THE IMPORTANCE OF MESH MODELING IN THE CALCULATION OF STRESS INTENSITY FACTOR

Alaattin AKTAŞ* and Mehmet AKTAŞ**
*Department of Mechanical Engineering, Kırıkkale University, Kırıkkale
** Engineering Faculty of Uşak, Afyon Kocatepe University, Afyon

Abstract- In this paper, stress intensity factor (SIF) of an infinite plate with a center notched hole is calculated by means of finite element method (FEM) using both triangular quadratic element (TQE) and triangular quarter point element (TQPE) (Fig. 2).The FE analyses are performed using Ansys5.4 program code. SIF also is calculated from linear elastic fracture mechanics. The obtained results are compared to each other and it is found that the results from the TQPE are closer to exact solution.

Keywords- Quarter point element, Fracture, Stress intensity factor, Crack

1. INTRODUCTION

Determination of the SIF using FEM is too complicated due to the stress singularity which exists inherently at the crack tip. However, using FEM is useful for not well defined crack geometries and semi-fictitious boundary conditions. Because, those problems are difficult to solve by analytical methods.

Cracks result from discontinuities in the materials. To get rid of stress singularity, the vicinity of the crack tip must be well meshed by the most appropriate element types. Therefore, more than 30 different special crack tip elements have been formulated to date by researchers. Generally, these elements are based on special displacement-function or stress-function approaches or on mixed formulation [1]. Gustavo *et al.* [2] have investigated the effect of the number of the elements used at the crack tip to find SIF. They meshed crack tips with 4,8,12 and 16 TQP elements in some standard examples. They concluded that the mesh with 12 elements is the most accurate and recommended that the length of the elements close to the crack tip taken minimum. Murti *et al.* [1] have used collapsed rectangular quarter point (CRQPE) and TQP elements to calculate SIF in some standard crack configurations. They show that TQPE give more accurate results than CRQPE.

In this study, SIF of the infinite plate with a notched hole is calculated by means of FEM. In the analyses, two element types are used at the vicinity of the crack tip. The same geometries and the same element sizes are used to compare the results calculated from the FEM and the results obtained from the linear elastic fracture mechanics (LEFM)

2. DEFINITION AND ANALYSIS OF THE PROBLEM

The well known infinite plate with a center notched hole is taken (Fig.1a) to compare
TQPE with TQE. The plate is subjected to tension stress, and has 1 mm thickness and is
made of 2024-T4 alluminum alloy. The mechanical properties of the material are taken
from ref. [3]. The notch length is 2 mm and a/r ratios are changed from 0.2 to 1 with an
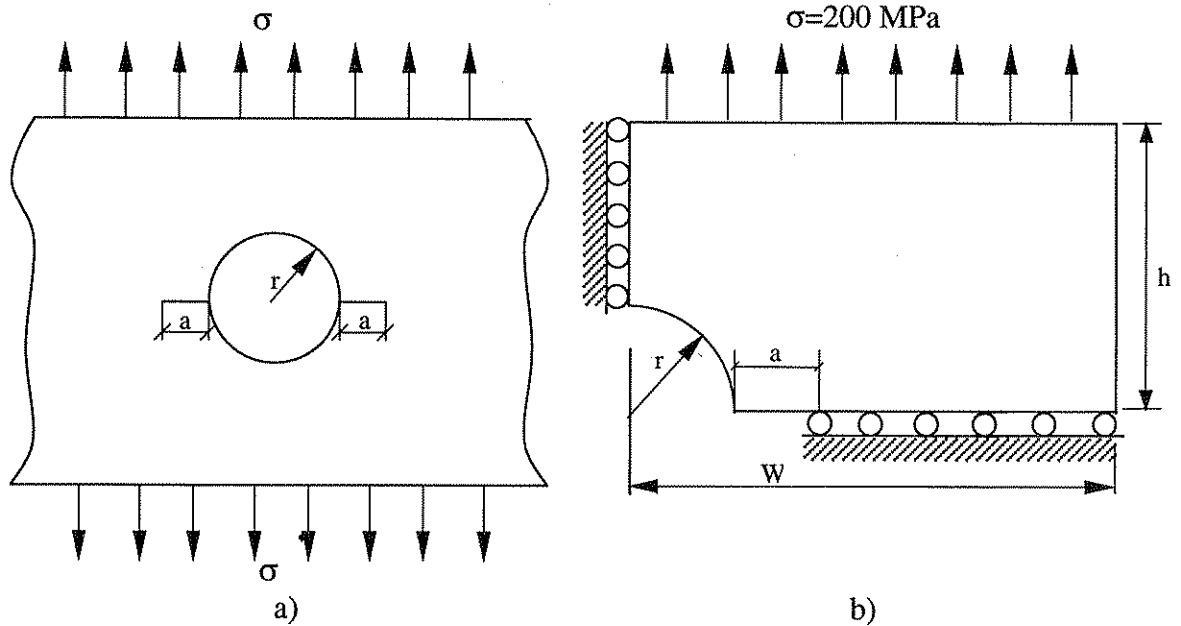increment of 0.2.

Figure 1. a)Geometry of the problem b) Modeling of the problem

2.1. Calculation of The Problem By Using Analytical Method

The problem under discussion has only opening-mode (i.e. Mode I). To calculate SIF of
this problem analytically, the formulas given in LEFM is used. This formula is [4];

$$K_I = \sigma \sqrt{\pi a} f\left(\frac{a}{r}\right) \tag{1}$$

The values of the $f\left(\dfrac{a}{r}\right)$ are taken from the ref. [5]. Then, the values of the SIF are

calculated. These values are given in fig. 5 for different a/r ratios. For instance, for

$\dfrac{a}{r} = 0.2$, the value of the $f\left(\dfrac{a}{r}\right)$ is equal to 2.41 and using Eq.1, the value of K_I is as to

equal $38.2\, MPa\sqrt{m}$.

2.2. Calculation of The Problem By Using FEM

In FE analyses, The plate is modeled with two different quadratic elements. The same
length and the same number of the elements around the crack-tip are used. The ratios

r/w and r/h are taken as 40 and 30 respectively (Fig.1b). Analyses are performed by using Ansys5.4 programme code. For TQPE, plane 2 and for TQE, shell 93 are used [6]. Only a quarter of the plate is modeled due to symmetry (fig.3). In the FE models 2827 elements and 5824 nodes are used.
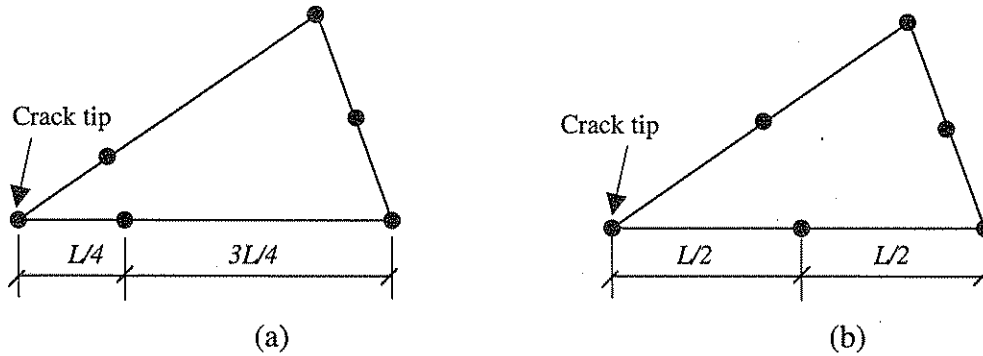

Figure 2. a) Collapsed triangular QPE, b) triangular QPE
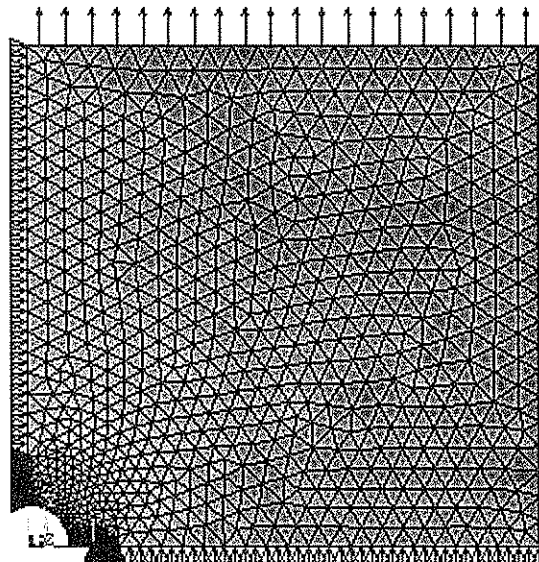

Figure 3. The mesh of the quarter plate

In FE analyses, the SIF's (Mode I or Mode II, or both) can be determined using various approaches. In some special crack-tip elements, the SIF's are obtained as the nodal parameters. The J-integral and the virtual crack extension method are among the common methods used to calculate these SIF's. However, the simplest and most direct method is known as the crack opening displacement method. By equating the displacement function coefficients and the FE solutions along the crack face. In FE analyses SIF can be calculated using the following formulas [1];

$$K_I = \sqrt{\frac{2\pi}{l_c}} \frac{G}{(\kappa + 1)} [4(u'_a - u'_d) - (u'_b - u'_c)] \qquad (2)$$

$$K_{II} = \sqrt{\frac{2\pi}{l_c}} \frac{G}{(\kappa+1)} [4(v_a' - v_d') - (v_b' - v_c')] \qquad (3)$$

where G=shear modulus; l_c= crack-tip element length κ=(3-4v) for plane strain and (3-v)/(1+v) for plane stress respectively; v=Poisson's ratio; and $u_a', u_b', u_c', u_d', v_a', v_b', v_c'$ and v_d' have been defined in figure 4.

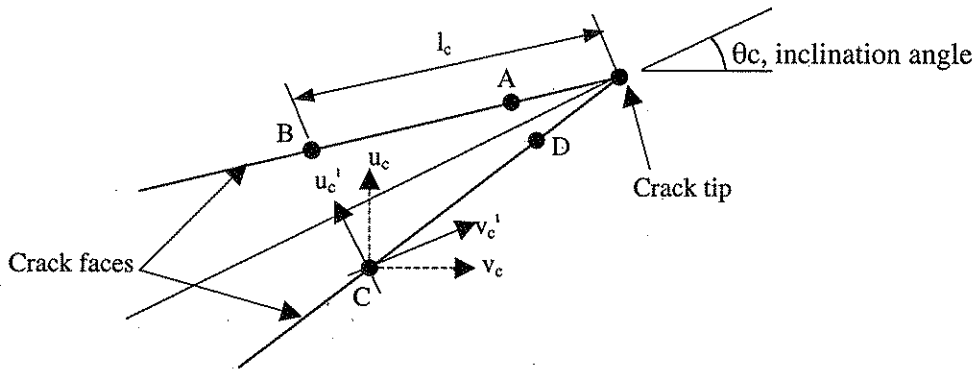These formulas are valid for static and dynamic analyses as long as the crack remains stationary [1].

Figure 4. The description of the displacement at the crack-face

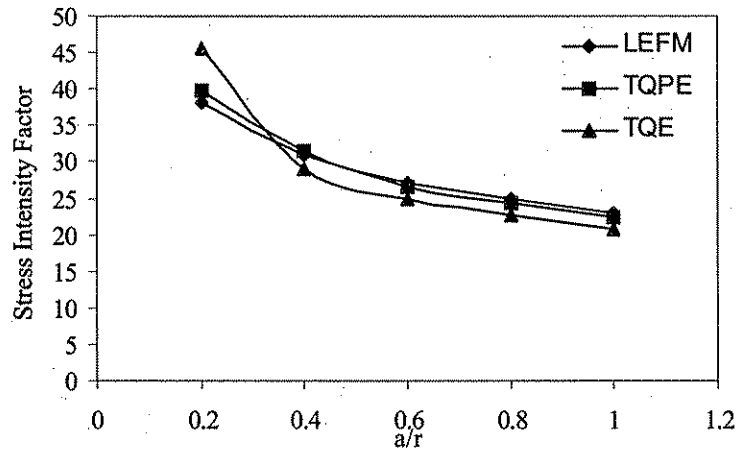By using Eq.2, the SIF was obtained and the results are given n grafics for each a/r ratios in figure 5.

Figure 5. Distribution of the SIF with respect to a/r ratios

3. CONCLUSIONS

In fig. 5, The results obtained from the TQPE are closer to the results obtained from the LEFM than the results obtained from the TQE. Therefore, it is once again proved that the TQPE elements should be used around the crack-tip for SIF calculation in FE analyses. But bearing in mind the small error brought about, the TQE can also be used in meshing.

If one looks at the variation of the SIF values with respect to *a/r* ratios, it may be concluded that the SIF values decreases as the *a/r* ratio increases (fig 5).

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