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Computational results on the compound binomial risk model with nonhomogeneous claim occurrences

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ABSTRACT

The aim of this paper is to give a recursive formula for non-ruin (survival) probability when the claim occurrences are nonhomogeneous in the compound binomial risk model. We give recursive formulas for non-ruin (survival) probability and for distribution of the total number of claims under the condition that the claim occurrences are nonhomogeneous.

1. Introduction

The compound binomial risk model, which is of special importance in actuarial studies, was first presented by Gerber [1]. Afterwards, the model was investigated by Shiu [2], Willmot [3] and Dickson [4]. Some further extensions and properties of the model have been studied by Cheng et al. [5], Yuen and Guo [6], Cossette et al. [7,8], Egido Dos Reis [9], and Liu and Zhao [10], Zhang et al. [11] and Eryilmaz [12].

The compound binomial risk model is a discrete time version of the classical risk model. The corresponding surplus process of an insurance company can be defined as

$$U_t = u + ct - \sum_{i=1}^{t} Y_i, \quad t = 0, 1, \dots, \tag{1.1}$$

where $U_0 = u$ is an initial capital, c is the periodic premium income and Y_i is the eventual claim amount in period i. Let X_i $(i \geq 1)$ be a sequence of independent random variables which represents successive individual claim amounts. Let I_i be a binary random variable representing the claim occurrences. That is,

$$I_i = \begin{cases} 1, & \text{if a claim occurs in period } i \\ 0, & \text{otherwise.} \end{cases} \tag{1.2}$$

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Eq. (1.1) can also be written as

$$U_t = u + t - \sum_{i=1}^{N_t} X_i, \quad t = 0, 1, \ldots \tag{1.3}$$

where N_t is the number of claims arrived up to time t and $c = 1$.

If the random variables $I_1, I_2, \ldots$ are independent with $P(I_j = 1) = p$ and $P(I_j = 0) = 1 - p$, then the interclaim times $W_1, W_2, \ldots$ are independent and

$$\begin{aligned} P\left(W_j = t\right) &= P\left(I_1 = 0, \ldots, I_{t-1} = 0, I_t = 1\right) \\ &= pq^{t-1}, \quad t = 1, 2, \ldots, j \geq 1. \end{aligned} \tag{1.4}$$

Let T be a random variable which represents the random time to ruin. That is,

$$T = \inf\{U_t \leq 0, t = 1, 2 \ldots\}. \tag{1.5}$$

In this case

$$\psi(u) = P(T < \infty | U_0 = u)$$

denotes the probability of ruin where u is the initial reserve. The finite time ruin probability is denoted by

$$\psi(u, n) = P(T \leq n | U_0 = u) \tag{1.6}$$

and finite time non-ruin (survival) probability, as a complement of (1.6), denoted by

$$\begin{aligned} \phi(u, n) &= 1 - \psi(u, n) \\ &= P(T > n | U_0 = u) \\ &= P(U_t > 0, \ t = 1, 2, \ldots, n). \end{aligned} \tag{1.7}$$

The above mentioned compound binomial risk model has been widely studied when the binary variables $I_1, I_2, \ldots$ are independent and identical. That is, the claim occurrences in each period are independent and have the same probability of p. In this study, we consider the case when the claim occurrences are independent but nonidentical with $P(I_i = 1) = p_i, \ i \geq 1$. In the latter case, the random variable N_t no longer has a binomial distribution.

The remaining part of the paper is organized as follows. In Section 2, we introduce the distribution of time to ruin with nonhomogeneous claim occurrences probability. In Section 3, we present distribution of total number of claims with nonhomogeneous claim occurrences. Finally, in Section 4, we give numerical examples to demonstrate the application of our model.

2. Distribution of time to ruin

Let $f(x) = P(X_i = x)$ be the probability mass function of individual claim amount for $i \geq 1$.

Theorem 2.1. *Let' $P(I_i = 1) = p_i$ and $P(I_i = 0) = q_i$ for $i \geq 1$ where $I_1, I_2, \ldots$ r.v.'s are independent. Then*

$$\begin{aligned} P_u(T > n) &= \phi^{(1,n)}(u) \\ &= \begin{cases} 1, & n = 0 \\ \displaystyle\sum_{t=1}^{n} p_t \prod_{i=1}^{t-1} q_i \sum_{x=1}^{u+t-1} f(x) P_{u+t-x}(T^{(t+1,n)} > n - t) + \left(1 - \sum_{t=1}^{n} p_t \prod_{i=1}^{t-1} q_i\right), & n > 0 \end{cases} \end{aligned} \tag{2.1}$$

where $T^{(t+1,n)}$ represents ruin time after the t-th period. That is $T^{(t+1,n)}$ appears as a function of $(I_{t+1}, I_{t+2}, \ldots)$.

Proof. Under the model assumption, the probability function of W_1, waiting time until the first claim, can be written as

$$\begin{aligned} P(W_1 = t) &= P(I_1 = 0, \ldots, I_{t-1} = 0, I_t = 1) \\ &= q_1 \ldots q_{t-1} p_t \\ &= p_t \prod_{i=1}^{t-1} q_i. \end{aligned}$$

By conditioning on W_1, probability of ruin after n period is

$$\begin{aligned} P_u(T>n) &= \sum_{t=1}^{\infty} P_u(T>n|W_1=t)P(W_1=t) \\ &= \sum_{t=1}^{n} P_u(T>n|W_1=t)P(W_1=t) + \sum_{t=n+1}^{\infty} P_u(T>n|W_1=t)P(W_1=t) \\ &= \sum_{t=1}^{n} P_u(T>n|W_1=t)\, p_t \prod_{i=1}^{t-1} q_i + \sum_{t=n+1}^{\infty} P(W_1=t). \end{aligned} \tag{2.2}$$

Clearly,

$$\begin{aligned} \sum_{t=n+1}^{\infty} P(W_1=t) &= 1 - \sum_{t=1}^{n} P(W_1=t) \\ &= 1 - \sum_{t=1}^{n} p_t \prod_{i=1}^{t-1} q_i. \end{aligned}$$

Then,

$$\begin{aligned} P_u(T>n|W_1=t) &= \sum_{x=1}^{\infty} P_{u+t-x}\left(u+t-X>0, X=x, T^{(t+1,n)}>n-t\right) \\ &= \sum_{x=1}^{u+t-1} f(x)\, P_{u+t-x}(T^{(t+1,n)}>n-t). \end{aligned} \tag{2.3}$$

Using (2.3) in (2.2) one obtains, it can be reached to

$$\begin{aligned} P_u(T>n) &= \sum_{t=1}^{\infty} P_u(T>n|W_1=t)\, P(W_1=t) \\ &= \sum_{t=1}^{n} p_t \prod_{i=1}^{t-1} q_i \sum_{x=1}^{u+t-1} f(x)\, P_{u+t-x}(T^{(t+1,n)}>n-t) + \left(1 - \sum_{t=1}^{n} p_t \prod_{i=1}^{t-1} q_i\right). \end{aligned} \tag{2.4}$$

Thus, the proof is completed. ■

Corollary 2.1. *Let $P(I_i=1)=p$ and $P(I_i=0)=1-p$ for $i\geq 1$ in Theorem 2.1. Then*

$$P_u(T>n) = \begin{cases} 1 & \text{if } n=0 \\ \displaystyle\sum_{t=1}^{n} pq^{t-1} \sum_{x=1}^{u+t-1} f(x)P_{u+t-x}(T>n-t) + (1-p)^n & \text{if } n>0. \end{cases}$$

which is the result in [1].

Below, we compute $P(T>n)$ using Theorem 2.1 for $n=1,2,3$.

- For $n=1$

$$P_u(T>1) = P(X_1\leq u)p_1 + q_1. \tag{2.5}$$

- For $n=2$

$$P_u(T>2) = q_1q_2 + q_1p_2P(X_1\leq u+1) + p_1q_2P(X_1\leq u) + p_1p_2\sum_{x=1}^{u} f(x)P(X_2\leq u+1-x). \tag{2.6}$$

- For $n=3$

$$\begin{aligned} P_u(T>3) &= q_1q_2q_3 + q_1q_2p_3P(X_1\leq u+2) + q_1p_2q_3P(X_1\leq u+1) + p_1q_2q_3P(X_1\leq u+2) \\ &\quad + q_1p_2p_3\sum_{x=1}^{u+1} P(X_2\leq u+2-x)f(x) + p_1q_2p_3\sum_{x=1}^{u} P(X_2\leq u+2-x)f(x) \\ &\quad + p_1p_2q_3\sum_{x=1}^{u} P(X_2\leq u+1-x)f(x) + p_1p_2p_3\sum_{x=1}^{u}\sum_{y=1}^{u+1-x} P(X_3\leq u+2-x-y)f(x)f(y). \end{aligned} \tag{2.7}$$

To verify the above results, in the following we also compute $P_u(T > n)$ for $n = 1, 2, 3$ considering all possible cases in terms of claim occurrences.

- For $n = 1$

$$\begin{aligned} P_u\left(T > 1\right) &= P(I_1 = 1, X_1 \le u) + P(I_1 = 0) \\ &= P\left(X_1 \le u\right) p_1 + q_1 \end{aligned} \tag{2.8}$$

- For $n = 2$

$$\begin{aligned} P_u\left(T > 2\right) &= P\left(I_1 = 0, I_2 = 0\right) + P\left(I_1 = 0, I_2 = 1, X_1 \le u+1\right) \\ &\quad + P\left(I_1 = 1, I_2 = 0, X_1 \le u\right) + P\left(I_1 = 1, I_2 = 1, X_1 \le u, X_1 + X_2 \le u+1\right) \\ &= q_1q_2 + q_1p_2P\left(X_1 \le u+1\right) + p_1q_2P\left(X_1 \le u\right) + p_1p_2P\left(X_1 \le u, X_1 + X_2 \le u+1\right) \\ &= q_1q_2 + q_1p_2P\left(X_1 \le u+1\right) + p_1q_2P\left(X_1 \le u\right) + p_1p_2\sum_{x=1}^{u} f(x)P\left(X_2 \le u+1-x\right). \end{aligned} \tag{2.9}$$

- For $n = 3$

$$\begin{aligned} P_u\left(T > 3\right) &= P\left(I_1 = 0, I_2 = 0, I_3 = 0\right) + P\left(I_1 = 0, I_2 = 0, I_3 = 1, X_1 \le u+2\right) \\ &\quad + P\left(I_1 = 0, I_2 = 1, I_3 = 0, X_1 \le u+1\right) + P\left(I_1 = 1, I_2 = 0, I_3 = 0, X_1 \le u\right) \\ &\quad + P\left(I_1 = 0, I_2 = 1, I_3 = 1, X_1 \le u+1, X_1 + X_2 \le u+2\right) \\ &\quad + P\left(I_1 = 1, I_2 = 0, I_3 = 1, X_1 \le u, X_1 + X_2 \le u+2\right) \\ &\quad + P\left(I_1 = 1, I_2 = 1, I_3 = 0, X_1 \le u, X_1 + X_2 \le u+1\right) \\ &\quad + P\left(I_1 = 1, I_2 = 1, I_3 = 1, X_1 \le u, X_1 + X_2 \le u+1, X_1 + X_2 + X_3 \le u+2\right) \\ &= q_1q_2q_3 + q_1q_2p_3P\left(X_1 \le u+2\right) + q_1p_2q_3P\left(X_1 \le u+1\right) + p_1q_2q_3P\left(X_1 \le u+2\right) \\ &\quad + q_1p_2p_3\sum_{x=1}^{u+1} P\left(X_2 \le u+2-x\right) f(x) + p_1q_2p_3\sum_{x=1}^{u} P\left(X_2 \le u+2-x\right) f(x) \\ &\quad + p_1p_2q_3\sum_{x=1}^{u} P\left(X_2 \le u+1-x\right) f(x) + p_1p_2p_3\sum_{x=1}^{u}\sum_{y=1}^{u+1-x} P\left(X_3 \le u+2-x-y\right) f(x)f(y). \end{aligned} \tag{2.10}$$

3. Total number of claims

Let $P(I_i = 1) = p_i$ and $P(I_i = 0) = 1 - p_i = q_i$ for $i \ge 1$. The pmf of $N_n = \sum_{i=1}^{n} I_i$ can be computed recursively from the following equation. The proof of (3.1) is easy when $k = 0$ and $k = n$. The equation for $1 \le k \le n-1$ can be obtained by first conditioning on I_n and using independence of $I_1, I_2, \ldots, I_n$.

$$P\left(N_n = k\right) = \begin{cases} p_nP\left(N_{n-1} = k-1\right) + q_nP\left(N_{n-1} = k\right), & 1 \le k \le n-1 \\ \prod_{i=1}^{n} q_i, & k = 0 \\ \prod_{i=1}^{n} p_i, & k = n. \end{cases} \tag{3.1}$$

The following theorem gives a recursive formula for the conditional distribution of N_n given $T > n$ when the claim occurrences are identical.

Theorem 3.1. *For $u = 1, 2, \ldots$ and $k = 0, 1, 2, \ldots, n$*

$$P_u(N_n = k | T > n) = \frac{\beta\left(u; n, k\right)}{\phi(u, n)}$$

where

$$\beta\left(u;n,k\right)=\begin{cases}1, \quad n=0 \text{ and } k=0\\ q^{n}, \quad n\geq k \text{ and } k=0\\ \sum_{t=1}^{n}pq^{t-1}\sum_{x=1}^{u+t-1}f(x)\beta\left(u+t-x;n-t,k-1\right), \quad n\geq k \text{ and } k>0\end{cases} \tag{3.2}$$

[12].

In the following, we extend Theorem 3.1 to the case when the claim occurrences are nonidentical.

Theorem 3.2. *For* $u = 1, 2, \ldots$ *and* $k = 0, 1, 2, \ldots, n$

$$P_u(N_n = k|T > n) = \frac{\beta^{(1,n)}\left(u;k\right)}{\phi^{(1,n)}(u)}$$

where

$$\beta^{(1,n)}\left(u;k\right)=\begin{cases}1, \quad n=0, k=0\\ \prod_{i=1}^{n}q_i, \quad n\geq k, k=0\\ \sum_{t=1}^{n}p_t\prod_{i=1}^{t-1}q_i\sum_{x=1}^{u+t-1}f\left(x\right)\beta^{(t+1,n-t)}\left(u+t-x;k-1\right), \quad n\geq k,\ k>0.\end{cases} \tag{3.3}$$

Proof. For $n = 0$ and $k = 0$, the proof is clear. $P(I_1 = 0, \ldots, I_n = 0) = \prod_{i=1}^{n} q_i$ can be obtained for $n \geq k$ and $k = 0$ by using Eq. (3.1). Let W_1 denote the waiting time for the first claim. Then, for $n \geq k$ and $k > 0$

$$\begin{aligned}&P_u\left(N_n = k, T > n|W_1 = t\right)\\ &= \sum_{t=1}^{\infty}\sum_{x=1}^{\infty}\left(P_{u+t-x}u+t-X>0, X=x, N_{n-t}^{(t+1,n)}=k-1, T^{(t+1,n)}>n-t\right)P\left(W_1=t\right)\\ &= \sum_{t=1}^{n}\sum_{x=1}^{u+t-1}f\left(x\right)\left(P_{u+t-x}N_{n-t}^{(t+1,n)}=k-1, T^{(t+1,n)}>n-t\right)P\left(W_1=t\right).\end{aligned}$$

Thus the proof is completed. ■

Expansion of Eq. (3.3) for some selected values as follows:

- For $n = 1$ and $k = 0$

$$\beta^{(1,1)}\left(u,0\right)=q_1.$$

- For $n = 1$ and $k = 1$

$$\begin{aligned}\beta^{(1,1)}\left(u,1\right) &= p_1\sum_{x=1}^{u}f(x)\beta^{(2,0)}\left(u+1-x;k\right)\\ &= p_1\sum_{x=1}^{u}f(x).\end{aligned}$$

- For $n = 2$ and $k = 0$

$$\beta^{(1,2)}\left(u,0\right)=q_1q_2.$$

- For $n = 2$ and $k = 1$

$$\begin{aligned}\beta^{(1,2)}\left(u,1\right) &= p_1\sum_{x=1}^{u}f(x)\beta^{(2,1)}\left(u+1-x;0\right)+q_1p_2\sum_{x=1}^{u+1}f(x)\beta^{(3,0)}\left(u+2-x;0\right)\\ &= p_1\sum_{x=1}^{u}f(x)\left[q_2\right]+q_1p_2\sum_{x=1}^{u+1}f(x).\end{aligned} \tag{3.4}$$

- For $n = 2$ and $k = 2$

$$\beta^{(1,2)}(u,1) = p_1\sum_{x=1}^{u} f(x)\beta^{(2,1)}(u+1-x;1) + q_1p_2\sum_{x=1}^{u+1} f(x)\beta^{(3,0)}(u+2-x;1)$$

$$= p_1\sum_{x=1}^{u} f(x)\left[p_2\sum_{y=1}^{u+1-x} f(y)\right] + q_1p_2\sum_{x=1}^{u+1} f(x). \quad (3.5)$$

- For $n = 3$ and $k = 0$

$$\beta^{(1,3)}(u,0) = q_1q_2q_3.$$

- For $n = 3$ and $k = 1$

$$\beta^{(1,3)}(u,1) = p_1\sum_{x=1}^{u} f(x)\beta^{(2,2)}(u+1-x;0) + q_1p_2\sum_{x=1}^{u+1} f(x)\beta^{(3,1)}(u+2-x;0)$$

$$+ q_1q_2p_3\sum_{x=1}^{u+2} f(x)\beta^{(4,0)}(u+3-x;0)$$

$$= p_1\sum_{x=1}^{u} f(x)[q_2q_3] + q_1p_2\sum_{x=1}^{u+1} f(x)[q_3] + +q_1q_2p_3\sum_{x=1}^{u+2} f(x).$$

- For $n = 3$ and $k = 2$

$$\beta^{(1,3)}(u,2) = p_1\sum_{x=1}^{u} f(x)\beta^{(2,2)}(u+1-x;1) + q_1p_2\sum_{x=1}^{u+1} f(x)\beta^{(3,1)}(u+2-x;1)$$

$$+ q_1q_2p_3\sum_{x=1}^{u+2} f(x)\beta^{(4,0)}(u+3-x;1)$$

$$= p_1\sum_{x=1}^{u} f(x)\left[p_2q_3\sum_{y=1}^{u+1-x} f(y) + q_2p_3\sum_{y=1}^{u+2-x} f(y)\right] + q_1p_2\sum_{x=1}^{u+1} f(x)\left[p_3\sum_{y=1}^{u+2-x} f(y)\right] + q_1q_2p_3\sum_{x=1}^{u+2} f(x).$$

- For $n = 3$ and $k = 3$

$$\beta^{(1,3)}(u,3) = p_1\sum_{x=1}^{u} f(x)\beta^{(2,2)}u+1-x;2 + q_1p_2\sum_{x=1}^{u+1} f(x)\beta^{(3,1)}(u+2-x;2)$$

$$+ q_1q_2p_3\sum_{x=1}^{u+2} f(x)\beta^{(4,0)}(u+3-x;2)$$

$$= p_1\sum_{x=1}^{u} f(x)\left[p_2p_3\sum_{y=1}^{u+1-x} f(y)\sum_{z=1}^{u+2-x-y} f(z) + q_2p_3\sum_{y=1}^{u+2-x} f(y)\right] + q_1q_2p_3\sum_{x=1}^{u+2} f(x) \quad (3.6)$$

where $\beta^{(3,1)}(u+2-x;2) = 0$.

4. Numerical results

In this section we present numerical results when the claim size distribution is geometric with the following cdf and pdf

$$F(x) = 1 - \alpha^x, \quad x = 1, 2, \ldots \quad (4.1)$$

$$f(x) = (1-\alpha)\alpha^{x-1}, \quad x = 1, 2, \ldots \quad (4.2)$$

It is clear that

$$E(X) = \frac{1}{1-\alpha}, \quad 0 < \alpha < 1. \quad (4.3)$$

Assume that an insurance company is faced nonhomogeneous claim occurrence probabilities in each period (e.g. month) in a year. Four different cases are considered with different values of α and u in a finite time model for monitoring the characteristics of non-ruin probabilities in finite time. So, it is given in Tables 2 and 3 where $P(I_i = 1) = p_i$ for $i = 1, \ldots, 12$.

Table 1
Claim occurrence probabilities.

Case1 (C1)	Case2 (C2)
$p_i = 0.01 * i,\ i = 1, \ldots, 12$	$p_i = 0.01 * (12 - i + 1),\ i = 1, \ldots, 12$
Case3 (C3)	Case4 (C4)
$p_i = \begin{cases} 0.1, & i = 1, \ldots, 6 \\ 0.2, & i = 7, \ldots, 12 \end{cases}$	$p_i = \begin{cases} 0.2, & i = 1, \ldots, 6 \\ 0.1, & i = 7, \ldots, 12 \end{cases}$

For instance, it can be seen from Table 1 that probability of claim occurrences in a year is increasing from 0.01 to 0.12 in each month for Case 1, 0.2 for first six months and 0.1 for second six months for Case 4. So it is clear that by all these cases non-identical probabilities in periods are provided.

4.1. Non-ruin probabilities in finite time

In this section, non-ruin probabilities in finite time ($\phi^{(1,n)}(u)$) are calculated for nonhomogeneous claim occurrences in different periods based on four generated scenarios which are given in Table 1.

Table 2
$\phi^{(1,n)}(u)$ survival probabilities in finite time.

n	Cases							
	C1	C2	C3	C4	C1	C2	C3	C4
	$u = 2, \alpha = 1/5$				$u = 2, \alpha = 2/5$			
1	0.9996	0.9952	0.9960	0.9920	0.9984	0.9808	0.9840	0.9680
2	0.9994	0.9936	0.9946	0.9882	0.9971	0.9721	0.9763	0.9501
3	0.9994	0.9930	0.9941	0.9863	0.9963	0.9679	0.9723	0.9393
4	0.9993	0.9928	0.9940	0.9854	0.9957	0.9659	0.9702	0.9324
5	0.9993	0.9928	0.9939	0.9849	0.9954	0.9649	0.9689	0.9279
6	0.9993	0.9928	0.9938	0.9847	0.9952	0.9644	0.9682	0.9248
7	0.9993	0.9928	0.9938	0.9846	0.9951	0.9642	0.9674	0.9237
8	0.9993	0.9928	0.9938	0.9846	0.9950	0.9641	0.9666	0.9231
9	0.9993	0.9928	0.9938	0.9846	0.9950	0.9640	0.9661	0.9227
10	0.9993	0.9928	0.9938	0.9846	0.9949	0.9640	0.9657	0.9225
11	0.9993	0.9928	0.9938	0.9846	0.9949	0.9640	0.9653	0.9224
12	0.9993	0.9928	0.9938	0.9846	0.9949	0.9640	0.9651	0.9223

Table 3
$\phi^{(1,n)}(u)$ survival probabilities in finite time.

n	Cases							
	C1	C2	C3	C4	C1	C2	C3	C4
	$u = 4, \alpha = 3/5$				$u = 8, \alpha = 4/5$			
1	0.9987	0.9844	0.9870	0.9741	0.9983	0.9799	0.9966	0.9664
2	0.9971	0.9742	0.9780	0.9533	0.9956	0.9633	0.9929	0.9342
3	0.9956	0.9674	0.9715	0.9368	0.9923	0.9500	0.9891	0.9040
4	0.9943	0.9630	0.9667	0.9234	0.9886	0.9393	0.9856	0.8758
5	0.9932	0.9601	0.9632	0.9124	0.9846	0.9310	0.9823	0.8496
6	0.9923	0.9583	0.9605	0.9033	0.9805	0.9247	0.9793	0.8254
7	0.9915	0.9572	0.9564	0.8995	0.9763	0.9199	0.9780	0.8141
8	0.9908	0.9565	0.9525	0.8968	0.9719	0.9165	0.9770	0.8044
9	0.9902	0.9562	0.9490	0.8948	0.9674	0.9141	0.9762	0.7959
10	0.9897	0.9560	0.9457	0.8932	0.9627	0.9126	0.9756	0.7884
11	0.9891	0.9559	0.9429	0.8920	0.9579	0.9118	0.9751	0.7818
12	0.9887	0.9559	0.9403	0.8911	0.9529	0.9114	0.9747	0.7758

Fig. 1 shows the graph of non-ruin probabilities in finite time with different values of parameters in selected cases. So, by Fig. 1, it can be seen the effects of the different probabilities and initial reserve to the non-ruin (survival) probability.

Fig. 2 shows the survival probabilities in finite time for a selected domain of u and α simultaneously.

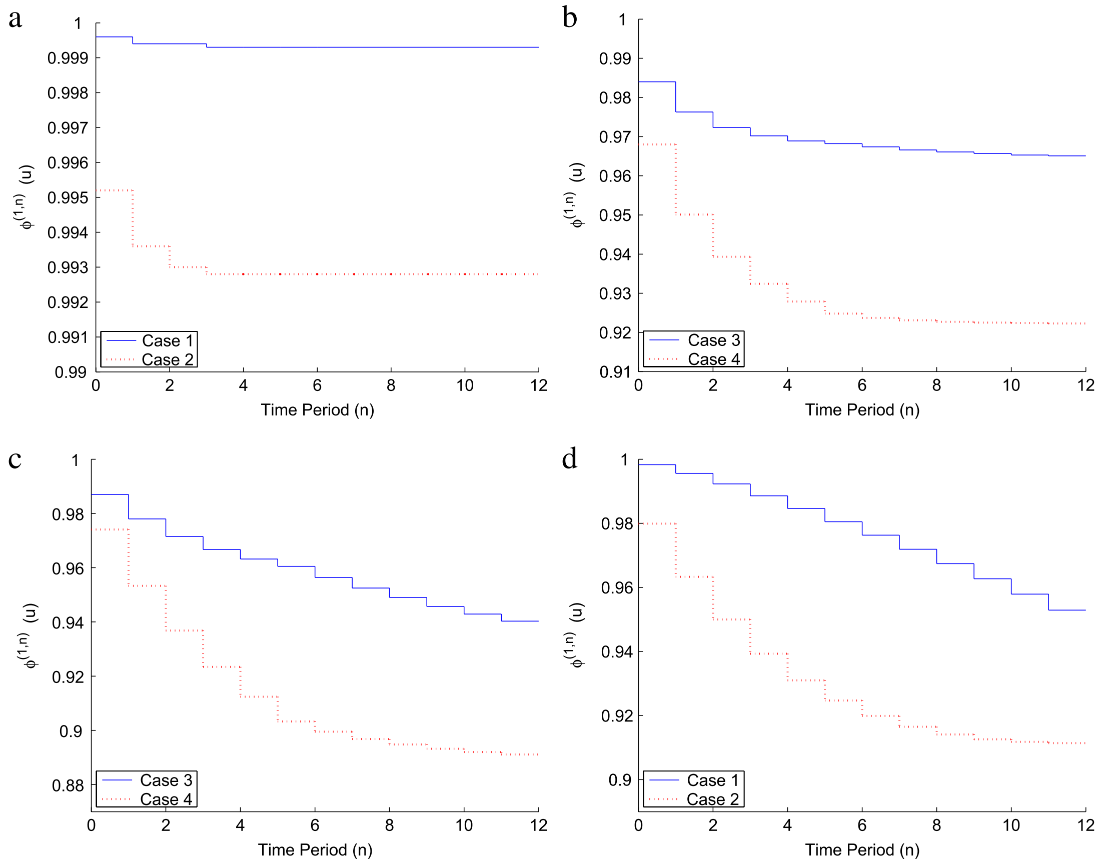

Fig. 1. Non-ruin probabilities in finite time for: (a) $u = 2$ and $\alpha = 1/5$, (b) $u = 2$ and $\alpha = 2/5$ (c) $u = 4$ and $\alpha = 3/5$, (d) $u = 8$ and $\alpha = 4/5$.

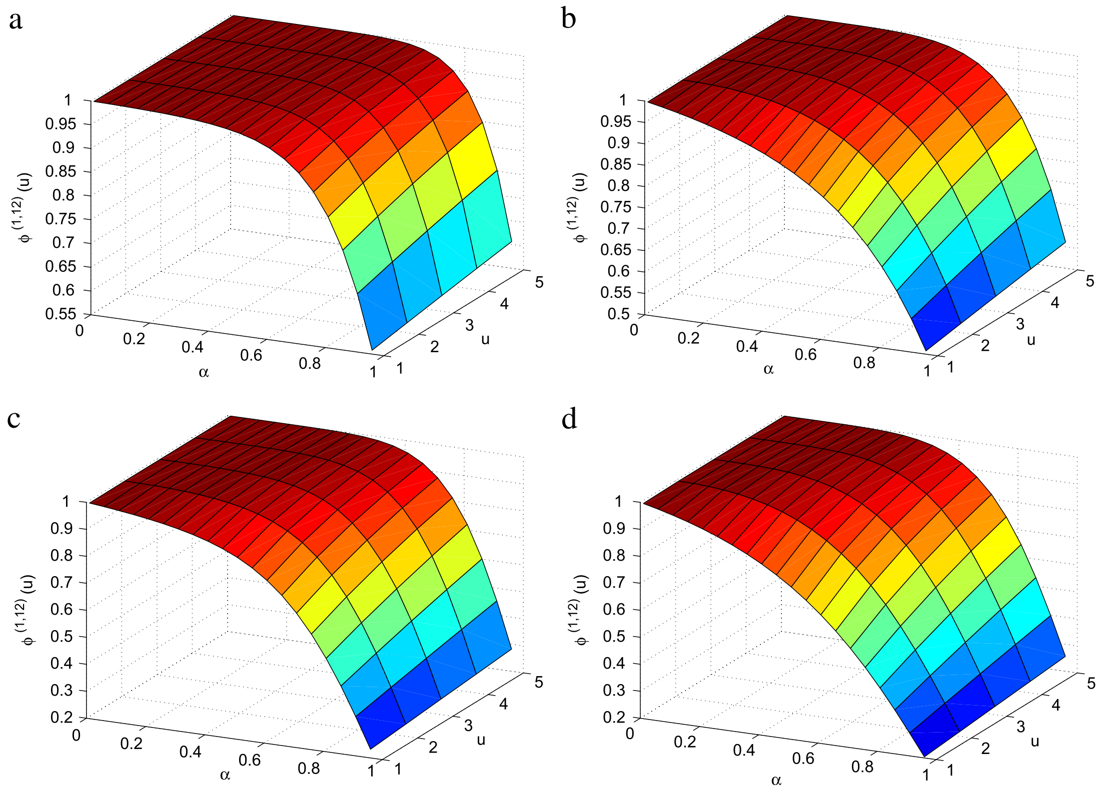

Fig. 2. Survival probabilities in Cases: (a) Case 1, (b) Case 2 (c) Case 3, (d) Case 4.

4.2. Conditional expected value of total claim numbers

Let $F(x)$ and $f(x)$ be defined as in (4.1) and (4.2) respectively. In this case, the conditional expected value of N_n given $T > n$ with respect to different values of α is presented in Table 4.

Table 4
Conditional expected values of N_n.

u	Cases	$\alpha = 1/5$	$\alpha = 3/5$	$\alpha = 4/5$
2	Case 1	0.7777	0.7463	0.6191
	Case 2	0.7708	0.6642	0.4925
	Case 3	1.7912	1.6399	1.2836
	Case 4	1.7855	1.5294	1.1404
4	Case 1	0.7773	0.7670	0.6674
	Case 2	0.7796	0.7320	0.5756
	Case 3	1.8067	1.7110	1.4019
	Case 4	1.7990	1.6581	1.3028
8	Case 1	0.7761	0.7730	0.7198
	Case 2	0.7788	0.7712	0.6767
	Case 3	1.7941	1.7687	1.5525
	Case 4	1.7937	1.7594	1.4956

5. Conclusion

This study presents the classical discrete time risk model when the claim occurrences in each period are independent but nonidentically distributed (non-homogeneous case), provides a recursive formula for the survival (non-ruin) probability and represents the conditional distribution of total number of claims in the non-homogeneous case. By expanding, all these provided results and recursive formula can be used by the related branches of an insurance company.

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References

[1] H.U. Gerber, Mathematical fun with the compound binomial process, Astin Bull. 18 (1988) 161–168.
[2] E.S.W. Shiu, The probability of eventual ruin in the compound binomial model, Astin Bull. 19 (2) (1989) 179–190.
[3] G.E. Willmot, Ruin probabilities in the compound binomial model, Insurance Math. Econom. 12 (2) (1993) 133–142.
[4] D.C.M. Dickson, Some comments on the compound binomial model, Astin Bull. 24 (1994) 33–35.
[5] S. Cheng, H.U. Gerber, E.S.W. Shiu, Discounted probabilities and ruin theory in the compound binomial model, Insurance Math. Econom. 26 (2–3) (2000) 239–250.
[6] K.C. Yuen, J. Guo, Ruin probabilieties for time-correlated claims in compound binomial model, Insurance Math. Econom. 29 (1) (2001) 47–57.
[7] H. Cossette, D. Landriault, E. Marceau, Ruin probabilities in the compound Markov binomial model, Scand. Actuar. J. 4 (2003) 301–323.
[8] H. Cossette, D. Landriault, E. Marceau, The compound binomial model defined in a markovian environment, Insurance Math. Econom. 35 (2) (2004) 425–443.
[9] A.D. Egido Dos Reis, The compound binomial model revisted. Astin Colloquia, Bergen Norway, 2004. http://www.actuaries.org/ASTIN/Colloquia/Bergen/EgidiodosReis.pdf.
[10] G. Liu, J. ve Zhao, Joint distributions of some actuarial random vectors in the compound binomial model, Insurance Mathe. Econom. 40 (1) (2007) 95–103.
[11] M.J. Zhang, X.J. Nan, S. Wang, Ruin probabilities the risk model with two compound binomial processes, J. Appl. Math. Inform. 26 (1–2) (2008) 191–201.
[12] S. Eryilmaz, On distributions of runs in the compound binomial risk model, Methodol. Comput. Appl. Probab. (2012) http://dx.doi.org/10.1007/s11009-012-9303-x.