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Research Article

Simultaneous Approximation for Generalized Srivastava-Gupta Operators

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We introduce a new Stancu type generalization of Srivastava-Gupta operators to approximate integrable functions on the interval $(0,\infty)$ and estimate the rate of convergence for functions having derivatives of bounded variation. Also we present simultenaous approximation by new operators in the end of the paper.

1. Introduction

To approximate integrable functions on the interval $(0,\infty)$, Srivastava and Gupta [1] introduced a general sequence of linear positive operators $G_{n,c}$ as follows:

$$G_{n,c}(f;x) = n\sum_{k=1}^{\infty} p_{n,k}(x;c)\int_0^{\infty} p_{n+c,k-1}(t;c) f(t)\,dt + p_{n,0}(x;c) f(0), \quad (1)$$

for a function $f \in H_\alpha(0,\infty)$, where $H_\alpha(0,\infty)$ $(\alpha \geq 0)$ is the class of locally integrable functions defined on $(0,\infty)$ and satisfying the growth condition

$$|f(t)| \leq Mt^{\alpha} \quad (M > 0;\ \alpha \geq 0;\ t \longrightarrow \infty), \quad (2)$$

$$p_{n,k}(x;c) = \frac{(-x)^k}{k!}\phi_{n,c}^{(k)}(x), \quad (3)$$

$$\phi_{n,c}(x) = \begin{cases} e^{-nx}, & c = 0 \\ (1+cx)^{-n/c}, & c \in \mathbb{N} := \{1,2,3,\ldots\}. \end{cases} \quad (4)$$

The general sequence of operators $G_{n,c}$ has many interesting properties in approximation theory, which is an interesting area of research in the present era, and several researchers have studied these operators; we can mention some important studies on these operators (see [1–3]). In [4], author introduced King and Stancu type generalization of Srivastava-Gupta operators and presented some direct results. Also, Verma and Agrawal [5] introduced a new generalization of Srivastava-Gupta operators and studied the rate of convergence for the functions having the derivatives of bounded variation (BV). The rate of convergence for the functions having the derivatives of (BV) is an active area of research and many researchers studied this direction. We refer the readers to [6–10] and references therein.

Stancu [11, 12] introduced generalizations of Bernstein polynomials with one and two parameters (resp.), satisfying the condition $0 \leq \alpha \leq \beta$, as

$$s_n^{\alpha}(f,x) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right)\binom{n}{k}\frac{\prod_{s=0}^{k-1}(x+\alpha s)\prod_{s=0}^{n-k-1}(1-x+\alpha s)}{\prod_{s=0}^{n-1}(1+\alpha s)}, \quad 0 \leq x \leq 1,$$

$$s_n^{\alpha,\beta}(f,x)=\sum_{k=0}^{n}f\left(\frac{k+\alpha}{n+\beta}\right)\binom{n}{k}x^k(1-x)^{n-k}\qquad 0\le x\le 1, \tag{5}$$

for any $f\in C[0,1]$. Stancu type generalization of approximation operators present better approach depending on α,β. Therefore, this kind of generalizations and their approximation properties have been studied intensively. We refer the readers to [13–17] and references therein. Mishra et al. [18, 19], V. N. Mishra, and L. N. Mishra [20] have established very interesting results on approximation properties of various functional classes using different types of positive linear summability operators.

The purpose of this paper is to introduce a new Stancu type generalization of the operators defined in [5] as

$$G_{n,r,c}^{(\alpha,\beta)}(f;x)=\frac{n\Gamma((n/c)+r)\Gamma((n/c)-r+1)}{\Gamma((n/c)+1)\Gamma(n/c)}\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)f\left(\frac{nt+\alpha}{n+\beta}\right)dt. \tag{6}$$

By the definition of operators, it is clear that $G_{n,r,c}^{(\alpha,\beta)}(f;x)$ is positive and linear. For $\alpha=\beta=0$, $G_{n,r,c}^{(0,0)}(f;x)$ reduces to operators defined in [5]. In this study we obtain the rate of convergence for the functions having the derivatives of bounded variation. Also, in the end of the paper, we study the simultaneous approximation.

2. Auxiliary Results

In order to prove our main results, we need the following lemmas.

Lemma 1. *Let the mth order moment be defined as*

$$\begin{aligned}U_{n,r,m}^{\alpha,\beta}(x)&=G_{n,c}^{(\alpha,\beta)}((t-x)^m;x)\\&=(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)\\&\quad\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^m dt,\end{aligned} \tag{7}$$

where $n,m\in\mathbb{N}\cup\{0\}$, and then, for $n>(m+r+1)c$, we have the following recurrence relation:

$$\begin{aligned}&(n-(r+m+1)c)(n+\beta)U_{n,r,m+1}(x)\\&=nx(1+cx)\left[\left(U_{n,r,m}^{\alpha,\beta}(x)\right)'+mU_{n,r,m-1}^{\alpha,\beta}(x)\right]\\&\quad+U_{n,r,m}^{\alpha,\beta}(x)\\&\quad\times[(m+r+(n+rc)x)n+(\alpha-(n+\beta)x)\\&\qquad\times(n-(r+2m+1)c)]\\&\quad+U_{n,r,m-1}^{\alpha,\beta}(x)\\&\quad\times\left[\frac{cm(\alpha-(n+\beta)x)^2-mn(\alpha-(n+\beta)x)}{n+\beta}\right],\end{aligned}$$

$$\begin{aligned}U_{n,r,0}^{\alpha,\beta}(x)&=1,\\U_{n,r,1}^{\alpha,\beta}(x)&=\frac{\alpha-(n+\beta)x}{n+\beta}+\frac{n(r+(n+rc)x)}{(n-(r+1)c)(n+\beta)},\\U_{n,r,2}^{\alpha,\beta}(x)&=\frac{nx(1+cx)}{(n-(r+1)c)(n-(r+2)c)(n+\beta)^2}\\&\quad+\left(\frac{\alpha}{n+\beta}-x+\frac{n(r+(n+rc)x)}{(n-(r+1)c)(n+\beta)}\right)\\&\quad\times\frac{n(1+r+(n+rc)x)}{(n-(r+2)c)(n+\beta)}\\&\quad+\left(\frac{\alpha}{n+\beta}-x+\frac{n(r+(n+rc)x)}{(n-(r+1)c)(n+\beta)^2}\right)\\&\quad\times(\alpha-(n+\beta)x)\\&\quad+\frac{(\alpha-(n+\beta)x)(c(\alpha-(n+\beta)x)-n)}{(n-(r+2)c)(n+\beta)^2}.\end{aligned} \tag{8}$$

Furthermore, $U_{n,r,m}^{\alpha,\beta}(x)$ is polynomial of degree m in x and

$$U_{n,r,m}^{\alpha,\beta}(x)=O\left((n+\beta)^{-[(m+1)/2]}\right). \tag{9}$$

Proof. By definition of $U_{n,r,m}^{\alpha,\beta}(x)$, taking the derivative of $U_{n,r,m}^{\alpha,\beta}(x)$, we get

$$\begin{aligned}&\left(U_{n,r,m}^{\alpha,\beta}(x)\right)'\\&=-(n-rc)m\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)\\&\quad\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m-1}dt\\&\quad+(n-rc)\sum_{k=0}^{\infty}p'_{n+rc,k}(x;c)\\&\quad\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m}dt\\&=-mU_{n,r,m-1}^{\alpha,\beta}(x)+(n-rc)\sum_{k=0}^{\infty}p'_{n+rc,k}(x;c)\\&\quad\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m}dt.\end{aligned} \tag{10}$$

Hence, using the identity

$$x(1+cx)p'_{n+rc,k}(x;c)=(k-(n+rc)x)p_{n+rc,k}(x;c) \tag{11}$$

we have

$$x(1+cx)\left[\left(U_{n,r,m}^{\alpha,\beta}(x)\right)' + mU_{n,r,m-1}^{\alpha,\beta}(x)\right]$$
$$= (n-rc)\sum_{k=0}^{\infty}(k-(n+rc)x)\,p_{n+rc,k}(x;c)$$
$$\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^m dt$$
$$= (n-rc)\sum_{k=0}^{\infty}k p_{n+rc,k}(x;c)\tag{12}$$
$$\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^m dt$$
$$-(n+rc)\,xU_{n,r,m}^{\alpha,\beta}(x)$$
$$= I-(n+rc)\,xU_{n,r,m}^{\alpha,\beta}(x).$$

We can write I as

$$I = \left[(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)\right.$$
$$\times\int_0^\infty [k+r-1-(n-(r-1)c)t]\,p_{n-(r-1)c,k+r-1}$$
$$\times(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^m dt$$
$$+(n-rc)(n-(r-1)c)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)\tag{13}$$
$$\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\,t\left(\frac{nt+\alpha}{n+\beta}-x\right)^m dt$$
$$-(r-1)(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)$$
$$\left.\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^m dt\right]$$
$$= I_1+I_2-(r-1)\,U_{n,r,m}^{\alpha,\beta}(x).$$

To estimate I_2 using $t=((n+\beta)/n)[(((nt+\alpha)/(n+\beta))-x)-((\alpha/(n+\beta))-x)]$, we have

$$I_2 = \frac{(n-(r-1)c)(n+\beta)}{n}$$
$$\times\left[(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)\right.$$
$$\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m+1} dt$$
$$-\left(\frac{\alpha}{n+\beta}-x\right)(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)$$
$$\left.\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^m dt\right],$$

$$I_2 = \frac{(n-(r-1)c)(n+\beta)}{n}$$
$$\times\left[(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)\right.$$
$$\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m+1} dt$$
$$-\left(\frac{\alpha}{n+\beta}-x\right)$$
$$\times\left((n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)\right.$$
$$\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)$$
$$\left.\left.\times\left(\frac{nt+\alpha}{n+\beta}-x\right)^m dt\right)\right]$$
$$= \frac{(n-(r-1)c)(n+\beta)}{n}$$
$$\times\left[U_{n,r,m+1}^{\alpha,\beta}(x)-\left(\frac{\alpha}{n+\beta}-x\right)U_{n,r,m}^{\alpha,\beta}(x)\right].\tag{14}$$

Next to estimate I_1 using the equality

$$t(1+ct)\,p'_{n-(r-1)c,k+r-1}(t;c)$$
$$= [(k+r-1)-(n-(r-1)c)t]\,p_{n-(r-1)c,k+r-1}(t;c),\tag{15}$$

we have

$$I_1 = (n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)$$
$$\times\int_0^\infty p'_{n-(r-1)c,k+r-1}(t;c)\,t\left(\frac{nt+\alpha}{n+\beta}-x\right)^m dt$$
$$+c(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)$$
$$\times\int_0^\infty p'_{n-(r-1)c,k+r-1}(t;c)\,t^2\left(\frac{nt+\alpha}{n+\beta}-x\right)^m dt$$
$$= \mathcal{J}_1+\mathcal{J}_2.\tag{16}$$

Putting $t = ((n+\beta)/n)[(((nt+\alpha)/(n+\beta))-x)-((\alpha/(n+\beta))-x)]$, we get

$$\begin{aligned}\mathcal{J}_1 = {} & \frac{n+\beta}{n} \\ & \times \left[(n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \right. \\ & \quad \times \int_0^{\infty} p'_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m+1} dt \\ & \quad - \left(\frac{\alpha}{n+\beta}-x\right)(n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \\ & \quad \left. \times \int_0^{\infty} p'_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m} dt \right].\end{aligned} \tag{17}$$

Now integrating by parts, we get

$$\begin{aligned}\mathcal{J}_1 = {} & -(m+1)(n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \\ & \times \int_0^{\infty} p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m} dt \\ & + m\left(\frac{\alpha}{n+\beta}-x\right)(n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \\ & \times \int_0^{\infty} p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m-1} dt \\ = {} & -(m+1) \\ & \times \left[(n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \right. \\ & \quad \left. \times \int_0^{\infty} p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m} dt \right] \\ & + m\left(\frac{\alpha}{n+\beta}-x\right) \\ & \times \left[(n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \right. \\ & \quad \left. \times \int_0^{\infty} p_{n-(r-1)c,k+r-1}(t;c)\left(\frac{nt+\alpha}{n+\beta}-x\right)^{m-1} dt \right] \\ = {} & -(m+1)U_{n,r,m}^{\alpha,\beta}(x) \\ & + m\left(\frac{\alpha}{n+\beta}-x\right)U_{n,r,m-1}^{\alpha,\beta}(x).\end{aligned} \tag{18}$$

Proceeding in a similar manner, we obtain the estimate $\mathcal{J}_2$ as

$$\begin{aligned}\mathcal{J}_2 = {} & -\frac{c(m+2)(n+\beta)}{n}U_{n,r,m+1}(x) \\ & + \frac{2c(m+1)(n+\beta)}{n}\left(\frac{\alpha}{n+\beta}-x\right)U_{n,r,m}(x) \\ & - \frac{cm(n+\beta)}{n}\left(\frac{\alpha}{n+\beta}-x\right)^2 U_{n,r,m-1}^{\alpha,\beta}(x).\end{aligned} \tag{19}$$

Combining the equations, we have

$$\begin{aligned}& (n-(r+m+1)c)(n+\beta)U_{n,r,m+1}^{\alpha,\beta}(x) \\ & \quad = nx(1+cx)\left[\left(U_{n,r,m}^{\alpha,\beta}(x)\right)' + mU_{n,r,m-1}^{\alpha,\beta}(x)\right] \\ & \qquad + U_{n,r,m}^{\alpha,\beta}(x) \\ & \qquad \times [(m+r+(n+rc)x)n + (\alpha-(n+\beta)x) \\ & \qquad\quad \times (n-(r+2m+1)c)] + U_{n,r,m-1}^{\alpha,\beta}(x) \\ & \qquad \times \left[\frac{cm(\alpha-(n+\beta)x)^2 - mn(\alpha-(n+\beta)x)}{n+\beta}\right]\end{aligned} \tag{20}$$

which is the desired result.

Moments for $m = 0, 1, 2$ can be easily obtained by using the above recurrence relation. □

Remark 2. For sufficiently large n, $C > 2$, and $x \in (0, \infty)$, it can be seen from Lemma 1 that

$$U_{n,r,2}^{\alpha,\beta}(x) \le \frac{C\sigma_{r,c}^{\alpha,\beta}(x)}{n+\beta}, \tag{21}$$

where $\sigma_{r,c}^{\alpha,\beta}(x) = [x(1+cx) + x(\alpha + \beta x + r(1+cx))]$ for the convenient notation.

Remark 3. By using Cauchy-Schwarz inequality, it follows from Remark 2 that, for sufficiently large n, $C > 2$, and $x \in (0, \infty)$,

$$\begin{aligned}& (n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \\ & \quad \times \int_0^{\infty} p_{n-(r-1)c,k+r-1}(t;c)\left|\frac{nt+\alpha}{n+\beta}-x\right| dt \\ & \quad \le \left[U_{n,r,2}^{\alpha,\beta}(x)\right]^{1/2} \le \sqrt{\frac{C\sigma_{r,c}^{\alpha,\beta}(x)}{n+\beta}}.\end{aligned} \tag{22}$$

Lemma 4. *Let $x \in (0,\infty)$ and $C > 2$; then, for sufficiently large n, we have*

$$\lambda_{n,r}(x,y) = (n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \times \int_0^y p_{n-(r-1)c,k+r-1}(t;c)\,dt \leq \frac{Cx(1+cx)}{n(x-y)^2}, \quad 0 \leq y \leq x,$$
$$1-\lambda_{n,r}(x,z) = (n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \times \int_z^{\infty} p_{n-(r-1)c,k+r-1}(t;c)\,dt \leq \frac{Cx(1+cx)}{n(z-x)^2}, \quad x \leq z \leq \infty. \tag{23}$$

Proof. We give the proof for only first inequality, and the other is similar. Using Remark 2 with $\alpha = \beta = 0$, for sufficiently large n and $0 \leq y \leq x$ and $((nt+\alpha)/(n+\beta)) \leq t$, we have

$$\begin{aligned}\lambda_{n,r}(x,y) &= (n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \times \int_0^y p_{n-(r-1)c,k+r-1}(t;c)\,dt \\ &\leq (n-rc)\sum_{k=0}^{\infty} p_{n+rc,k}(x;c) \times \int_0^y p_{n-(r-1)c,k+r-1}(t;c)\frac{(t-x)^2}{(y-x)^2}dt \\ &\leq \frac{Cx(1+cx)}{n(x-y)^2}.\end{aligned} \tag{24}$$

□

Lemma 5. *Suppose f is s times differentiable on $[0,\infty)$ such that $f^{(s-1)}(t) = O(t^{\alpha})$, for some integer $\alpha > 0$ as $t \to \infty$. Then, for any $r, s \in \mathbb{N}_0$, and $n > \max\{\alpha, r+s\}$, we have*

$$D^s G_{n,r,c}^{(\alpha,\beta)}(f;x) = \left(\frac{n}{n+\beta}\right)^s G_{n,r+s,c}^{(\alpha,\beta)}(f;x)(D^s f, x). \tag{25}$$

Proof. Using the identity

$$p'_{n,k}(x) = n\left[p_{n+c,k-1}(x,c) - p_{n+c,k}(x,c)\right]. \tag{26}$$

One can observe that, even in case $k = 0$, the above identity is true with the condition $p_{n+c,\text{negative}}(x,c) = 0$. Thus, applying (26), we have

$$\begin{aligned}D\left[G_{n,r,c}^{(\alpha,\beta)}\right](f;x) &= \frac{n\Gamma((n/c)+r)\Gamma((n/c)-r+1)}{\Gamma((n/c)+1)\Gamma(n/c)}\sum_{k=0}^{\infty} Dp_{n+rc,k}(x;c) \times \int_0^{\infty} p_{n-(r-1)c,k+r-1}(t;c) f\left(\frac{nt+\alpha}{n+\beta}\right)dt \\ &= \frac{n\Gamma((n/c)+r)\Gamma((n/c)-r+1)}{\Gamma((n/c)+1)\Gamma(n/c)} \times \sum_{k=0}^{\infty}(n+rc)\left[p_{n+(r+1)c,k-1}(x,c) - p_{n+(r+1)c,k}(x,c)\right] \times \int_0^{\infty} p_{n-(r-1)c,k+r-1}(t;c) f\left(\frac{nt+\alpha}{n+\beta}\right)dt \\ &= \frac{n(n+rc)\Gamma((n/c)+r)\Gamma((n/c)-r+1)}{\Gamma((n/c)+1)\Gamma(n/c)} \times \sum_{k=0}^{\infty} p_{n+(r+1)c,k}(x,c) \times \int_0^{\infty}\left[p_{n-(r-1)c,k+r}(t;c) - p_{n-(r-1)c,k+r-1}(t;c)\right] \times f\left(\frac{nt+\alpha}{n+\beta}\right)dt \\ &= \frac{-n(n+rc)\Gamma((n/c)+r)\Gamma((n/c)-r+1)}{(n-rc)\Gamma((n/c)+1)\Gamma(n/c)} \times \sum_{k=0}^{\infty} p_{n+(r+1)c,k}(x,c) \times \int_0^{\infty} Dp_{n-rc,k+r}(t;c) f\left(\frac{nt+\alpha}{n+\beta}\right)dt \\ &= \frac{n^2\Gamma((n/c)+r+1)\Gamma((n/c)-r)}{(n+\beta)\Gamma((n/c)+1)\Gamma(n/c)} \times \sum_{k=0}^{\infty} p_{n+(r+1)c,k}(x,c) \times \int_0^{\infty} p_{n-rc,k+r}(t;c) Df\left(\frac{nt+\alpha}{n+\beta}\right)dt \\ &= \frac{n}{(n+\beta)}\left[G_{n,r+1,c}^{(\alpha,\beta)}\right](Df;x),\end{aligned} \tag{27}$$

which means that the identity is satisfied for $s = 1$. Let us suppose that the result holds for $s = m$; that is,

$$\begin{aligned}
&D^m G_{n,r,c}^{(\alpha,\beta)}(f;x) \\
&= \left(\frac{n}{n+\beta}\right)^m G_{n,r+m,c}^{(\alpha,\beta)}(f;x)(D^m f,x) \\
&= \left(\frac{n}{n+\beta}\right)^m \\
&\quad \times \frac{n\Gamma((n/c)+r+m)\Gamma((n/c)-r-m+1)}{\Gamma((n/c)+1)\Gamma(n/c)} \\
&\quad \times \sum_{k=0}^{\infty} p_{n+(r+m)c,k}(x;c) \\
&\quad \times \int_0^{\infty} p_{n-(r+m-1)c,k+r+m-1}(t;c) D^m f\left(\frac{nt+\alpha}{n+\beta}\right) dt.
\end{aligned} \tag{28}$$

Also, from (26) we can write

$$\begin{aligned}
&D^{m+1} G_{n,r,c}^{(\alpha,\beta)}(f;x) \\
&= \left(\frac{n}{n+\beta}\right)^m \\
&\quad \times \frac{n\Gamma((n/c)+r+m)\Gamma((n/c)-r-m+1)}{\Gamma((n/c)+1)\Gamma(n/c)} \\
&\quad \times \sum_{k=0}^{\infty} D p_{n+(r+m)c,k}(x;c) \\
&\quad \times \int_0^{\infty} p_{n-(r+m-1)c,k+r+m-1}(t;c) D^m f\left(\frac{nt+\alpha}{n+\beta}\right) dt \\
&= \left(\frac{n}{n+\beta}\right)^m \\
&\quad \times \frac{n\Gamma((n/c)+r+m)\Gamma((n/c)-r-m+1)}{\Gamma((n/c)+1)\Gamma(n/c)} \\
&\quad \times \sum_{k=0}^{\infty} (n+(r+m)c) \\
&\quad \times [p_{n+(r+m+1)c,k-1}(x,c) - p_{n+(r+m+1)c,k}(x,c)] \\
&\quad \times \int_0^{\infty} p_{n-(r+m-1)c,k+r+m-1}(t;c) D^m f\left(\frac{nt+\alpha}{n+\beta}\right) dt \\
&= \left(\frac{n}{n+\beta}\right)^m \\
&\quad \times \frac{cn\Gamma((n/c)+r+m+1)\Gamma((n/c)-r-m+1)}{\Gamma((n/c)+1)\Gamma(n/c)} \\
&\quad \times \sum_{k=0}^{\infty} p_{n+(r+m+1)c,k}(x;c) \\
&\quad \times \int_0^{\infty} [p_{n-(r+m-1)c,k+r+m}(t;c) \\
&\qquad - p_{n-(r+m-1)c,k+r+m-1}(t;c)] D^m f\left(\frac{nt+\alpha}{n+\beta}\right) dt \\
&= -\left(\frac{n}{n+\beta}\right)^m \\
&\quad \times \frac{cn\Gamma((n/c)+r+m+1)\Gamma((n/c)-r-m+1)}{\Gamma((n/c)+1)\Gamma(n/c)} \\
&\quad \times \sum_{k=0}^{\infty} p_{n+(r+m+1)c,k}(x;c) \\
&\quad \times \int_0^{\infty} \frac{D p_{n-(r+m)c,k+r+m}(t;c)}{n-(r+m-1)c} D^m f\left(\frac{nt+\alpha}{n+\beta}\right) dt
\end{aligned} \tag{29}$$

and, integrating by parts the last integral, we have

$$\begin{aligned}
&D^{m+1} G_{n,r,c}^{(\alpha,\beta)}(f;x) \\
&= \left(\frac{n}{n+\beta}\right)^{m+1} \\
&\quad \times \frac{n\Gamma((n/c)+r+m+1)\Gamma((n/c)-r-m)}{\Gamma((n/c)+1)\Gamma(n/c)} \\
&\quad \times \sum_{k=0}^{\infty} p_{n+(r+m+1)c,k}(x;c) \\
&\quad \times \int_0^{\infty} p_{n-(r+m)c,k+r+m}(t;c) D^{m+1} f\left(\frac{nt+\alpha}{n+\beta}\right) dt.
\end{aligned} \tag{30}$$

Hence we have

$$\begin{aligned}
D^{m+1} G_{n,r,c}^{(\alpha,\beta)}(f;x) &= \left(\frac{n}{n+\beta}\right)^{m+1} \\
&\quad \times G_{n,r+m+1,c}^{(\alpha,\beta)}(f;x)(D^{m+1} f,x),
\end{aligned} \tag{31}$$

in which the result is true for $s = m + 1$, and hence by mathematical induction the proof of the lemma is completed. □

3. Main Results

Throughout the paper by $DB_q(0,\infty)$ we denote the class of absolutely continuous functions f on $(0,\infty)$ (where q is a some positive integer) satisfying the conditions:

(i) $|f(t)| \le C_1 t^q$ and $C_1 > 0$,

(ii) the function f has the first derivative on the interval $(0,\infty)$ which coincide almost everywhere with a function which is of bounded variation on every finite subinterval of $(0,\infty)$. It can be observed that for all functions $f \in DB_q(0,\infty)$ we can have the representation

$$f(x) = f(c) + \int_c^x \psi(t)\, dt, \quad x \ge c \ge 0. \tag{32}$$

Theorem 6. *Let $f \in DB_q(0,\infty)$, $q>0$, and $x \in (0,\infty)$. Then, for $C>2$ and sufficiently large n, we have*

$$\left|\frac{(\Gamma(n/c))^2}{\Gamma((n/c)+r)\Gamma((n/c)-r)}G_{n,r,c}^{(\alpha,\beta)}(f;x)-f(x)\right| \le \frac{C(1+cx)}{n}\sum_{k=1}^{[\sqrt{n}]}\bigvee_{x-(x/k)}^{x+(x/k)}\left(f'_x(x)\right)+\frac{x}{\sqrt{n}}\bigvee_{x-(x/\sqrt{n})}^{x+(x/\sqrt{n})}\left(f'_x(x)\right) + \frac{C(1+cx)}{nx}\left|f(2x)-f(x)-xf'(x^+)+|f(x)|\right| + O(n^{-q})+\left|f'(x^+)\right|\sqrt{\frac{Cx(1+cx)}{n}} + \sqrt{\frac{C\sigma_{r,c}^{\alpha,\beta}(x)}{n+\beta}}\frac{\left|f'(x^+)-f'(x^-)\right|}{2}+\frac{\left|f'(x^+)+f'(x^-)\right|}{2} \times\left(\frac{(\alpha-\beta x)(n-c(r+1))+2nrcx+nxc+nr}{(n-(r+1)c)(n+\beta)}\right), \quad (33)$$

where C is a constant which may be different on each occurrence.

Proof. Using the mean value theorem, we have

$$\frac{(\Gamma(n/c))^2}{\Gamma((n/c)+r)\Gamma((n/c)-r)}G_{n,r,c}^{(\alpha,\beta)}(f;x)-f(x) = (n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c) \times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)\left[f\left(\frac{nt+\alpha}{n+\beta}\right)-f(x)\right]dt = \int_0^\infty\left(\int_x^{(nt+\alpha)/(n+\beta)}(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c) \times p_{n-(r-1)c,k+r-1}(t;c)f'(u)\,du\right)dt. \quad (34)$$

Also, using the identity

$$f'(u)=\frac{f'(x^+)+f'(x^-)}{2}+(f')_x(u) + \frac{f'(x^+)-f'(x^-)}{2}\operatorname{sgn}(u-x) + \left[f'(x)-\frac{f'(x^+)+f'(x^-)}{2}\right]\chi_x(u), \quad (35)$$

where

$$\chi_x(u)=\begin{cases}1, & u=x;\\ 0, & u\neq x,\end{cases} \quad (36)$$

we have

$$(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c) \times\int_0^\infty\left(\int_x^t\left[f'(x)-\frac{f'(x^+)+f'(x^-)}{2}\right]\chi_x(u)\,du\right) \times p_{n-(r-1)c,k+r-1}(t;c)\,dt=0. \quad (37)$$

Thus, using the above identities, we can write

$$\left|\frac{(\Gamma(n/c))^2}{\Gamma((n/c)+r)\Gamma((n/c)-r)}G_{n,r,c}^{(\alpha,\beta)}(f;x)-f(x)\right| \le\left|\int_0^\infty\left(\int_x^t(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c) \times p_{n-(r-1)c,k+r-1}(t;c) \times\left[\frac{f'(x^+)+f'(x^-)}{2}+(f')_x(u)\right]du\right)dt\right| +\left|\int_0^\infty\left(\int_x^t(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c) \times p_{n-(r-1)c,k+r-1}(t;c) \times\left[\frac{f'(x^+)-f'(x^-)}{2}\operatorname{sgn}(u-x)\right]du\right)dt\right|. \quad (38)$$

Also, it can be verified that

$$\left|\int_0^\infty\left(\int_x^t(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c) \times p_{n-(r-1)c,k+r-1}(t;c) \times\left[\frac{f'(x^+)-f'(x^-)}{2}\operatorname{sgn}(u-x)\right]du\right)dt\right| \le\frac{\left|f'(x^+)-f'(x^-)\right|}{2}[U_{n,r,2}(x)]^{1/2}, \quad (39)$$

$$\left|\int_0^\infty\left(\int_x^t(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c) \times p_{n-(r-1)c,k+r-1}(t;c) \times\left[\frac{f'(x^+)+f'(x^-)}{2}\right]du\right)dt\right| \le\frac{\left|f'(x^+)+f'(x^-)\right|}{2}U_{n,r,1}(x). \quad (40)$$

Combining (38)–(40), we get

$$\begin{aligned}
&\left|\frac{(\Gamma(n/c))^2}{\Gamma((n/c)+r)\Gamma((n/c)-r)}G_{n,r,c}^{(\alpha,\beta)}(f;x)-f(x)\right| \\
&\quad\leq\left|\int_x^\infty\left(\int_x^t f_x'(u)\,du\right)(n-rc)\right. \\
&\qquad\times\sum_{k=0}^\infty p_{n+rc,k}(x;c)\,p_{n-(r-1)c,k+r-1}(t;c)\,dt \\
&\qquad+\int_0^x\left(\int_x^t f_x'(u)\,du\right)(n-rc) \\
&\qquad\left.\times\sum_{k=0}^\infty p_{n+rc,k}(x;c)\,p_{n-(r-1)c,k+r-1}(t;c)\,dt\right| \qquad (41)\\
&\qquad+\frac{\left|f'(x^+)-f'(x^-)\right|}{2}\left[U_{n,r,2}(x)\right]^{1/2} \\
&\qquad+\frac{\left|f'(x^+)+f'(x^-)\right|}{2}U_{n,r,1}(x) \\
&\quad=\left|A_{n,r}^{\alpha,\beta}(f,x)+B_{n,r}^{\alpha,\beta}(f,x)\right|+\frac{\left|f'(x^+)-f'(x^-)\right|}{2} \\
&\qquad\times\left[U_{n,r,2}(x)\right]^{1/2}+\frac{\left|f'(x^+)+f'(x^-)\right|}{2}U_{n,r,1}(x).
\end{aligned}$$

Applying Remark 2 and Lemma 1 in above equation, we have

$$\begin{aligned}
&\left|\frac{(\Gamma(n/c))^2}{\Gamma((n/c)+r)\Gamma((n/c)-r)}G_{n,r,c}^{(\alpha,\beta)}(f;x)-f(x)\right| \\
&\quad\leq\left|A_{n,r}^{\alpha,\beta}(f,x)\right|+\left|B_{n,r}^{\alpha,\beta}(f,x)\right| \\
&\qquad+\sqrt{\frac{C\sigma_{r,c}^{\alpha,\beta}(x)}{n+\beta}}\frac{\left|f'(x^+)-f'(x^-)\right|}{2} \\
&\qquad+\frac{\left|f'(x^+)+f'(x^-)\right|}{2} \\
&\qquad\times\left(\frac{(\alpha-\beta x)(n-c(r+1))+2nrcx+nxc+nr}{(n-(r+1)c)(n+\beta)}\right).
\end{aligned} \qquad (42)$$

In order to complete the proof of the theorem, it suffices to estimate the terms $A_{n,r}^{\alpha,\beta}(f,x)$ and $B_{n,r}^{\alpha,\beta}(f,x)$. Applying Remark 2 with $\alpha=\beta=0$, we get

$$\begin{aligned}
&\left|A_{n,r}^{\alpha,\beta}(f,x)\right| \\
&\quad=\left|\int_x^\infty\left(\int_x^t f_x'(u)\,du\right)(n-rc)\right. \\
&\qquad\left.\times\sum_{k=0}^\infty p_{n+rc,k}(x;c)\,p_{n-(r-1)c,k+r-1}(t;c)\,dt\right| \\
&\quad\leq\left|(n-rc)\sum_{k=0}^\infty p_{n+rc,k}(x;c)\right. \\
&\qquad\left.\times\int_{2x}^\infty(f(t)-f(x))\,p_{n-(r-1)c,k+r-1}(t;c)\,dt\right|+\left|f'(x^+)\right| \\
&\qquad\times\left|(n-rc)\sum_{k=0}^\infty p_{n+rc,k}(x;c)\right. \\
&\qquad\quad\left.\times\int_x^{2x}p_{n-(r-1)c,k+r-1}(t;c)(t-x)\,dt\right| \\
&\qquad+\left|\int_x^{2x}f_x'(u)\,du\right|\left|1-\lambda_{n,r}(x,2x)\right| \\
&\qquad+\left|\int_x^{2x}\left|f_x'(t)\right|\cdot\left|1-\lambda_{n,r}(x,t)\right|dt\right| \\
&\quad\leq(n-rc)\sum_{k=0}^\infty p_{n+rc,k}(x;c) \\
&\qquad\times\int_{2x}^\infty p_{n-(r-1)c,k+r-1}(t;c)\,C_1t^{2q}dt \\
&\qquad+\frac{\left|f(x)\right|}{x^2}(n-rc)\sum_{k=0}^\infty p_{n+rc,k}(x;c) \\
&\qquad\times\int_0^\infty p_{n-(r-1)c,k+r-1}(t;c)(t-x)^2\,dt \\
&\qquad+\left|f'(x^+)\right|(n-rc)\sum_{k=0}^\infty p_{n+rc,k}(x;c) \\
&\qquad\times\int_{2x}^\infty p_{n-(r-1)c,k+r-1}(t;c)\left|t-x\right|dt \\
&\qquad+\frac{Cx(1+cx)}{nx^2}\left|f(2x)-(x)-xf'(x^+)\right| \\
&\qquad+\frac{C(1+cx)}{n}\sum_{k=1}^{[\sqrt{n}]}\bigvee_x^{x+(x/k)}\left(f_x'(x)\right) \\
&\qquad+\frac{x}{\sqrt{n}}\bigvee_x^{x+(x/\sqrt{n})}\left(f_x'(x)\right).
\end{aligned} \qquad (43)$$

For estimating the integral

$$(n-rc)\sum_{k=0}^\infty p_{n+rc,k}(x;c)\int_{2x}^\infty p_{n-(r-1)c,k+r-1}(t;c)\,C_1t^{2q}dt, \qquad (44)$$

we proceed as follows: since $t \geq 2x$ implies that $t \leq 2(t-x)$ so by Schwarz inequality and Lemma 1,

$$\begin{aligned}
&(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)\int_{2x}^{\infty}p_{n-(r-1)c,k+r-1}(t;c)\,C_1t^{2q}dt \\
&\quad\leq C_1 2^q(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c) \\
&\quad\times\int_0^{\infty}p_{n-(r-1)c,k+r-1}(t;c)\,C_1(t-x)^{2q}dt \\
&\quad\leq C_1 2^q U_{n,r,2q}(x)=O(n^{-q}) \quad \text{as } n\longrightarrow\infty.
\end{aligned} \tag{45}$$

By using Hölder's inequality and Remark 2 ($\alpha=\beta=0$), we get the estimate as follows:

$$\begin{aligned}
&\left|f'(x^+)\right|(n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c) \\
&\quad\times\int_{2x}^{\infty}p_{n-(r-1)c,k+r-1}(t;c)\,|t-x|\,dt \\
&\quad\leq\left|f'(x^+)\right| \\
&\quad\times\left((n-rc)\sum_{k=0}^{\infty}p_{n+rc,k}(x;c)\right. \\
&\quad\left.\times\int_0^{\infty}p_{n-(r-1)c,k+r-1}(t;c)\,(t-x)^2dt\right)^{1/2} \\
&\quad\leq\left|f'(x^+)\right|\sqrt{\frac{Cx(1+cx)}{n}}.
\end{aligned} \tag{46}$$

Collecting the estimates from (43)–(46), we obtain

$$\begin{aligned}
\left|A_{n,r}^{\alpha,\beta}(f,x)\right| &\leq O(n^{-q})+\left|f'(x^+)\right| \\
&\quad\times\sqrt{\frac{Cx(1+cx)}{n}}+\frac{C(1+cx)}{nx} \\
&\quad\times\left|f(2x)-f(x)-xf'(x^+)+|f(x)|\right| \\
&\quad+\frac{C(1+cx)}{n}\sum_{k=1}^{[\sqrt{n}]}\bigvee_{x}^{x+(x/k)}\left(f'_x(x)\right) \\
&\quad+\frac{x}{\sqrt{n}}\bigvee_{x}^{x+(x/\sqrt{n})}\left(f'_x(x)\right).
\end{aligned} \tag{47}$$

On the other hand, to estimate $B_{n,r}^{\alpha,\beta}(f,x)$ by applying Lemma 4 with $y=x-(x/\sqrt{n})$ and integration by parts, we have

$$\begin{aligned}
&\left|B_{n,r}^{\alpha,\beta}(f,x)\right| \\
&\quad=\left|\int_0^x\int_x^t f'_x(u)\,d_t\lambda_{n,r}(x,t)\right| \\
&\quad\leq\left(\int_0^y+\int_y^x\right)\left|f'_x(t)\right|\left|\lambda_{n,r}(x,t)\right|dt \\
&\quad\leq\frac{Cx(1+cx)}{n}\int_0^y\bigvee_t^x\left((f')_x\right)\frac{1}{(x-t)^2}dt \\
&\qquad+\int_y^x\bigvee_t^x\left((f')_x\right)dt \\
&\quad=\frac{Cx(1+cx)}{n}\int_1^{\sqrt{n}}\bigvee_{(x-(x/u))}^{x}\left((f')_x\right)du \\
&\qquad+\frac{x}{\sqrt{n}}\bigvee_{x-(x/\sqrt{n})}^{x}\left((f')_x\right) \\
&\quad\leq\frac{Cx(1+cx)}{n}\sum_{k=1}^{[\sqrt{n}]}\bigvee_{x-(x/k)}^{x}\left((f')_x\right) \\
&\qquad+\frac{x}{\sqrt{n}}\bigvee_{x-(x/\sqrt{n})}^{x}\left((f')_x\right),
\end{aligned} \tag{48}$$

where $u=(x/(x-t))$.

Combining (41), (47), and (48), we get the desired result. □

Corollary 7. *Let $f^{(s)}\in DB_q(0,\infty)$, $q>0$, and $x\in(0,\infty)$. Then, for $C>2$ and n sufficiently large, one has*

$$\begin{aligned}
&\left|\frac{(\Gamma(n/c))^2}{\Gamma((n/c)+r)\,\Gamma((n/c)-r)}\left(\frac{n+\beta}{n}\right)^s\right. \\
&\quad\left.\times D^sG_{n,r,c}^{(\alpha,\beta)}(f;x)-f^s(x)\right| \\
&\quad\leq\frac{C(1+cx)}{n}\sum_{k=1}^{[\sqrt{n}]}\bigvee_{x-(x/k)}^{x+(x/k)}\left((D^{s+1}f)_x\right) \\
&\qquad+\frac{x}{\sqrt{n}}\bigvee_{x-(x/\sqrt{n})}^{x+(x/\sqrt{n})}\left((D^{s+1}f)_x\right)+\frac{C(1+cx)}{nx} \\
&\qquad\times\left|f(2x)-(x)-xD^{s+1}f(x^+)+|f(x)|\right| \\
&\qquad+O(n^{-q})+\left|D^{s+1}f(x^+)\right|\sqrt{\frac{Cx(1+cx)}{n}}
\end{aligned}$$

$$+\sqrt{\frac{C\sigma_{r,c}^{\alpha,\beta}(x)}{n+\beta}}\frac{\left|D^{s+1}f(x^+)-D^{s+1}f(x^-)\right|}{2}$$
$$+\frac{\left|D^{s+1}f(x^+)+D^{s+1}f(x^-)\right|}{2}$$
$$\times\left(\frac{(\alpha-\beta x)(n-c(r+1))+2nrcx+nxc+nr}{(n-(r+1)c)(n+\beta)}\right), \quad (49)$$

where $\bigvee_a^b f_x$ *denotes the total variation of* f_x *on* $[a,b]$ *and the auxiliary function* $D^{s+1}f_x$ *is defined by*

$$D^{s+1}f_x(t)=\begin{cases}D^{s+1}f(t)-D^{s+1}f(x^-), & 0\le t\le x\\ 0, & t=x\\ D^{s+1}f(t)-D^{s+1}f(x^+), & x<t<\infty.\end{cases} \quad (50)$$

4. Conclusion

The results of our lemmas and theorems are more general rather than the results of any other previously proved lemmas and theorems, which will enrich the literature of applications of quantum calculus in operator theory and convergence estimates in the theory of approximations by positive linear operators. The researchers and professionals working or intend to work in areas of mathematical analysis and its applications will find this research paper to be quite useful. Consequently, the results so established may be found useful in several interesting situations appearing in the literature on mathematical analysis, pure and applied mathematics, and mathematical physics. Some interesting applications of the positive approximation linear operators can be seen in [21–24].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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