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FAULT DIAGNOSIS OF SHAFT- BALL BEARING SYSTEM USING ONE-WAY ANALYSIS OF VARIANCE

Mustafa Yuzukirmizi[1] and Hakan Arslan[2]

[1]Department of Industrial Engineering,
Kirikkale University, 71450, KIRIKKALE, TURKEY
[2]Dep. of Mechanical Engineering, Kirikkale University, 71450, KIRIKKALE, TURKEY
myuzukirmizi@kku.edu.tr, arslan71@yahoo.com

Abstract- Roller bearing is one of the most widely used and critical elements in rotating machinery. In consequence, bearing fault diagnosis in machines, as well as to discriminate the different fault conditions have been a great interest. In this study, firstly, analytical model of a shaft-ball bearing system is developed. The shaft is assumed to be perfectly rigid and uniform, and supported by two radial ball bearings. Then, the effect of localized defects on bearing running surfaces (i.e. surfaces of inner and outer rings and balls) on the shaft vibrations are obtained using the simulation program. Then, vibration signatures are analyzed by one-way analysis of variance (ANOVA) method. Finally, post-hoc tests are applied to differentiate the ball bearing element's localized defects in shaft-ball bearing simulation model.

Key Words- Ball bearing, Fault diagnosis, One-way analysis of variance

1. INTRODUCTION

Breakdowns in rotating elements cause time and economical losses due to malfunction of their components. The condition of the components should be monitored in order to not to encounter unexpected failures. Bearings are one of the most important and frequently used components in the rotating machinery. Ball bearings may contain manufacturing errors, mounting defects or damages which may also occur under working conditions. Most failures of rotating machinery have roots in the damage of rolling element bearings, such as fatigue crack, spalling on the races or rolling elements [1].

There is no clear criterion for the estimation of failure of a ball bearing. As long as the forces acting on a ball bearing are constant or changing slowly, the vibration level of shaft or ball bearing remains almost constant or changes slightly. But the vibrations will change when the defects and irregularities start to develop on the bearing elements. After detecting faulty element, it is possible to find out the type of the defect [2].
Single point defects begin as localized defects which include cracks, pits and spalls on the rolling surfaces on the raceways or rolling elements, and, as the rolling elements pass over these defect areas, small collisions occur producing mechanical shockwaves. This process occurs every time a defect collides with another part of the bearing [3].

2. RELATED LITERATURE

Many researchers have used vibration signature analysis techniques for rolling element bearing fault identification in case of single defect on bearing components. Time-domain and frequency-domain vibration analysis techniques were tested, but effective identification of bearing condition is, however, not typically straightforward.

The early diagnosis methods in time domain are based on a number of parameters, such as peak, root mean square (RMS), crest factor, and so on. Time domain analysis has been widely employed. [4] Successful results of Root Mean Square (RMS) [5] [6], Kurtosis [7] [8] [9], peak value [10], Crest Factor (CF) [11], and synchronous averaging [12], [13] have been reported in the low frequency range of < 5 kHz.

Many researchers, such as [7-9], have found the Kurtosis value to be more useful, when it is compared with the RMS, crest factor, and peak value. Peak and RMS can directly reflect the energy level of the vibration. The vibration data of a healthy bearing exhibits a normal distribution; thus, the kurtosis is equal to three. The propagation of damage in the bearing, which generates more peaks and increases the RMS, alters the Kurtosis level [14]. During the incipient failure condition of a bearing, the RMS value of the signal remains virtually unchanged, while an increase is noted in the peak value. As damage progresses, the RMS value increases, while the peak value does not necessarily increase. The ratio of peak value to RMS value of a signal is known as its crest factor and is an indicator of bearing condition [15].

Statistical moments recently played an important role in condition monitoring and diagnostics of rolling element bearings, and have attracted the attentions of many researchers. Statistical movements are descriptors of the shape of the amplitude distribution of vibration data collected from a bearing, and have some advantages over traditional time and frequency analysis [16]. Dyer and Stewart[17] first proposed the use of the fourth normalized central statistical moment kurtosis for bearing defect detection. White [9] studied the effectiveness of this method under a simulated condition. Several other studies [18-19], [5-6] have also shown the effectiveness of kurtosis in bearing defect detection. Because of the symmetry of the distribution, odd moments are zero for vibration signals of both healthy and damaged bearings.

Xinwen Niu et. al. [7] presented some new statistical moments for the early detection of bearing failure. They found the simulation and experimental tests show that the two new statistical parameters are preferred to the traditional third rectified moment and the fourth moment, respectively. Drona et. al. [8] have shown the interest of spectral subtraction for the improvement of the sensitivity of scalar indicators (crest factor, kurtosis) within the application of conditional maintenance by vibratory analysis on ball bearings. Furthermore they considered as the case of a bearing in good conditions of use, the distribution of amplitudes in the signal is of Gaussian kind.

The other new time domain method applied to bearing fault diagnosis is the Kolmogorov & Smirnov (K-S) test. It is a non-parametric and distribution free goodness-of-fit test. It is designed to test the null hypothesis in favor of the alternative hypothesis. Kar and Mohanty [20] employed the K-S test for quick analysis of time-domain signals in diagnosing rolling element ball bearing faults. The bearings are compared statistically with vibration signatures of good and faulty bearings using K-S test.

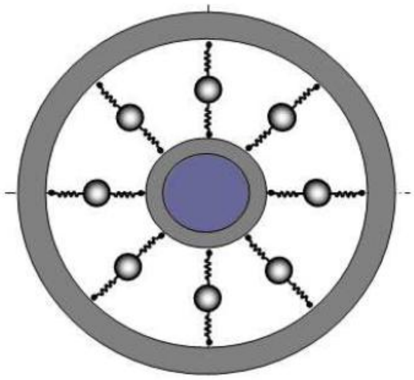


Figure 1. Proposed elastical model for a ball bearing

In this study, the vibration signatures received from roller bearing are analyzed by the Analysis of Variance technique. This method allows consideration of probability to derive statistical inferences about the groups of observations. It has been widely applied to scientific studies, and recent ones include in manufacturing material, medical, chemical, oceanographic and food [21-26].

3. SIMULATION ANALYSIS

The shaft-bearing assembly is considered as a mass-spring system and the model also incorporates masses of the balls. Because the system shows a non-linear characteristic under dynamic conditions, the contacts of balls to the inner and outer races are represented by nonlinear contact springs as shown in Fig 1.

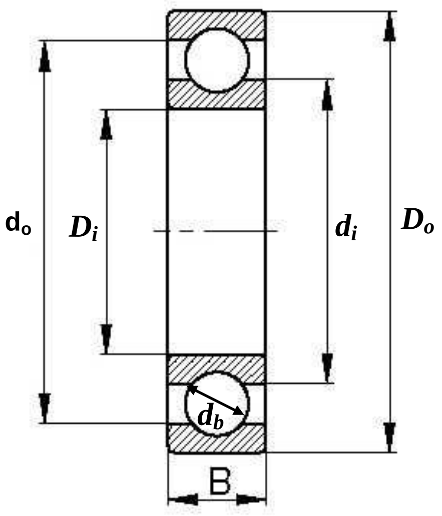
In order to the equations of motion, it is necessary to calculate the deflection of i^{th} ball for the calculation of contact forces acting on the shaft and the ball [2]. Subsequently, the equations of motion in radial and axial direction were obtained for shaft and ball bearing elements, and they were solved simultaneously with a computer simulation program as [2]. A defect in the form of crack or debris was assumed to be located on the running surface (inner race, outer race, ball surface). It was assumed to have 3 μm depth and 1 degree of width. The combination of these multi-localized defects then input to the simulation program and shaft vibrations were obtained.

The solution of the equations of motion for the shaft and balls are obtained using the Runge-Kutta iterative method, since they are non-linear and the direct substitution technique does not hold for them. For numerical solutions, the initial conditions and step sizes are very important for successive and economic computational solutions. The initial displacements of the shaft are assumed *as* $x_0 = 1,\mu m$, $y_0 = 0.l\ \mu m$ and $z_0 = \mu m$ and the vibrations in radial direction are obtained for 5000 rpm shaft speed.

In this study, the shaft is assumed to be perfectly rigid and uniform which is supported by a pair of radial ball bearings. The shaft and ball bearing are arranged as in Karacay and Akturk [27]. This arrangement is illustrated in Figure 2. The specifications of the bearings and the shaft are shown in Figure 3.

The bearings are force fitted to the spindle and in order to see the vibration characteristics of the bearing clearly, the axial preload was set to a value of 10 N, which is, a respectively small preload for the given bearing. An artificial viscous damping is introduced to the system in order to damp out the effect of transient vibrations. The value of this artificial damping is set to as low as 300 Ns/m in order to make sure that the magnitudes of vibrations will not be extremely affected. This value is also checked with the critical damping of the system and damping ratio ζ is found to be approximately 0.008. Simulation results are obtained for unloaded spindle (i.e. $Q_x=Q_y=Q_z=0\ N$)

Figure 2. Shaft-Ball bearing experimental arrangement (Karacay and Akturk[27])



Specifications of the bearings and the shaft

Inner race diameter (d_i): 18.3 mm

Outer race diameter (d_O): 25.8 mm

Outer race outside dia. (D_O): 32 mm

Bearing width (B): 10 mm

Ball diameter (d_b): 5.4 mm

Ball mass (m_b): 0.003 kg

Number of balls (m): 9

Mass of the shaft (M): 6.1 kg

Diameter of the shaft between the bearings (D_i): 12 mm

Length of the shaft: 640 mm

Figure 3. Dimensions of the FAG 6201 type ball bearing and the shaft

4. ANALYSIS OF VARIANCE

Fault diagnosis in FAG 6201 type roller bearing can be viewed as one factor problem where several treatments are tested for good or faulty. In this study, defect types of the roller bearing are taken as aggregate types and statistical tests are conducted.

It has been shown that the analysis of variance and subsequent tests are very appropriate procedures in evaluating such populations. The statistical approach used in this study is summarized in Figure 4. The procedure is certainly not a new technique for statisticians, however, to the best of our knowledge there has been no study on the applicability to the fault diagnosis of roller bearings.

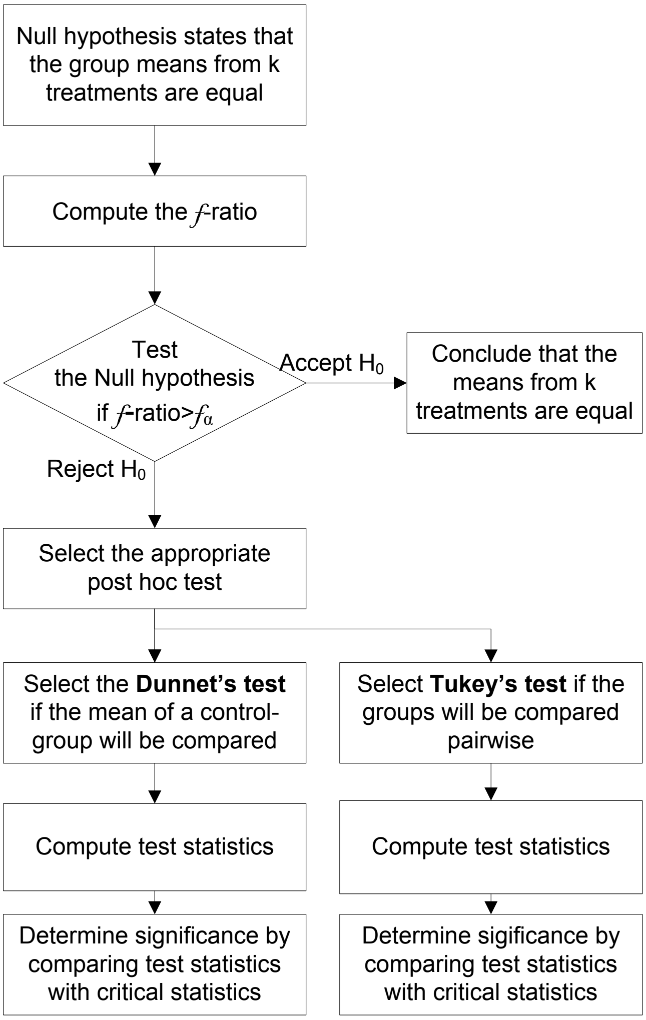


Figure 4. The procedure of statistical analysis

4.1. One- Way Analysis of Variance

In a k > 2 sample problem, where k is the number of samples, the model can be considered as follows:

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$$

$$H_a : \textit{At least one of the mean is different}$$

Where the samples are from different observations taken from populations with means $\mu_1, \mu_2, ..., \mu_k$. Furthermore, it will be interesting to make individual comparisons among these k population means. By analysis of variance, part of the goal is to determine if the differences between k sample means are what one would expect due to random variation alone or if indeed there is also contribution from systematic variation attributed to aggregate types.

The procedure, with k treatments and n observations in each, can be described as follows: Let y_{ij} denote the j th($j=1,\ldots,n$) observation from the i th($i=1,\ldots,k$) treatment, $\bar{y}_i$ is the mean of observations in the i th treatment and $\bar{y}$ is the mean of all k observations.

Usually, the computations in an analysis of variance problem are summarized in a tabular form as in Table 1:

Table 1. Summary of ANOVA computations

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Computed F
Control	k	SSA	$s_1^2 = \frac{SSA}{k-1}$	$\frac{s_1^2}{s^2}$
Error	$k(n-1)$	SSE	$s^2 = \frac{SSE}{k(n-1)}$	
Total	$nk-1$	SST		

where

$$SST = Totalsumofsquares = \sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij}-\bar{y})^2$$

$$SSA = Treatmentsumofsquares = n\sum_{i=1}^{k}(\bar{y}_i-\bar{y})^2$$

$$SSE = Errorsumofsquares = \sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij}-\bar{y}_i)^2$$

When Ho is true, the ratio

$$f = \frac{s_1^2}{s^2}$$

is a value of the random variable F having F-distribution with $k-1$ and $k(n-1)$ degrees of freedom. When H_0 is false, there is a one-tailed test with the critical region entirely in the right tail of the distribution. The null-hypothesis H_0 is rejected at the α-level of significance when $f > f_\alpha[k-1, k(n-1)]$

Another approach, the P-value, suggests that the evidence in favor of or against H_0 is given by: $P = P[F[k-1, k(n-1)] > f$

4.2. Comparing a Control Group by Dunnet's Test

Dunnet's test compares the mean of the control mean with the mean of the other groups, pairwise [28]. The test groups are not compared with each other , thus only k-l comparisons are made. In diagnosing a roller bearing fault after the rejection of null-hypothesis, applying this test is appropriate to determining if a bearing is good or not.

By selecting the good vibration data as the control group, other fault types can be compared and statistically significant differences can be detected.

The Dunnet's confidence intervals for comparisons with confidence coefficient of at least $1-\alpha$ are calculated as: $\bar{y}_{control} - \bar{y}_i \pm d_{\alpha;n-1;n-k}\sqrt{SSE(\frac{1}{n_{control}} + \frac{1}{n_i})}$

The values of $d_{\alpha;n-1;n-k}$ are calculated by Dunnet[9] and [10] and can be found in statistical books.

4.3. Multiple Comparison using Tukey's Test

The analysis of variance is a powerful procedure for testing the homogeneity of a set of means. However, if the null hypothesis is rejected and the stated alternative is accepted, it is still not known which of the population means are equal and which are different. One of the methods used for making paired comparisons is called Tukey's Test[31]. The method is based on the studentized range distribution, where it corrects for experiment wise error rate.

The Tukey confidence limits for all pairwise comparisons with confidence coefficient of at least $1-\alpha$ are: $\bar{y}_i - \bar{y}_j \pm \frac{q_{\alpha;\vartheta;v}}{\sqrt{2}}\hat{\sigma}_\varepsilon\sqrt{\frac{2}{n}}$

where $\hat{\sigma}_\varepsilon$ is the standard deviation of the entire design and q is the studentized range distribution($q = \frac{range_i}{s_j}$) with three factors:

α :Type I error rate or the probability of rejecting a true null hypothesis

ϑ :The number of degrees of freedom in the i^{th} sample(The one from which range was calculated)

:The number of degrees of freedom in the j^{th} sample(The one which standard deviation(s) is calculated.

The studentized range distribution can be found in [32], [33], [34]. Notice that the point estimator and the estimated variance are the same as those for a single pairwise comparison. The only difference between a single comparison is the multiple estimated standard deviation. Null hypothesis for Tukey's test states that all means being compared are from the same population, and hence the means should be normally distributed according to the central limit theorem.

5. NUMERICAL EXPERIMENTS

In this section, one-way ANOVA test is applied to roller bearing fault diagnosis using shaft vibration signatures followed by Tukey's pairwise comparisons. Based on simulation results of a FAG 6201 type roller bearing, data from seven treatments are collected. These treatments are good, single ball defect, double ball defect, single inner race defect, double inner race defect, 5° outer race defect, 5° and 115° outer race defect.(Table 2)

Table 2. Summary Data

Control	Level	N	Mean	St Dev.
1	Good	4999	1,2405	0,2534
2	Single Ball Defect	4999	0,5543	0,9907
3	Double Ball Defect	4999	0,5706	1,0923
4	Single Inner Race Defect	4999	0,7203	0,4345
5	Double Inner Race Defect	4999	0,647	0,7207
6	5± Outer Race Defect	4999	0,6692	0,6197
7	5± and 115 Outer Race Defect	4999	0,8186	0,103

In first step of the experimental procedure, the data is verified if the normality assumption is valid. Then, one-way ANOVA is carried if there exist statistically significant difference between groups. Based on the test result, pairwise comparisons are carried out to signify which of the groups are similar and which are different. The methods are fundamentals in a statistical analysis and the most related software can carry out this analysis. There is also specific software to carry out above analysis such as [35]. In this research, the data are analyzed in commercially available software Minitab version 13. As it is generally convention, it can be assumed that the Type I error of 5% which is a confidence level of 95%.

5.1. Normality Assumption

The above described methods are parametric tests. One of the main assumptions for the tests to be applied is the distributions of the groups are normal. The histogram of vibration data from each group with normal curve is plotted in Figure 5. Goodness-of-fit analysis of the normal probability plots also indicates validity of the assumption (Figure 6). Using least squares estimation, Pearson correlation coefficient which measure the strength of the linear relationship between the theoretical distribution and the collected data is very close to 1 with the lowest of .983.

5.2. Fault Diagnosis by One-way ANOVA

In this section, the vibration data from all treatments are analyzed and analysis-of variance technique is applied. The summary of all groups is displayed in Table 2. The box plots and dot plots of the data are shown in Figure 7. The null hypothesis of all means are equals is tested against the alternative hypothesis of at least one of the means is different. The resulting table is shown in Table 3. In this table, while F-value is 582.40 the p-value is less than 0.000. P-value suggests that the probability of the groups come from the same population is very slim. Based on this result, the null hypothesis is rejected, and concludes that there is a statistically significant difference between groups.

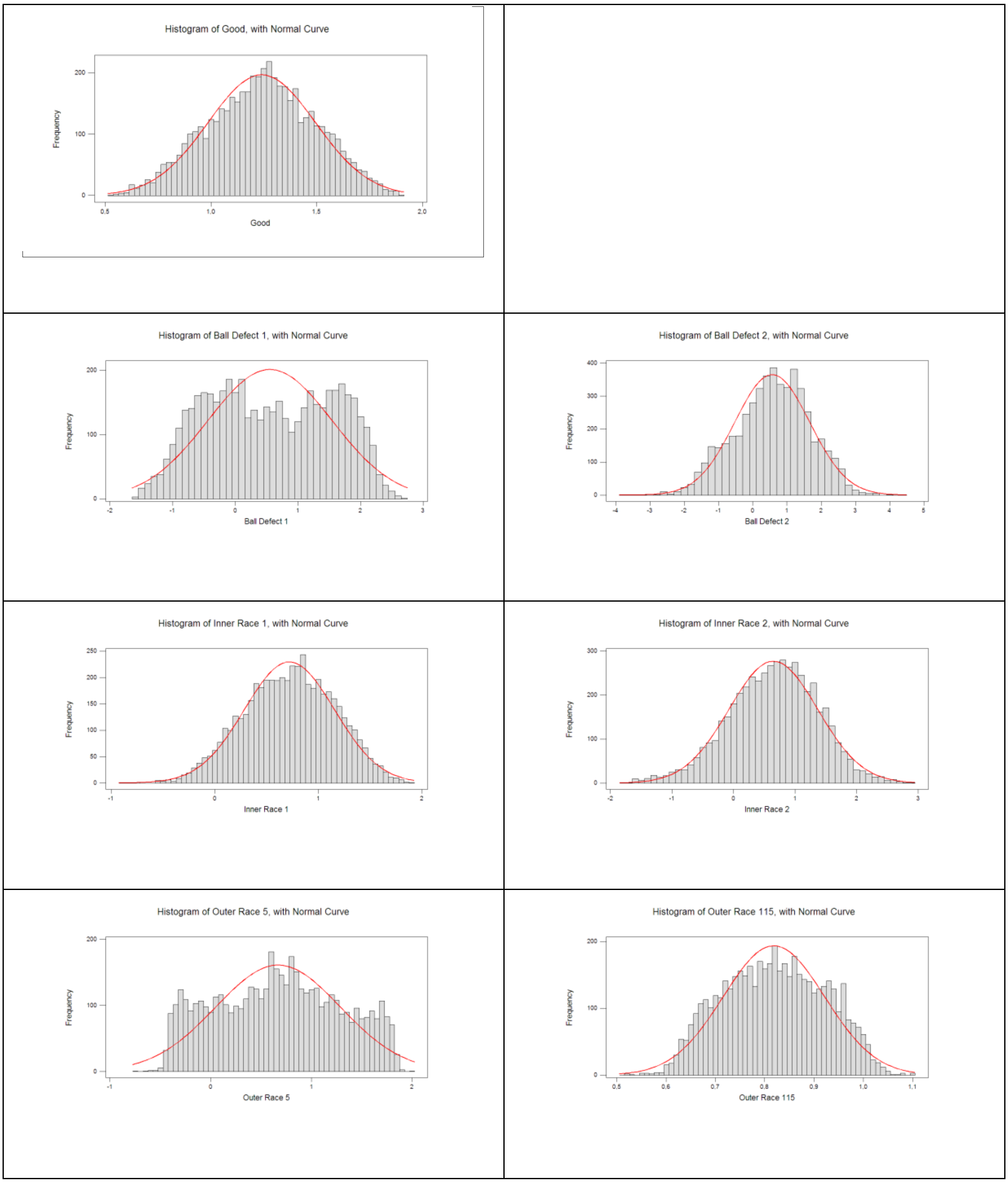


Figure 5. Histogram and normal plot of the rolling bearing

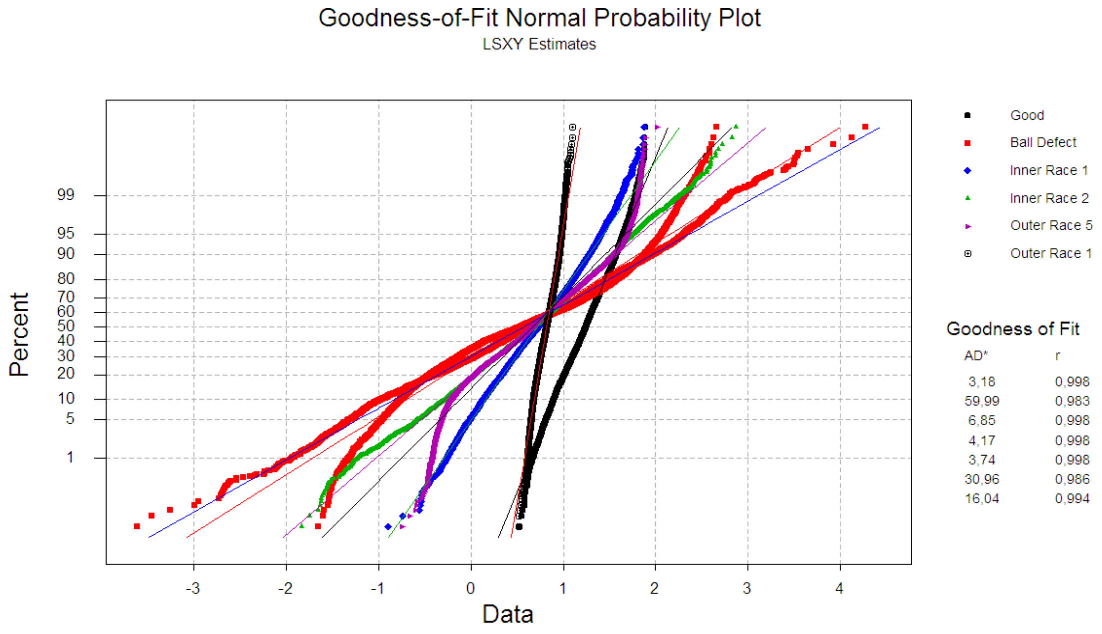


Figure 6. Good-of-Fit analysis of normal distribution

Table 3. ANOVA data

Source	DF	SS	MS	F	P
Control	6	1668,16	278,03	582,42	0,000
Error	34986	16701,10	0,48		
Total	34992	18369,26			

5.3. Good vs. Faulty Diagnosis

In this analysis, the control group of good bearing is compared with other groups using Dunnet's comparison. The intervals are confidence intervals for means minus the control mean of good. In this test, investigation was made for a significant difference between means. Since, neither of the intervals includes "0", conclusion was that they were not good. Resulting table with the decision outcome is summarized in Table 4.

Table 4. Comparison of other bearings against with good bearing

Level	Lower	Center	Upper	Decision
Single Ball Defect	-0,7217	-0,6862	-0,6507	Reject H_0, conclude faulty
Double Ball Defect	-0,7054	-0,6699	-0,6345	Reject H_0, conclude faulty
Single Inner Race Defect	-0,5557	-0,5202	-0,4848	Reject H_0, conclude faulty
Double Inner Race Defect	-0,6290	-0,5935	-0,5581	Reject H_0, conclude faulty
5° Outer Race Defect	-0,6068	-0,5713	-0,5358	Reject H_0, conclude faulty
5° and 115° Outer Race Defect	-0,4574	-0,4219	-0,3865	Reject H_0, conclude faulty

5.4. Fault Classification by Tukey's Comparisons

In ANOVA test, the null-hypothesis is rejected and concluded that least one of the means is different. It is still not known which populations are same which are different. In this section, Tukey's pairwise comparisons to the vibration data are applied. The resulting table is shown in Table 7. In this table, the lower and upper limits of confidence interval is displayed where they are calculated as column level mean minus row level mean. As in Dunnet's test, the observations suggest that the good bearing is significantly different from all other groups. The values in column 2 and 3 suggest that the ball defects are different from all other treatments. However, they are similar to themselves as single ball defect is not significantly different from the double ball defect. ANOVA analysis fails only in capturing the difference in double inner race defect and 5° outer race defect.

Table 5. Tukey's test results

	1	**2**	**3**	**4**	**5**	**6**
2	0,6455 0,7270					
3	0,6292 0,7107	-0,0570 0,0245				
4	0,4795 0,5610	-0,2067 -0,1252	-0,1904 -0,1089			
5	0,5528 0,6343	-0,1334 -0,0519	-0,1171 -0,0356	0,0326 0,1141		
6	0,5306 0,6121	-0,1556 -0,0742	-0,1394 -0,0579	0,0103 0,0918	-0,0630 0,0185	
7	0,3812 0,4627	-0,3050 -0,2235	-0,2888 -0,2073	-0,1391 -0,0576	-0,2124 -0,1309	-0,1901 -0,1086

6. CONCLUSION

In this study, a shaft which is assumed to be perfectly rigid and uniformly supported by a pair of radial ball bearings is considered. An analytical model of the system is developed and the effects of localized defects on bearing running surfaces (i.e. surfaces of inner and outer rings and balls) on ball bearing vibrations are obtained by using the simulation program.

Secondly, the vibration signatures caused by roller bearing faults have been analyzed. Initially, the analysis of variance is applied if there is a statistically significant difference between data groups. After determining the dissimilarity, it is proceeded with the post-hoc tests. Firstly, Dunnet's test is applied if the fault treatments are different from the control group of good bearing. Then, Tukey's pairwise comparison has been applied to identify which populations are different. The validity of the assumptions of the experimental approach has also been discussed. It has been shown that, ANOVA analysis and post-hoc tests can be conducted on the roller bearing vibration data. The methodology successfully distinguishes the good bearing from the faulty ones. Also, using an effective method for making paired comparisons, faulty bearings are separated according to their types.

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