McLatchie, J.E. Kitching, The (t, p) reaction on the even isotopes of neodymium, *Nucl. Phys*. A **186**, 603-619, 1972.
34. Y. Kawase, K. Okano, Identification of a new isotope^{154}Pr, *Z. Phys*.A **330**, 231-234, 1988.
35. D. Ecclesshall, M.J.L. Yates, J.J.Simpson, Energy levels of light nuclei, *Nucl.Phys.* **78***,* 481-488, 1966.
36. P.A. Crowley, J.R. Kerns, J.X. Saladin, Coulomb-Excitation Measurements on the Isotopes Nd^{144}, Nd^{146}, and Nd^{148}, *Phys. Rev.* C **3**, 2049-2053, 1971.
37. J.B. Gupta, Nuclear Structure of $^{144\text{-}150}$Nd in IBM-1, *J. Phys.* G **21**, 565-575, 1995.
38. J. Holden, N. Benczer-Koller et al. Single particle degrees of freedom in the transition from deformed to spherical Nd nuclei, *Phys. Rev.* C **63**, 024315, 2001.
39. P.R. Christiensen, G. Lovhoiden, J. Rasmussen, Elastic and inelastic deuteron scattering from ^{124}Te and ^{146}Nd, *Nucl. Phys.* A **149,** 302-322, 1970.
40. R. Bijker, R.F. Casten, N.V. Zamfir, E.A. MacCutchan, Test of X(5) for the γ degree of freedom, *Phys. Rev.* C **68**, 064304, 2003.